

Mode Coupling Instabilities Induced by a Periodic Coefficient of Friction

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Georg P. Ostermeyer
Technical Univ. of Braunschweig

Matthias Graf
Institute of Dynamics and Vibration

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ABSTRACT

To predict and minimize squeal propensity in brake systems, numerous models, which perform friction induced vibration, are available. However, today there is no model that can explain satisfactorily the dynamic behavior of brake systems. We argue that this is - among other effects - because existing models are based on decaying or constant coefficient of friction, although many investigations indicate rich dynamics of the coefficient of friction. Stability analysis of linear differential equations with periodic coefficients shows that instability regions change with rising amplitude of the periodic coefficients. If the periodic coefficient corresponds to the coefficient of friction, its periodicity can lower the stability of a minimal model.

INTRODUCTION

Brake noise with frequencies above 1000Hz is usually named as squeal [1]. Experimental investigations have shown how the sound of squeal is radiated by out-of-plane bending modes of the rotating disc, acting as a resonator with defined frequencies. Furthermore it is known that squeal propensity is favored by a high coefficient of friction, which leads to a strong coupling between normal forces and shear forces, a slow sliding velocity and low brake pressures, e.g. if a car rolls out in front of traffic lights. Furthermore, system stiffness, damping, eigenfrequencies, braking history, surface topography, tribological films and humidity have an influence. On the one hand, the dynamic behavior of all parts of the brake (except of the frictional contact itself) is widely analyzed, including periodic behavior of an unsymmetric disc [2]. On the other hand, dynamics of the boundary layer between pad and disc is still poorly understood due to its

complexity and spatial inaccessibility, but is expected to have a significant influence on the dynamics of the whole brake system.

MINIMAL MODELS FOR SQUEAL PHENOMENA IN BRAKE SYSTEMS

One way to model squeal phenomena are so-called “minimal models”: Mechanical models with a limited number of degrees of freedoms. Compared to e.g. FEM-models they are valuable for investigating specific excitation mechanisms, eigenfrequencies, parameter studies and for control design. Today three fundamental mechanisms are widely accepted leading from a frictional coupling in the brake to excited vibrations:

- Friction characteristic: A coefficient of friction with a negative slope in velocity can cause vibrations of a one-dimensional oscillator (stick-slip).
- Geometric instability: A model consisting of a beam- or rod-like element pressed against a moving surface develops instabilities for specific contact angles at a constant coefficient of friction (sprag-slip).
- Nonconservative restoring forces: A mechanical model with at least two degrees of freedom and a constant coefficient of friction can perform instable oscillations due to the self-directed energy flow between different modes that is initiated by the frictional coupling of normal forces and tangential forces.

Good overviews of the models available today are presented in e.g. [1], [3], [4] or [5]. Especially the last excitation mechanism is object of comprehensive research activities. This group of models was introduced by the model of North

[6] and is further developed today: Minimal models with a constant coefficient of friction count limited mode numbers and can comprise sophisticated mode shapes. Some of these models agree well with the geometry, stiffness, damping and masses of real brake systems. After linearization they lead to an equation of the form

$$\underline{\underline{M}}\ddot{\underline{\underline{x}}} + (\underline{\underline{D}} + \underline{\underline{G}})\dot{\underline{\underline{x}}} + (\underline{\underline{K}} + \underline{\underline{N}})\underline{\underline{x}} = \underline{\underline{0}}$$

with constant coefficients. The models contain an unsymmetrical stiffness term which enables instabilities even in the damped case. Usually a linear stability analysis is identified as a criterion whether a system will squeal or not: If a positive real part of an eigenvalue is found, the system is recognized as a squealing one. A root locus plot enables parameter studies with reference to stability and eigenfrequencies. Usually the smaller the gap between the system's eigenfrequencies is, the higher is the squeal propensity (mode locking). Although a broad variety of minimal models is available and the assembly of many comes close to a real brake system, their squealing prediction does not agree well with real brake systems and cannot fulfill industrial needs for designing silent brakes. One weakness of these models is that on the one hand, they comprise realistic kinetics and kinematics, but on the other hand a simple tribological model is applied: a constant coefficient of friction.

BASIC BEHAVIOR OF THE COEFFICIENT OF FRICTION IN BRAKE SYSTEMS

Friction between pad and rotating disc is a complex process and is still far away from being sufficiently understood. It is known that simple constant or velocity-decreasing cofs can already lead to friction induced instabilities, but experiments have demonstrated that the dynamics of the cof must be richer. In the following we discuss a couple of effects leading to time-dependent cofs.

A nearly harmonic coefficient of friction is commonly observed, which has a frequency that is proportional to the rotating speed of the wheel and therefore can cover wide frequency ranges, down to approx. 2 Hz (FIGs. 1 and 2). There are different possible reasons for this phenomenon ("judder"), e.g. different surface states on the disc after a period of rest, unsymmetric disc geometry (thickness variations, unequal wear loss distribution or misalignment) or thermoelastic instabilities, so-called hot-spots [1].

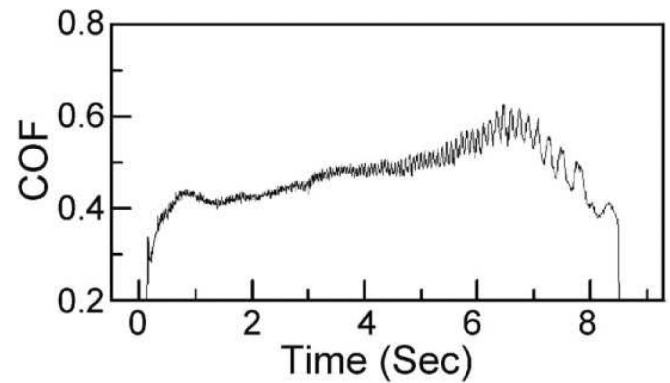


FIG. 1. Periodic cof in a braking test, 50km/h to stop at 220°C, adapted from [7] (datum lining material without any nanotube additives).

Furthermore the tribological layer between pad and disc can show a complex behavior [1]. The relative movement, pressure and heat continuously agglomerate wear debris to brittle contact plateaus and destroy them again. The more of the surface is covered with these hard layers, the higher is the observed coefficient of friction. Based on this flux balance of plateau generation and destruction we developed a dynamic friction law that describes the cof depending on frictional power (pressure times velocity) over time and accords well with experimental dynamometer tests [8] [9].

The growth and the destruction of hard frictional layers were experimentally observed by Eriksson, who linked the tribological behavior with squeal propensity [10]. With a periodic braking pressure, the coefficient of friction shows a hysteretic behavior (FIG. 3). According to the dynamic friction law [8], the wide scattering of experimental results can be understood as a reaction to a high-frequency noise of the braking pressure. For periodic excitations in pressure and relative velocity this dynamics provides a periodic cof with the same frequency as its excitation.

High-frequency tangential dynamics of the numerous contact plateau and their mutual coupling are theoretically analyzed in [11]. It was shown how coupling can lead to a synchronized state in the boundary layer that corresponds to a periodic coefficient of friction.

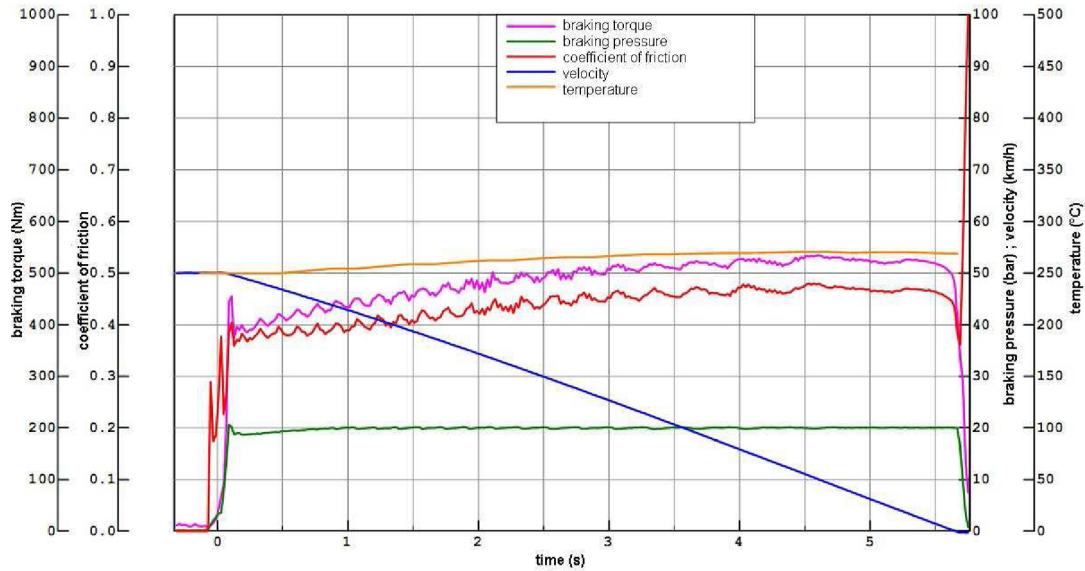


FIG. 2. Periodic coefficient of friction in a dynamometer test, data provided by Honeywell / Jurid.

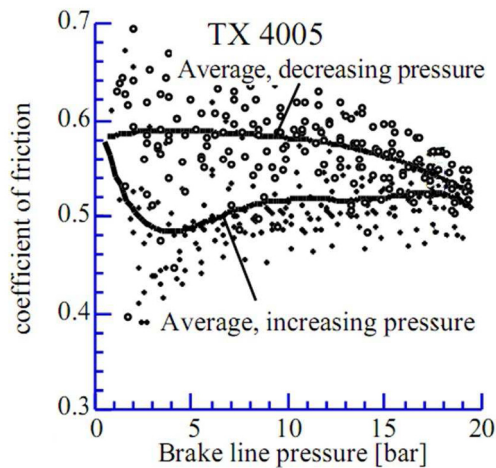


FIG. 3. Behavior of the coefficient of friction over braking pressure, adapted from [10].

Further dynamics of the coefficient of friction are expected, but it can already be recognized that a broad frequency scale is covered. Therefore, a harmonic coefficient of friction that oscillates with variable frequency and magnitude around a constant value

$$\mu = \mu_{av}(1 + \epsilon \cos(\omega t)) > 0 \forall t$$

will be assumed in the following investigations.

DISCUSSION OF THE 1D MATHIEU EQUATION

All minimal models that can be written as a linear matrix equation, include terms depending on the coefficient of friction μ and consequently comprise expressions e.g. of the form μx_i . If μ is not a constant, as usually assumed, but performs periodic oscillations in time, the analysis of the system's eigenvalues does not hold neither as a criterion for instability nor as a measure of squeal propensity. A linear differential equation that contains periodic coefficients is called "Hill equation" or, in the simpler case of harmonic coefficients, "Mathieu equation". To analyze the influence of a periodic factor on stability behavior, first the one-dimensional, undamped Mathieu equation of the form

$$\frac{d^2x}{dt^2} + (\sigma + \epsilon \cos(\omega t))x = 0$$

is discussed. The system has an eigenfrequency at $\sqrt{\sigma}$ and is excited with the amplitude ϵ at a frequency ω . A transformation in time leads to the classical form [12].

$$\frac{d^2x}{d\tau^2} + \left(\frac{\sigma}{\omega^2} + \frac{\epsilon}{\omega^2} \cos(\tau) \right) x = 0$$

The undisturbed system ($\epsilon = 0$) is stable for $\sigma > 0$ and otherwise unstable. The related stability map (FIG. 4) shows, how the harmonic parameter ($\epsilon \neq 0$) opens new instabilities in the parameter plane of the originally stable system.