

Effect of Local Mesh Refinement on Inverse Numerical Acoustics

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ABSTRACT

Inverse numerical acoustics is a method which reconstructs the source surface normal velocity from the sound measured in the near-field around the source. This is of particular interest when the source is rotating or moving, too light or too hot to be instrumented by accelerometers. The use of laser vibrometers is often of no remedy due to the complex shape of the source.

The Inverse Numerical Acoustics technique is based on the inversion of transfer relations (Acoustic Transfer Vectors) using truncated Singular Value Decomposition (SVD). Most of the time the system is underdetermined which results in a non unique solution. The solution obtained by the truncated SVD is the minimal solution in the RMS sense. This paper is investigating the impact of non homogeneities in the mesh density (local mesh refinement) on the retrieved solution for underdetermined systems. It will be shown that if transfer quantities are inverted as such, big elements get a higher weight in the inversion. An approach is proposed to alleviate this issue and is illustrated using multiple examples.

INTRODUCTION

In acoustic response simulation for noise radiating structures, the boundary condition for the acoustic simulation model is given by the structure's discretized surface normal velocities. Depending on the nature of the problem, the acoustic radiation is computed directly from the velocity boundary conditions (after solving the boundary problem) or through the use of transfer relations between the normal structural velocity and the acoustic pressure. Such transfer relations are usually called Acoustic Transfer Vectors (ATVs). The actual response for any given set of normal surface velocities can then be readily computed by multiplying them with the ATVs to obtain the acoustic pressure response.

Often the confidence in the quality of the acoustic model, build using a BEM or FEM approach, is higher compared to the confidence one has in the accuracy of simulated structural surface vibrations. For an engine application some uncertainties can be operational temperature (distribution), oil distribution, damping of the structural FE model, accuracy and completeness of structural loads definition.

The general objective of inverse numerical acoustics for noise radiating structures is to retrieve or complete the (partially) unknown surface normal velocities, based on measured responses around the noise radiating structure and ATVs. As the ATVs are obtained using only the acoustic model, inverse numerical acoustics doesn't require a complete definition of the structural FE model. Only the surface of the structure should be replicated in the acoustic model, either exact or in a coarser way. This is a clear benefit for this approach.

However, as will be discussed further in this paper, it is usually impractical to measure the pressure response in as many microphone points as there are acoustic nodes on the structure's surface. Therefore in inverse numerical acoustics one usually solves an underdetermined problem by inverting the SVD decomposition of the ATV matrix at each frequency. This doesn't change too much the technique's capability of finding an equivalent source that is able to reproduce the measured near-field pressure responses within a small relative error. However, as the problem is underdetermined, many of such solutions exist. The objective is to find back the actual or most likely surface vibrations. As shown also in [5], the solution found depends on the configuration of microphone points and singular value truncation before inversion. This paper will start by summarizing some of these requirements on the positioning and spatial resolution of the measurement points.

The acoustic model can contain different element (BEM) or boundary face (FEM) sizes depending on the typology of the structural surface it captures. The mesh size near or on the radiating surface not only depends on maximum frequency of interest but also on the irregularities of the surface modeled (in order to capture accurately the surface normal direction) or the level of detail desired in order to pinpoint sources on the surface. This paper will explain how non homogeneities in the mesh density will influence the result if the ATV matrix is directly used for SVD based inversion. A preconditioning to apply on the ATV matrix to attenuate these influences and obtain a physically meaningful result will be shown.

ATV MATRIX CONDITION, SVD TRUNCATION AND REGULARIZATION

The inverse numerical acoustic examples shown in this paper will all be based on the Boundary Element Method, which is a commonly used approach for solving the acoustic radiation problem. However a FEM approach can also be used to compute the acoustic transfer relations. Once the ATV are computed using the BEM approach, the relation between the m measured near field pressures and the n surface normal velocities of the radiating structure is given by (1).

$$[p]_{m \times 1} = [ATV]_{m \times n} \cdot [v_{\perp}]_{n \times 1} \quad (1)$$

As it is impractical to measure the near field pressure in as many microphone points as there are surface nodes in the acoustic model, equation (1) represents an underdetermined problem for the unknown surface velocities, $m < n$. Given this underdetermination, an infinite number of solutions exist for $v_{\perp}$. Using the Moore-Penrose inverse ($^+$) of the ATV matrix, the obtained solution for $v_{\perp}$ will be the unique solution that minimizes the 2 norm, $\|v_{\perp}\|_2$. Equation (2) presents the Moore-Penrose solution for $v_{\perp}$. Equation (3) shows the Moore-Penrose solution can be obtained using Singular Value Decomposition of the ATV matrix. When all singular values are retained in the computations, (2) and (3) provide the exact same solution.

$$[ATV]_{m \times n}^+ \cdot [p]_{m \times 1} = [v_{\perp}]_{n \times 1} \quad (2)$$

$$[V]_{n \times n} \cdot [\Sigma]_{n \times m}^+ \cdot [U]_{m \times m}^T [p]_{m \times 1} = [v_{\perp}]_{n \times 1} \quad (3)$$

When plotting the solution of $v_{\perp}$ on the radiating surface, it can be observed that for many frequencies, the Moore-

Penrose solution will present a purely mathematical solution rather than also a physically meaningful solution. As there are too few equations to find the exact solution, there is some 'freedom' for the surface nodes to receive whichever velocity that renders the total solution minimal in norm and conform with (1). Therefore very abrupt changes in the velocity solution can occur for nodes lying close too each other on the surface. This effect can be understood by looking at the singular values matrix $[\Sigma]_{m \times n}$.

$$[\Sigma]_{m \times n}^+ = \begin{bmatrix} \sigma_1^{-1} & & & & \\ & \sigma_2^{-1} & & & \\ & & \sigma_3^{-1} & & \\ & & & \ddots & \\ & & & & \sigma_m^{-1} \end{bmatrix} \quad (4)$$

As shown in (4), after inversion, the small singular values become large and will dominate the solution. Small changes in the singular vectors will therefore be translated in large changes on the solution. The singular vectors belonging to the smallest singular values are however containing the least important information of the ATV matrix. They generate 'noise' on the solution for $v_{\perp}$ and give the total solution a spiky appearance. To obtain a solution for $v_{\perp}$ that is smoother in appearance and therefore physically more meaningful, a number of small singular values can be discarded using a threshold α (5). The condition number of the ATV matrix is given by the ratio of the largest versus smallest (retained) singular value and influences how smooth the result for $v_{\perp}$ will be after inversion of the ATV matrix. By truncation, the condition number will always be smaller than α^{-1} . By choosing an appropriate α we can make sure to obtain a smooth normal velocity result.

$$\frac{\sigma_i}{\sigma_{\max=1}} < \alpha \Rightarrow \sigma_i \quad \kappa(ATV) = \frac{\sigma_{\max}(ATV)}{\sigma_{\min}(ATV)} \leq \alpha^{-1} \quad (5)$$

Fig 1 shows the difference between two choices of regularization for engine velocities at 4680 RPM, 30th order. Near field simulated pressures were used as 'measurement' data. The result on the right is clearly preferred over the result on the left.

(See Figure 1 after last section of paper)

Of course, we can set the Singular Value threshold very high in order to make sure a smooth looking solution is obtained.

However, with each singular value discarded a part of the information in the ATV matrix disappears as well. This is shown in Fig 2. A typical L shaped curve is obtained for residual error versus solution norm (top). The normal velocity solution norm drops, the solution becomes smoother, when removing more singular values from the ATV matrix (increasing the α threshold). On the other hand the price paid is the error between the reconstructed near field pressures and the measured near field pressures. The optimal solution is to use the threshold that smoothens as much as possible the result with an as small as possible residual error. This optimum, as shown in Fig 2, is the corner point of the L-curve.

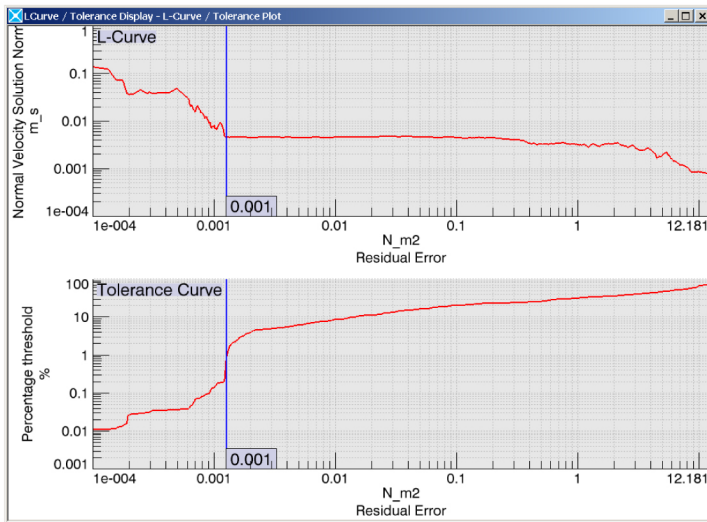


Fig 2. L-curve (top) representing change of solution norm versus residual error for different α threshold values

MICROPHONE POINTS CONFIGURATION

The goal of microphone point choice is to obtain good conditioning of the ATV matrix before applying any truncation measure. If we can assure good ATV matrix conditioning to begin with, less information needs to be removed in order to get a plausible result. In order to achieve a well conditioned ATV matrix, some guidelines need to be taken into account. These are well described in for example [5]. To summarize, each row in the ATV matrix should bring new information, all ATV rows should bring complete information. Therefore:

- The distance between the microphone points and the surface should be $< \lambda/3$ for the max frequency of interest. For 2 microphone points close to each other but far away from the radiating structure, the ATV almost doesn't change and therefore no new information is added to the ATV matrix by including both microphone points

• The distance between microphone points should be $< \lambda/2$ as to obtain enough information

• The distance between microphone points should not be too small as to avoid including the same information twice in the ATV matrix (linearly dependent rows)

• Make sure to have a good coverage of the radiating structure: all radiating surfaces should be in line of sight by the union of all microphone points

Fig 4 shows the ATV matrix amplitude (in P / Volume Velocity format) for several microphone point configurations positioned over a radiating plate. Frequency of interest is 500 Hz. It is clearly noticeable how the ATV matrix rows become less linearly dependent going from right (large microphone to surface distance) to left (small microphone to surface distance). Also, reading top to bottom, it can be seen how less and less microphone points will result in a matrix relatively less populated with high values.

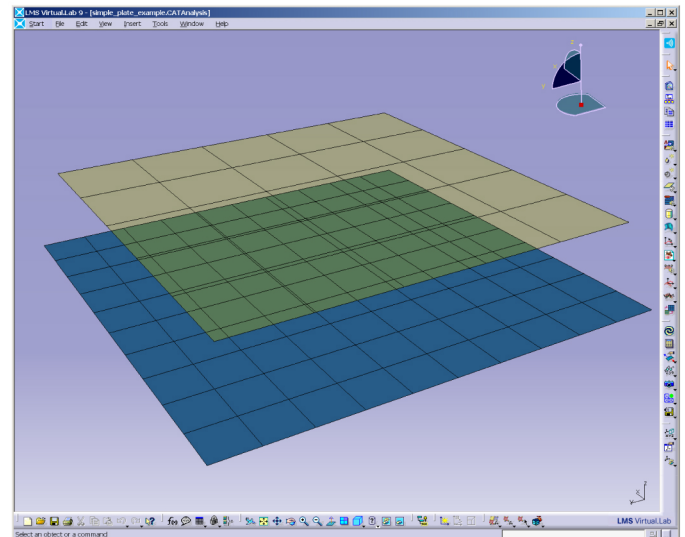


Fig 3. Small plate model (1 m × 1 m), 121 BEM nodes, 100 BEM elements with microphone point configuration

(See Figure 4 after last section of paper)

THE EFFECT OF LOCAL MESH REFINEMENT ON INA

In order to demonstrate the effect of mesh refinement on the surface velocity estimation, a simple example was first considered (Fig. 5). A ring structure was modeled using 3 arrays of 240 surface nodes. The first 120 nodes of each array were positioned every 1 degree of the ring from 1 to 120 degrees. The remaining 120 nodes of each array had 2 degrees spacing. 20 microphone points are positioned according to the recommendations mentioned before to guarantee a low ATV matrix condition number.

(See Figure 5 after last section of paper)

Table 1. Ring Model setup

ring model	
nr microphone points	20
nr of BEM nodes	240 X 3
frequency	1000 Hz
radius structure	200 mm
radius microphone points	240 mm
distance between microphone points	75 mm
distance to surface	40 mm
$\lambda/2$	170 mm
$\lambda/3$	113 mm

Only velocities on the nodes on the mid section of the ring were considered. The 2 remaining layers of nodes were put to zero velocity. Microphone point 1000 is positioned at the left in Fig 5 and the numbering is continued counterclockwise up to 1019. The first BEM node on the middle ring has node ID 1 is positioned at the left near microphone point 1000. Numbering is also done counterclockwise. Fig 6 shows the ATV matrix at 1000 Hz.

(See Figure 6 after last section of paper)

For the microphone points near the smaller BEM elements, it can be observed how the smaller ATV values for these elements are compensated by the number of elements ‘seen’ by the microphone point: a boundary condition of unit surface velocity in all surface nodes will result in equal pressure in every response point. Fig 6 also shows how a good positioning of microphone points has led to a well conditioned ATV matrix as the rows are approaching linear complete independency. Therefore the solution for the surface normal velocities near a microphone point will depend almost completely on the value of only that microphone point. In extremis, the Inverse Numerical Acoustic problem for such a group of r surface nodes near a microphone point with pressure p could be written as:

$$p_{1 \times 1} = \begin{bmatrix} ATV_{p,1} & ATV_{p,2} & \cdots & ATV_{p,r} \end{bmatrix}_{1 \times r} \cdot \begin{bmatrix} v_{\perp 1} \\ v_{\perp 2} \\ \vdots \\ v_{\perp r} \end{bmatrix}_{r \times 1} \quad (6)$$

Using the Moore-Penrose Inverse (SVD based computation without regularization), the solution found for $v_{\perp}$ will be of minimum norm $\|v_{\perp}\|_2$.

For the problem with only 1 microphone point given by (6), the solution with no regularization for r nearby surface nodes is unique and given by:

$$v_{\perp i} = p \cdot \left(\frac{ATV_{p,i}^*}{\sum_{k=1}^r ATV_{p,k} \cdot ATV_{p,k}^*} \right) \quad (7)$$

The local solution for each surface node is therefore (as expected) inversely proportional to the average ATV for all r nearby nodes but directly proportional to its own ATV magnitude towards the nearby microphone point.

A problem arises in case the microphone point is located near a local mesh refinement. For the ring structure example considered here, there are two such microphones points: point 1000 (left) and 1007 (–120 degrees located from point 1000). For these acoustic field points, the acoustic transfer function for the r nearest surface nodes can vary strongly when expressed as pressure/normal velocity, purely because of the differences in element size at the surface, a factor 2 in this case: the larger BEM elements will be twice as efficient and have an ATV twice as high compared to the smaller BEM elements. Therefore, the solution of the underdetermined local problem will favor velocities twice as high for the nodes belonging to the larger elements compared to the nodes belonging to the smaller elements. However, from a physical perspective, a more homogeneous velocity solution would be expected.

As the distance to the microphone point is almost equal for all r nodes and smaller than $\lambda/3$, the acoustic transfer function expressed as pressure / volume velocity is clearly almost constant over the r surface nodes. As any homogeneous discretization for the surface nearby a microphone point will result in an equal normal velocity for all r nodes, there is no reason this should be different for an inhomogeneous discretization.

PRECONDITIONING OF THE ATV MATRIX TO ATTENUATE THE EFFECT OF LOCAL MESH REFINEMENT

In order to investigate if a preconditioning of the ATV matrix could attenuate or remove the effect of mesh refinement, equation (6) for the local problem is rewritten as:

$$p_{1 \times 1} = \begin{bmatrix} \frac{ATV_{p,1}}{S_1} & \frac{ATV_{p,2}}{S_2} & \dots & \frac{ATV_{p,r}}{S_r} \end{bmatrix}_{1 \times r} \cdot \begin{bmatrix} S_1 v_{\perp 1} \\ S_2 v_{\perp 2} \\ \vdots \\ S_r v_{\perp r} \end{bmatrix}_{r \times 1} \quad (8)$$

S_i are the individual scaling factors for each ATV. With prerequisite that if all acoustic transfer functions expressed as pressure / volume velocity are equal, the normal velocity solution in all nodes should be equal as well:

$$\forall i, \frac{ATV_{p,i}}{A_i} = \frac{p}{v_{\perp i} \cdot A_i} = C \Rightarrow v_{\perp i} = v_{\perp} \quad (9)$$

A_i represents the nodal area of surface node i and C is a complex constant. Combining the solution of (8) and the prerequisite for the solution (9), it can be found that the scaling factors should equal the square root of the nodal areas for each surface node:

$$\left. \begin{aligned} v_{\perp i} &= p \cdot \left(\frac{\frac{ATV_{p,i}^*}{S_i^2}}{\sum_{k=1}^r \frac{ATV_{p,k} \cdot ATV_{p,k}^*}{S_k^2}} \right) = v_{\perp} \Rightarrow \forall i, \frac{ATV_{p,i}^*}{S_i^2} = cte \\ \forall i, \frac{ATV_{p,i}}{A_i} &= C \Rightarrow \forall i, \frac{ATV_{p,i}^*}{A_i} = C^* \end{aligned} \right\} \Rightarrow \forall i, S_i = \sqrt{A_i} \quad (10)$$

To look if the same preconditioning of the ATV matrix used for the local problem will also work for the global problem, following equations were solved for the ring structure problem. No regularization was used.

$$\begin{aligned} [p]_{m \times 1} &= [ATV]_{m \times n} \cdot [S]_{n \times n} \cdot [S]_{n \times n}^{-1} \cdot [v_{\perp}]_{n \times 1} \\ [p]_{m \times 1} &= [sATV]_{m \times n} \cdot [S]_{n \times n}^{-1} \cdot [v_{\perp}]_{n \times 1} \end{aligned} \quad (11)$$

$$\begin{aligned} [sV]_{n \times n} \cdot [s\Sigma]_{n \times m}^+ \cdot [sU]_{m \times m}^T \cdot [p]_{m \times 1} &= [S]_{n \times n}^{-1} \cdot [v_{\perp}]_{n \times 1} \\ [S]_{n \times n} \cdot [sATV]_{n \times m}^+ \cdot [p]_{m \times 1} &= [v_{\perp}]_{n \times 1} \end{aligned} \quad (12)$$

S is a diagonal scaling matrix and $sATV$ represents the column scaled ATV matrix. Fig 7 shows $[S]_{n \times n} \cdot [sATV]_{n \times m}^+$ for the ring structure for scaling with respectively the unity matrix, nodal area, square rooted nodal area.

(See Figure 7 after last section of paper)

The left matrix amplitude plot shows that indeed the larger elements near a local mesh refinement will get more energy assigned in case no ATV matrix preconditioning is used: highest velocities will go to the nodes of the larger elements near node 240 and node 1. For scaling with the nodal area, an equal volume velocity solution will be found resulting in higher velocities for the nodes belonging to the smaller elements near the local mesh refinement. As can be seen in the right picture of Fig 7, scaling with the square rooted nodal area will have most chance of returning a homogeneous velocity solution.

To look at the velocity solution differences for the ring structure example, a unit normal velocity was first applied for each middle ring surface node. A forward response computation using the ATV matrix was used to obtain p . Afterwards the three scaling approaches were used to compute the surface normal velocity. Fig 8 affirms the square rooted nodal area as most appropriate scaling. Notice also how that large differences between the three solutions only occur near the boundaries of the mesh refinement.

(See Figure 8 after last section of paper)

EXAMPLE: INA FOR A BOX MODEL

In order to further validate the new preconditioning approach, an inverse numerical acoustics problem was solved for a box model without and with local mesh refinement. In Fig 9, the elements in red are half the size of the elements in dark blue, which are on their turn half the size of the elements in pale blue.

(See Figure 9 after last section of paper)

Table 2. Box Model setup

box model	
nr microphone points near field	108
nr microphone points far field	98
nr of BEM nodes	1034
frequency	1500 - 2400 Hz
distance between near field microphone points	70 mm
distance to surface	40 mm
smallest $\lambda/2$	71 mm
smallest $\lambda/3$	47 mm

To validate the accuracy of the new approach with preconditioning, a first comparison is made between unity scaling and square rooted nodal area scaling on the model with homogeneous mesh density. A unity normal panel velocity is assigned to all BEM elements to compute a set of pressures. An impedance of 416 kg / m² s was applied as internal boundary condition to remove internal resonance in