

A 3D Semi-Empirical On-Road Transient Tire Model

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ABSTRACT

To realistically predict the dynamics of a vehicle, the forces and moments in the contact patch must be accurately computed. A two-dimensional semi-empirical transient tire model was previously developed in the Advanced Vehicle Dynamics Lab (AVDL) at Virginia Tech, and extended the capabilities of the steady-state tire model also developed at AVDL. In this paper, a three-dimensional semi-empirical transient tire model is presented. The tire structure is modeled by an elastic ring supported on a spring and damper system. The elastic ring represents the belt ring and the spring and damper system represents the sidewall and the tread element. The analysis of the deformation of the tire structure with camber angle is performed on a flat surface to obtain the geometry of the contact patch and the normal pressure distribution. The forces and the moments are formulated using empirical data and based on theoretical mechanics. Illustrative simulations were performed for two scenarios: a driving/braking maneuver and a lane changing maneuver. The simulation results were compared with the 2D semi-empirical transient tire model results and with the steady-state tire model results and validated based on available data from the literature.

INTRODUCTION

Tire dynamic behavior plays an important role in vehicle dynamics studies since almost all forces and moments acting on the vehicle are generated at the tire contact patch and transmitted through the tire structure. Therefore, tire models that accurately describe this dynamic behavior are needed for vehicle dynamic simulation. However, it is not easy to develop the tire model due to the complexity of the tire structure. It consists of several layers of synthetic polymer,

many flexible filaments of high modulus cord and glass fiber, bonded to a matrix of low modulus polymeric material [1, 2]. The tire structure may be classified into two parts: hyperelastic components and laminated rubber composites. The hyperelastic components, such as tire belts, beads, and carcass, are soft and practically incompressible such as: tread caps, shoulders, inner liner, and bead fillers. The laminated rubber composites normally are stiff and have a highly orthotropic behavior [3].

Many researchers developed tire models for vehicle dynamic simulations, such as the magic formula tire model, MF-SWIFT model, UniTire model, FTire model family, and CDTire model family [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The magic formula tire model [4, 5] consists of formulas that express the longitudinal force, the lateral force and the aligning moment in steady-state and transient conditions. The formulas contain many coefficients which must be acquired from the experimental data by curve fitting technique. The MF-SWIFT model [6, 7, 8] is composed of a rigid ring tire model and the magic formula model, which can describe tire handling behavior under pure and combined slip conditions and the tire vibration behavior at relatively high frequency excitation. The UniTire model [9, 10, 11, 12] is a semi-physical tire model, which satisfies the required accuracy but has a simple form and needs few parameters. It consists of a steady-state part and a non-steady state part. The steady-state part focuses on the dynamic friction coefficient between the tire and the road surface and the direction of the forces generated at the contact patch. The non-steady state part was developed through an approximation of the transfer function. The FTire model [13, 14] is in fact a family of three tire models: FETire, FTire, and RTire. They combined the coarse-mesh finite element tire model with the spatial non-linear flex/rigid ring model to describe the tire behavior. It

consists of two parts: the structure model and the tread model. The FTire model is used in various applications with extensions such as thermal model, wear model, and tire misuse model. The CDTire model [15, 16] is a family of three tire models: CDTire20 (Rigid Ring), CDTire30 (Single Flexible Ring), and CDTire40 (Multiple Flexible Rings). The applications of the CDTire model are focused on ride comfort and durability & reliability studies. The CDTire can be used only with small deflections of belts and with smooth tread surface tires.

In the past few years, AVDL developed a steady-state tire model using analytical mechanics starting from Fiala's tire model [17] and experimental data from NHTSA, as well as in-house testing [18]. Next, a two-dimensional semi-empirical tire model for transient maneuvers was developed to extend the capabilities of the steady-state tire model to capture the transient behavior of tire without additional testing [19]. Recently, the three-dimensional semi-empirical on-road transient tire model has been developed. This model can simulate in-plane and out-of-plane tire forces and moments in steady-state and transient conditions on a flat surface. The camber angle effects are taken into account.

In this paper, the overall description and model assumptions are given. It describes in details how the contact patch geometry and the contact pressure distribution are calculated; the analytical-empirical force and moment formulas are presented. Simulation results are illustrated and benchmarked against available results from the literature.

3D SEMI-EMPIRICAL ON-ROAD TRANSIENT TIRE MODEL

Development of the tire model

The 3D semi-empirical transient tire model developed in this paper is based on observation of experimental data, on the quasi-static semi-empirical tire model developed by Chan [18] and on the 2D semi-empirical transient tire model previously developed by the authors [19]. Throughout this paper, we used nomenclature, definitions, and the tire coordinate system based on the SAE standards [20].

The tire model is developed based on the following assumptions.

1. The tire contact patch is a trapezoidal shape with camber angle and is a rectangular shape with zero camber angle, which, according to our observations and the literature [1, 2, 18, 21] are reasonable assumptions.
2. The friction coefficients between the tire and the road are approximated based on observations of empirical data which vary with the normal force, the longitudinal slip ratio, and the slip angle.

3. The relationship between the longitudinal slip ratio and the effective longitudinal slip ratio for low frequency input is presented by using the first order differential equation, as shown in [equation \(1\)](#) [22].

$$a_s \dot{s}_c + s_c = s \quad (1)$$

4. The relationship between the slip angle at the wheel center and the slip angle at the tire contact patch in the transient steering is expressed by using the second order differential equation as in [equation \(2\)](#) [23, 24].

$$a_2 \ddot{\alpha}_c + a_1 \dot{\alpha}_c + \alpha_c = \alpha \quad (2)$$

5. The belt is laminated rubber composited which normally very stiff, so we can assume that the belt ring is flexible but

not extensible. The inextensibility condition yields $(\cdot = \frac{d}{d\theta})$

$$w = v' \quad (3)$$

The transient tire model was developed by modeling the tire structure as a flexible belt ring supported by a spring and damper system on a flat surface, as shown in [Figure 1](#).

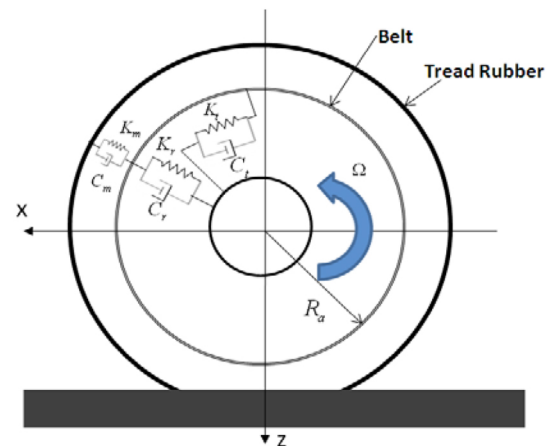


Figure 1. Flexible belt ring supported by a spring and damper system on a flat surface

The spring and damper system represents the side wall and the tread element of the tire. The tire model consists of two components: a physical component and an empirical component, so it can be considered a semi-empirical model. The inputs are the slip angle, the longitudinal slip ratio, the camber angle, the speed, the vertical load, and the road friction. Viscoelastic properties of the tire cause a lag in the slip at the contact patch which we considered can be realistically described by a 2nd order differential equation.

The physical component has two main parts: It involves the calculation of the contact patch geometry and the calculation of the contact pressure distribution. As stated above, we assume that the contact patch has a trapezoidal shape when the camber angle is not zero and has a rectangular shape when the camber angle is zero. The length of the contact patch on a flat surface can be derived from the geometric diagram. The pressure distribution can be calculated from the deflection of tire structure (geometric deformation (z), elastic deformation (ζ), and additional deformation (w^*)).

Geometry of contact patch

Tire contact patch length

The tire contact patch length on flat and curved surfaces can be simply approximated from a geometric diagram as shown in [Figure 2](#).

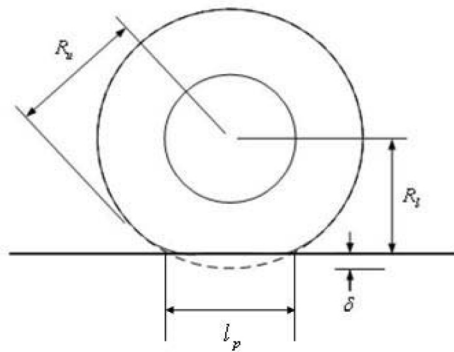


Figure 2. Geometrical relations between tire contact length on a flat surface

From [Figure 2](#), the relationship between R_u , R_t , l_p , and δ can be expressed using the Pythagorean Theorem as:

$$l_p = 2\sqrt{2R_u\delta - \delta^2} \quad (4)$$

Where

$$\delta = \frac{F_z}{K_z} \quad (5)$$

$$K_z = \left(C_{z1}(1 - C_{z1}) \operatorname{sech} \left(C_{z2} \frac{|\tan(\alpha_c)|}{1 - |\tan(\alpha_c)|} \right) \right) K_{z0} \quad (6)$$

Contact patch area with camber angle

The contact area of the belt has a quasi-trapezoidal shape slightly bent in the camber direction as shown in [Figure 3](#).

The deformation of radial tire having the camber angle γ , deflection δ , central angles θ_1 and θ_2 , where subscripts 1 and 2 means the front and rear edges of the contact belt respectively, and the asterisk (*) expresses the deformed state of position p , s , and r . From [Figure 3](#), we can calculate the taper angle β of the trapezoidal contact area as expressed in [equation 7](#).

$$\tan(\beta) = \frac{p_1^* r_1^* - p_2^* s_2^*}{b} \approx \frac{R_t(\theta_1 - \theta_2)}{b} \quad (7)$$

Also, we know that:

$$\tan(\gamma) = \frac{R_t(\cos(\theta_2) - \cos(\theta_1))}{b} \approx \frac{R_t(\theta_1^2 - \theta_2^2)}{2b} \quad (8)$$

Then, substituting [equation \(7\)](#) into [equation \(8\)](#) yields

$$\beta = \tan^{-1} \left(\frac{\tan(\gamma)}{\theta_0} \right) \quad ; \theta_0 = \frac{\theta_1 + \theta_2}{2} \quad (9)$$

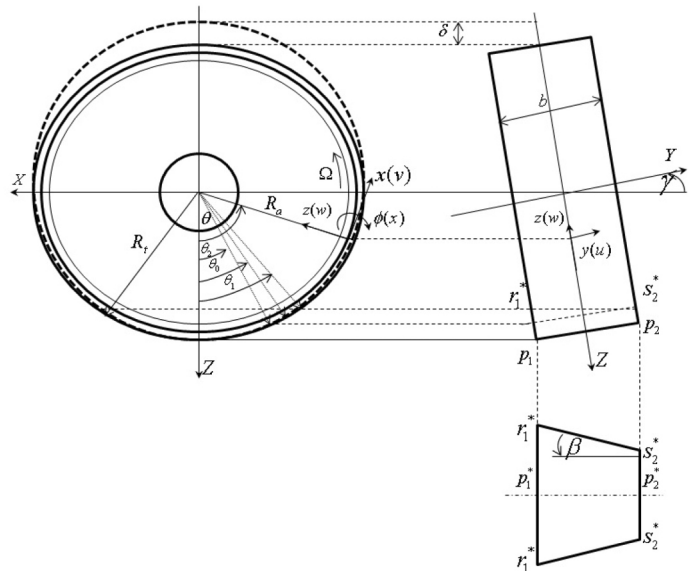


Figure 3. Tire structure deformations with γ camber angle

Contact pressure distribution and vertical force

Analysis of the deformation of the tire structure on the road surface with the camber angle is very complex due to large deformation and nonlinear properties of the tire. Many studies tried to solve this problem. Böhm analyzed the torsional and bending deformation of the belt ring supported

by the elastic foundation under the inflation pressure without considering the contact deformation and rotating acceleration [25]. Gong [26] developed the one-dimensional contact pressure using a flexible ring model, and then Kim and Savkoor [27] included the damping effects and the finite friction into account. Shiobara et al. [28] developed a one-dimensional contact pressure distribution for rolling tire. Kim et al. [29, 30] extended it to a two-dimensional contact problem on flat and uneven roads.

Contact-free belt deformation analysis

The equilibrium equations of forces and moments of the contact-free belt element in radial, tangential, and lateral can be derived from Figure 4, as shown in equations (10), (11), (12), (13), (14), respectively.

$$\left(\begin{matrix} i \\ \end{matrix} \right) = \frac{\partial^i}{\partial \theta^i}$$

The equilibrium equation in z direction

$$v^{(6)} + \alpha_1 v^{(4)} + \alpha_2 v^{(3)} + \alpha_3 v^{(2)} + \alpha_4 v^{(1)} + \alpha_5 v = 0 \quad (10)$$

Where

$$\begin{aligned} \alpha_1 &= 2 - \frac{R_a^2}{D_y} \bar{N}_x b + \frac{R_a^4}{D_y} \rho b \Omega^2 & \alpha_4 &= \frac{R_a^4}{D_y} C_r \Omega \\ \alpha_2 &= -\frac{R_a^4}{D_y} C_r \Omega & \alpha_5 &= -\frac{R_a^4}{D_y K_t} + \frac{R_a^4}{D_y} \rho b \Omega^2 \\ \alpha_3 &= 1 + \frac{R_a^4}{D_y} K_r - \frac{R_a^2}{D_y} \bar{N}_x b - 6 \frac{R_a^4}{D_y} \rho b \Omega^2 \end{aligned} \quad (11)$$

The equilibrium equation in x direction

$$u^{(2)} - \gamma_1 \phi^{(2)} + \gamma_2 \phi = 0 \quad (12)$$

The equilibrium equation in y direction

$$u^{(4)} + \kappa_1 u^{(2)} + \kappa_2 u + \kappa_3 \phi^{(2)} + \kappa_4 \phi = 0 \quad (13)$$

Where

$$\begin{aligned} \gamma_1 &= \frac{(\Gamma - I R_a^2 \Omega^2) R_a}{\Gamma + D_z} & \kappa_2 &= \frac{K_s R_a^4}{D_z} \\ \gamma_2 &= \frac{(D_z + \bar{K}_r R_a^2 b^2 / 4) R_a}{\Gamma + D_z} & \kappa_3 &= \frac{R_a (D_z + \Gamma)}{D_z} \\ \kappa_1 &= -\frac{\bar{N}_x b R_a^2 + \Gamma - \rho R_a^4 b \Omega^2}{D_z} & \kappa_4 &= -\frac{\bar{N}_x R_a^3 b}{D_z} \end{aligned} \quad (14)$$

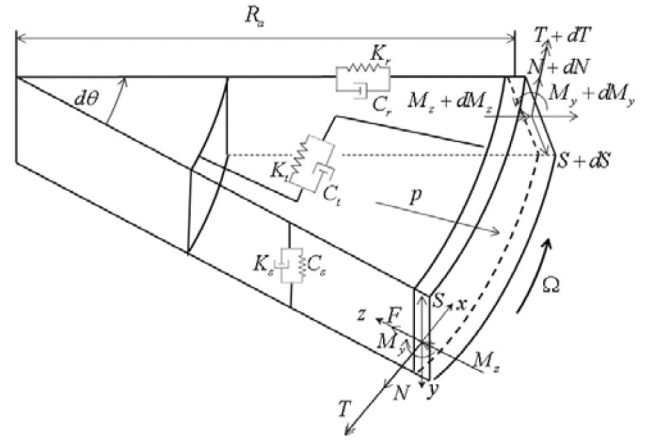


Figure 4. Forces and moments applied to the contact-free belt element in motion

Substituting equation 12 into equation 13 yields

$$\Phi^{(6)} + \beta_1 \Phi^{(4)} + \beta_2 \Phi^{(2)} + \beta_3 \Phi = -K_1 \theta - K_2 \quad (15)$$

Where

$$\begin{aligned} \beta_1 &= \kappa_1 + \frac{\gamma_2}{\gamma_1} + \frac{\kappa_3}{\gamma_1} \\ \beta_2 &= \kappa_2 + \frac{\kappa_1 \gamma_2}{\gamma_1} + \frac{\kappa_4}{\gamma_1} \\ \beta_3 &= \frac{\kappa_2 \gamma_2}{\gamma_1} \end{aligned}$$

K_1, K_2 = Integral constants of Eq. (17)

$$(16)$$

And

$$\phi(\theta) = \Phi^{(2)} \quad (17)$$

$$u(\theta) = -\gamma_1 \Phi^{(2)} - \gamma_2 \Phi + K_1 \theta + K_2 \quad (18)$$

Solving equations (10), (15), (17) and (18) and using the inextensibility condition, we get the solutions of w, v, u, ϕ as

$$w(\psi) = \sum_{i=1}^6 C_i e^{\lambda_i \theta}$$

$$v(\psi) = \sum_{i=1}^6 C_i \lambda_i e^{\lambda_i \theta}$$

$$\begin{aligned} \phi(\psi) &= B_1 e^{-\mu_1 \theta} + e^{-v_1 \theta} (B_2 \cos(v_2 \theta) + B_3 \sin(v_2 \theta)) \\ u(\psi) &= B_1 g_1 e^{-\mu_1 \theta} + B_2 e^{-v_1 \theta} (g_2 \cos(v_2 \theta) - g_3 \sin(v_2 \theta) + \dots \\ &\quad + B_3 e^{-v_1 \theta} (g_2 \sin(v_2 \theta) + g_3 \cos(v_2 \theta) + K_1 \theta + K_2 \end{aligned} \quad (19)$$

Where

λ_i = roots of characteristic equation of equation (10)

$\pm \mu_i$ = real roots of characteristic equation of equation (15) ; $\mu_i > 0$

$\pm (v_1 \pm v_2)$ = complex roots of characteristic equation of equation (15) ; $v_1, v_2 > 0$

$$g_1 = \gamma_1 + \frac{\gamma_2}{\mu_1^2}, \quad g_2 = \gamma_1 + \gamma_2 \frac{v_1^2 - v_2^2}{(v_1^2 + v_2^2)^2}, \quad g_3 = \gamma_2 \frac{2v_1^2 v_2}{(v_1^2 + v_2^2)^2} \quad (20)$$

To find the integral constants C_{1-6} , B_{1-3} , $K_{1,2}$, we used the boundary conditions at the contact end and the symmetry condition. After we obtained the solution for w , v , u , ϕ , we could calculate the vertical deflection of the contact-free belt at the contact end, transverse shearing force, twisting torque, and in-plane bending moment which are needed in contact belt deformation analysis, as shown in the following equations.

The vertical deflection of the contact-free belt at the contact end is

$$\eta_0 = z_0 + \zeta_0 = [R_a(1 - \cos \theta_0)] + [w_0 \cos \theta_0 + v_0 \sin \theta_0] \quad (21)$$

The transverse shearing force is

$$F = \frac{D_y}{R_a} (v^{(4)} + v^{(2)}) \quad (22)$$

The twisting torque is

$$T = \frac{\Gamma}{R_a} \left(\phi' - \frac{u'}{R_a} \right) \quad (23)$$

The in-plane bending moment is

$$M_{in} = -D_z \left(\frac{u''}{R_a^2} + \frac{\phi}{R_a} \right) \quad (24)$$

Contact belt deformation analysis

The in-plane shapes of the deformed belt and the deformed tread surface is assumed to be slightly bent trapezoids [31], as shown in Figure 5. We can define the relative displacement between the belt and the tread surface as:

$$U = A \left(1 - \frac{x^2}{\bar{x}^2} \right) \quad (25)$$

A is the maximum relative displacement between the belt and the tread surface.

$$A = \frac{R_a \sin^4 \theta_1}{4 \cos \theta_1} \sin \gamma \cos \gamma \quad (26)$$

$\bar{x}$ is a geometrical function of the oblique side of the trapezoidal contact area.

$$\bar{x}(y) = \frac{\bar{x}_1 + \bar{x}_2}{2} + \frac{\bar{x}_2 - \bar{x}_1}{2} y \quad (27)$$

Then the contact shear stress in circumferential and lateral directions can be calculated by

$$S_x = G\theta, \quad S_y = G \frac{U}{H} \quad (28)$$

Therefore, the camber thrust can be given by the following equation

$$Q_y = \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\bar{x}}^{\bar{x}} S_y dx dy = \frac{G}{12H} R_a^2 b \frac{\sin^4 \theta_1}{\cos \theta_1} (\sin \theta_1 + \sin \theta_2) \sin 2\gamma \quad (29)$$

Next, we considered the deformation of the contact belt in the vertical direction. The total deflection of the contact belt (w^*) is the superimposed solution from two parts: the standard flat deflection (w_0) and the additional deflection ($\bar{w}$) which are described mathematically as:

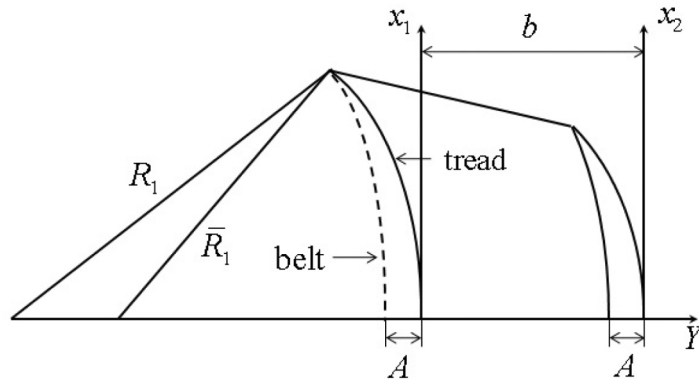


Figure 5. Deformation of the belt associated with the relative displacement to the tread surface

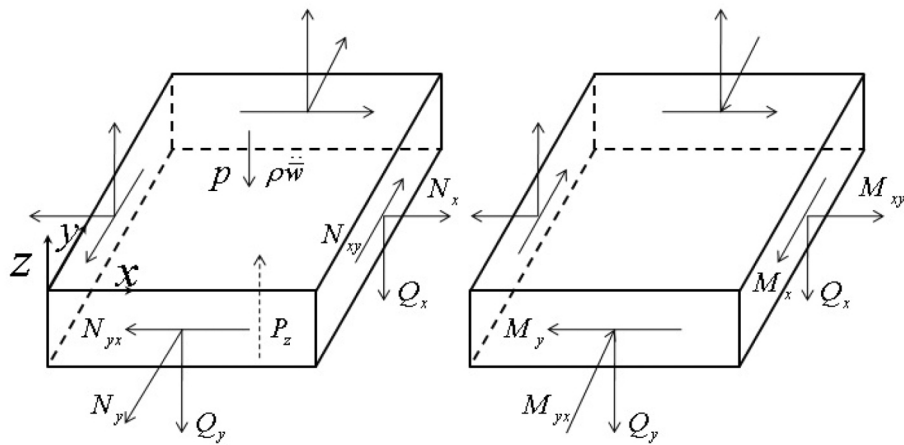


Figure 6. Forces and moments applied to the contact belt element in motion

$$w_0(x, y) = -w_1 - w_2 y \quad (30)$$

$$\bar{w}(x, y) = f(y)(\bar{x}^2(y) - x^2)^2 \quad (31)$$

Where

$$f(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 \quad (32)$$

The belt structure in the contact area has been approximated by a flat plate with pressure distribution P_z . The equilibrium equation of the contact belt in the vertical direction is obtained from Figure 6 as:

$$D_{xx} \bar{w}_{xxxx} + 2(D_{xy} + 2D_{ss}) \bar{w}_{xxyy} + D_{yy} \bar{w}_{yyyy} = -p + P_z + \dots + N_x \bar{w}_{xx} + N_y \bar{w}_{yy} + 2N_{xy} \bar{w}_{xy} - \rho V^2 \bar{w}_{xx} \quad (33)$$

After we determine the standard flat deflection, and the additional deflection as shown in [Appendix A](#), the normal pressure distribution is described by the following equation.

$$P_z(x, y) = -K_m w^*(x, y) - C_m V \bar{w}_y(x, y) \quad (34)$$

The tire vertical force is a function of the normal contact pressure distribution acting on the tire contact patch. We can calculate the vertical force by integrating the normal contact pressure distribution over the contact area.

$$F_z = \int_{-b/2}^{b/2} \int_{-\bar{x}}^{\bar{x}} P_z(x, y) dx dy \quad (35)$$

When the vehicle is accelerating or braking, the vertical force at the front wheel and at the rear wheel change due to the longitudinal load transfer. In this model, the load transfer between the front and rear wheels while accelerating or braking is taken into account, as shown below.