

Simulation of the Dynamical Behavior of Elastic Multi-Body Systems with Bolted, Rough Contact Interfaces

2010-01-1422

Published
06/09/2010Johannes Sebastian Hölzl
Graz University of TechnologyWalter Sextro
Univ. of Paderborn

Copyright © 2010 SAE International

ABSTRACT

For many technical applications it is necessary to avoid or to reduce vibrations. Factors benefiting from vibration reduction are for example the durability of the application, which is increased, as well as cost expenses and the level of noise, which are both decreased. Rough, bolted interfaces are common in most machines and can be used as damping devices with some effort. Perhaps in future such contact surfaces could be used as damping devices at the interfaces of an automotive engine or exhaust system. Nevertheless it is difficult to predict the effect of a change in contact interface parameters on the dynamic behavior of the entire mechanical system. Therefore a method for calculating the steady state behavior of elastic multi-body systems was developed. The basis of this method is a finite element model of each contacting unit. On each model a modal reduction is applied in order to reduce the degrees of freedom. The method of Ritz and Galerkin is used to calculate the frequency response functions of the reduced system including contact. This method allows the calculation of the steady-state behavior of elastic multi-body systems with rough, bolted contact interfaces, by taking the pre-stress of the bolts, the roughness and the friction coefficient of the contacting surfaces as parameters. In order to verify calculated results, a test stand was built. The comparison of calculated and experimental results showed a satisfying correlation.

INTRODUCTION

The dynamic behavior of many mechanical systems is affected by friction in joints. As a result of this, many scientific studies and publications concerning this topic have

been made. Gaul and Nitsche [1] give a global overview of technical progress and literature about this subject matter. In their overview several contact formulations are presented, which provide phenomenological and constitutive contact laws. Phenomenological contact formulations are based on experiments. Forces normal to the contact surfaces and forces in tangential direction are linked together. An example therefore is the Coulomb friction law. Constitutive contact formulations are based on physical models. The contact behavior is described local. In most constitutive contact models the contact forces depend on the relative displacement of the corresponding contact surfaces. Furthermore Gaul and Nitsche [1] provide the reader with several techniques for solving mathematical models for nonlinear coupled structures, such as contacting elastic bodies with rough surfaces. One method is the numerical integration of the systems equations. Unfortunately this method is extremely time consuming and can take hours up to several days. Whereas an exact solution for the steady state can be calculated for systems with friction and less degrees of freedom (DOFs) under harmonic excitation, this is no longer possible for systems with a larger number of DOFs. An approximation of the exact solution can be obtained by using the method of the Harmonic Balance (HBM).

The HBM was often used to calculate the dynamics of a bladed disc assembly with friction dampers in turbo machinery. Publications of Sextro [2, 3], Götting [4] and Panning [5] are few examples for studies concerning this research topic. Finite Elements Models (FEMs) are used to describe the dynamics of elastic bodies with complex geometry, such as turbine blades. In order to reduce the DOFs

of the FEMs a modal condensation is done. In Götting [4] und Panning [5] the calculation of the nonlinear contact forces is based on a contact model which was introduced by Sextro [3]. Taking the influence of the surface texture into account, the contact forces depend on the relative displacements of the two corresponding contact points. With the HBM the contact law can be piecewise linearized. Depending on the amplitudes of the relative displacements substitute stiffnesses can be pre-calculated and stored into arrays. In this way very fast an approximate solution for the steady state of coupled elastic structures under harmonic excitation in the frequency domain can be calculated.

Genzo, Sextro and Panning [6, 7] use the presented method to calculate the dynamical behavior of coupled, elastic bodies with extended, rough surfaces under harmonic excitation. A comparison of measurement and calculation shows good correlation, if the contact is smooth over the whole surface.

When using the HBM it has to be assumed, that harmonic relative displacements lead into harmonic contact forces. Due to the non-linear contact description the contact forces contain further spectral components. Poudou und Pierre [8] use the so called Hybrid Frequency Time Domain Method (HFT). With this method also higher frequency parts of the contact forces, and thus of the system response can be taken into account. The Galerkin method is applied on the equations of motion of the system of coupled elastic structures under periodic excitation. This application leads to a set of non-linear equations for the calculation of the system response in the frequency domain. Thereby the solution is approximated by a Fourier series. With increasing number of terms of the Fourier series the accuracy of the approximate solution increases. The Fourier-coefficients of the non-linear contact forces must be calculated via the loop way of numerical integration of the contact model in the time domain. The disadvantage of the HFT method is the time-consuming numerical integration in the time domain. Thus this method is restricted for calculations with few contact points.

In Hölzl [9] a new method was found to pre-calculate the Fourier-coefficients of the contact forces under certain conditions. With this method an approximate solution of nonlinear coupled structures under harmonic excitation and considering higher frequency parts can be calculated efficiently.

MODELING

EQUATIONS OF MOTION

The equations of motion for two linear elastic bodies are

$$\underbrace{\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} \ddot{\mathbf{x}}_1 \\ \ddot{\mathbf{x}}_2 \end{bmatrix}}_{\ddot{\mathbf{x}}} + \underbrace{\begin{bmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{bmatrix}}_{\mathbf{D}} \underbrace{\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix}}_{\dot{\mathbf{x}}} + \dots + \underbrace{\begin{bmatrix} \mathbf{K}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix}}_{\mathbf{F}}, \quad (1)$$

with the positive definite mass matrix $\mathbf{M}$ and the positive semi definite stiffness matrix $\mathbf{K}$. Using the Rayleigh hypothesis for the structural damping matrix

$$\mathbf{D} = \alpha \mathbf{M} + \beta \mathbf{K} \quad (2)$$

is also positive definite. The coupling of the two elastic bodies is done by the external forces $\mathbf{F}$. In order to reduce the DOFs of the system a modal condensation is applied. One can calculate the eigenfrequencies ω_i and the mass normalized eigenvectors $\hat{\mathbf{x}}_i$ by solving the eigenvalue problem

$$(\mathbf{E}\omega^2 - \mathbf{M}^{-1}\mathbf{K})\hat{\mathbf{x}} = \mathbf{0} \quad (3)$$

of the undamped system. Collecting the mass normalized eigenvectors in the so-called modal matrix

$$\mathbf{T} = [\hat{\mathbf{x}}_1 \quad \hat{\mathbf{x}}_2 \quad \dots \quad \hat{\mathbf{x}}_m], \quad (4)$$

as well as taking the first m modes into account and applying the modal transformation

$$\mathbf{w} = \mathbf{T}\mathbf{x} \quad (5)$$

on Eq. (1), leads to the modal reduced equations of motion of the two elastic bodies

$$\ddot{\mathbf{w}} + \mathbf{D}_M \dot{\mathbf{w}} + \mathbf{K}_M \mathbf{w} = \mathbf{T}^{-1} \mathbf{F}. \quad (6)$$

CONTACT MODEL

Sextro [3] introduced a three dimensional contact model (Fig. 1).

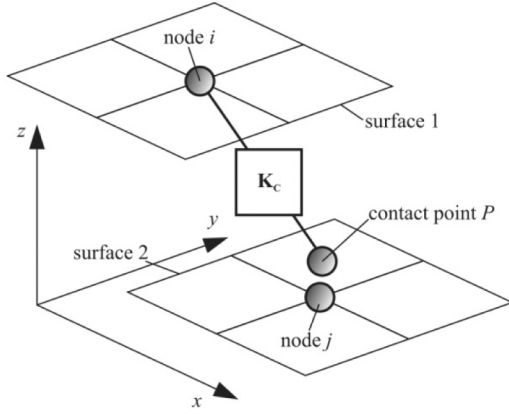


Figure 1. Three dimensional contact element.

A massless contact point P is elastically coupled via the contact stiffnesses

$$\mathbf{K}_C = \begin{bmatrix} c_{Rx} & 0 & 0 \\ 0 & c_{Ry} & 0 \\ 0 & 0 & c_N \end{bmatrix} \quad (7)$$

with node i on the contact surface 1. Coulomb friction acts between the contact point P and node j on the contact surface 2. Depending on the motions of the nodes i and j the contact point can be found in three distinct states. The three states are governed by the following equations:

1. Separation state - The contact point P and the node j are not in contact, so the node i and the point P move in the same way:

$$\begin{aligned} x_p &= x_i \\ y_p &= y_i \\ z_p &= z_i \end{aligned} \quad (8)$$

2. Stick state - In this state the contact point P follows the motion of the node j :

$$\begin{aligned} \dot{x}_p &= \dot{x}_j \\ \dot{y}_p &= \dot{y}_j \\ z_p &= z_j \end{aligned} \quad (9)$$

3. Slip state - Relative motion between the contact point P and node j occurs. The tangential contact force acts against this relative motion:

$$\begin{aligned} -c_{Rx}(x_i - x_j) - F_N \mu \frac{\dot{x}_i - \dot{x}_j}{v_{rel}} &= 0 \\ -c_{Ry}(y_i - y_j) - F_N \mu \frac{\dot{y}_i - \dot{y}_j}{v_{rel}} &= 0 \end{aligned} \quad (10)$$

The relative velocity between node i and the node j is

$$v_{rel} = \sqrt{(\dot{x}_i - \dot{x}_j)^2 + (\dot{y}_i - \dot{y}_j)^2}. \quad (11)$$

Because of the influence of the surface texture the contact stiffnesses c_{Rx} , c_{Ry} and c_N and therefore the normal force F_N are nonlinear dependent on the relative displacement

$$\Delta z = z_i - z_j. \quad (12)$$

For all three states the tangential contact forces can be calculated by

$$F_{Rx} = -c_{Rx}(x_i - x_j) \quad (13)$$

and

$$F_{Ry} = -c_{Ry}(y_i - y_j). \quad (14)$$

During numerical integration it is necessary to accurately determine the transitions between the three states (Tab. 1).

(See Table 1 after last section of paper)

INFLUENCE OF SURFACE TEXTURE

The surface texture of the two contacting bodies affects the contact behavior. Fig. 2 shows an example surface profile and the according accumulated height distribution.

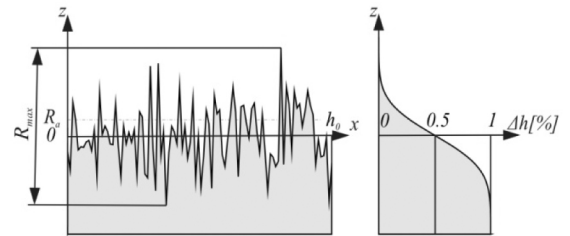


Figure 2. Example surface profile and according accumulated height distribution

Sextro [3] approximates the normalized accumulated height distribution for grinded surfaces by