

Drivability Analysis of Heavy Goods Vehicles

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ABSTRACT

The paper presents linear and non-linear driveline models for Heavy Goods Vehicles (HGVs) in order to evaluate the main parameters for optimal tuning, when considering the drivability. The implemented models consider the linear and non-linear driveline dynamics, including the effect of the engine inertia, the clutch damper, the driveshaft, the half-shafts and the tires. Sensitivity analyses are carried out for each driveline component during tip-in maneuvers. The paper also analyses the overall frequency response using Bode diagrams and natural frequencies.

It is demonstrated that the most basic model capable of taking into account the first order dynamics of the driveline must consider the moments of inertia of the engine, the transmission and the wheels, the stiffness and the damping properties of the clutch damper, driveshaft and half-shafts, and the tires (which link the wheel to the equivalent inertia of the vehicle). The results show that the first natural frequency of the drivetrain increases as a function of the half-shaft stiffness and the gear number, and the overall damping decreases as a function of the longitudinal slip stiffness of the tire. The vehicle payload has a significant effect not only on the steady-state acceleration, but also on the overall system dynamics (frequencies and damping).

INTRODUCTION

Several HGV driveline models have been presented and discussed in the literature [1,2,3,4,5,6]. Reference [1] describes experimental testing procedures, heavy truck modeling, and the implementation of a linear model capturing the significant torsion resonances in the driveline. Optimization has been the main focus of the other models. In these models the optimal configuration of the driveline, either by using statistical methods or simulation tools, is selected in

order to achieve the lowest fuel consumption [2-3]. Article [4] is focused on benchmarking techniques to screen design options until a favorable balance of performance is indicated. Reference [5] concentrates on the optimization of an offline identification process for driveline systems and on establishing an optimal model fit. References [6,7,8,9,10,11] show procedures for the experimental and theoretical assessment of vehicle drivability, including the implementation of empirical and advanced algorithms for its optimization. However, a detailed study of the influence of the individual component (tires, half-shafts, clutch damper, driveshaft, gearbox and differential) parameters on the overall first order dynamics of the vehicle and powertrain is absent. Although reference [12] presents a complete model for vehicle drivability analysis and some time histories of vehicle response, no general trend is derived as a function of vehicle parameters.

This activity should help to understand the real influence of changing each element of the driveline in terms of the overall performance of the system. This research is based on a linear driveline model, adopted for the simulation of the basic dynamics and for the response analysis in the frequency domain, and a non-linear one, adopted for advanced simulations in the time domain. The paper describes the driveline model of the truck, based on the formulations presented and experimentally validated in literature [13,14,15,16] for passenger cars and on the torque balance applied to the individual driveline components [17].

The first part of the paper describes the schematic of the driveline model and the mathematical formulation of the linear dynamic equation of each component. The fundamental equations of the linear driveline model are derived, for a condition of fixed gear ratio, and are then arranged in a state-space formulation to implement the model [14]. The second part of the article presents the analysis of the results,

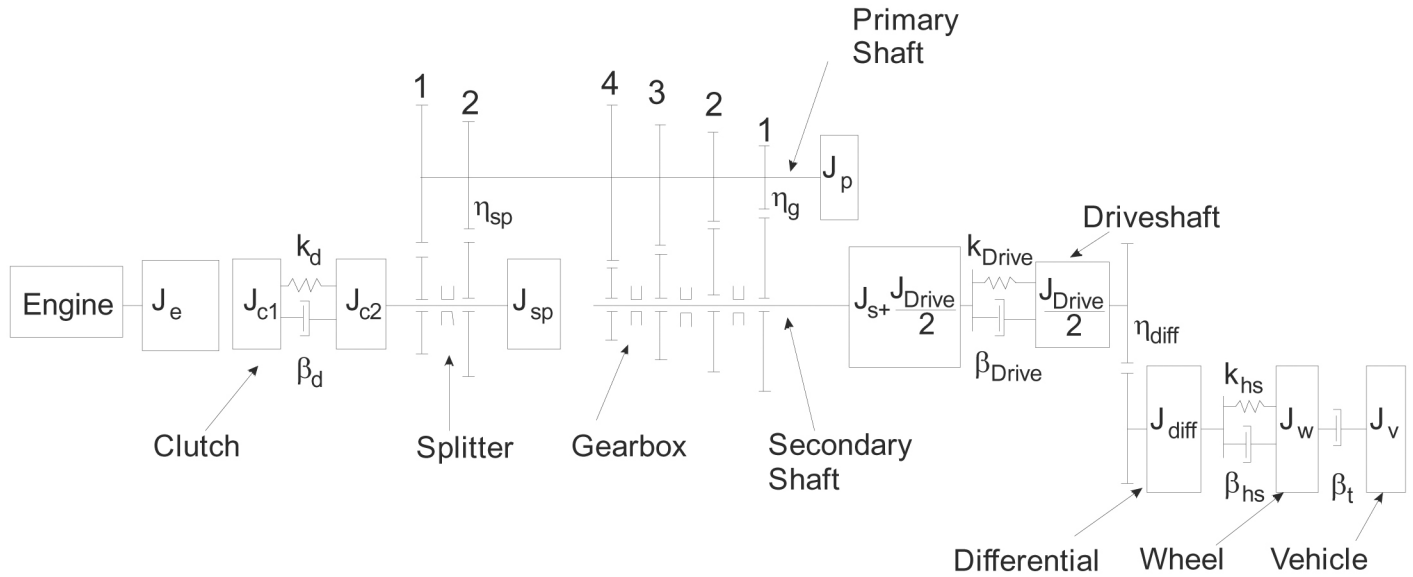


Figure 1. Linear model layout, including splitter and gearbox

comparisons and conclusions. The sensitivity analysis of the system is presented in different plots in both time and frequency domains. Furthermore the effect of the non-linearities on the system drivability is investigated.

THE LINEAR MODEL

Figure 1 is the schematic of the simulated vehicle layout. The vehicle is equipped with a 2-ratio splitter and a 4-ratio gearbox, for a total number of 8 ratios. The driver uses the gearbox only, and not the splitter, when the vehicle is unladen, whereas the combination of the splitter and the gearbox is necessary in the condition of a significantly laden vehicle. The differential is located on the driven axle and is connected to the gearbox through a driveshaft.

The full truck model, shown above, includes the dynamics of the clutch damper, splitter, gearbox, driveshaft, half-shafts, the driven tires/wheels and the vehicle, where an equivalent half-shaft and equivalent wheel for each driven axle are considered; this is consistent with the typical layout of a HGV driven axle, characterized by symmetric half-shafts. The full transmission dynamics can be described as a system of 5 differential equations (5 degree-of-freedom model) considering the main torsionally flexible elements of the powertrain, which are the clutch damper, the driveshaft, the half-shaft and the tire. According to the layout of the driveline system shown in Figure 1, the dynamic equations for the transmission components are derived in the following paragraphs.

Engine

By applying Newton's second law to the engine rotation, it is:

$$J_1 \ddot{\theta}_e = T_e - T_c \quad (1)$$

where J_1 is the sum of the moments of inertia of the engine J_e and the clutch disk J_{c1} ($J_1 = J_{c1} + J_e$), T_c is the torque of the clutch damper, T_e is the engine torque (derived through the engine map as a function of torque demand and engine speed) and $\ddot{\theta}_e$ is engine angular acceleration. In a linear model, T_c can be written as:

$$T_c = k_d (\theta_e - \theta_{sp}) + \beta_d (\dot{\theta}_e - \dot{\theta}_{sp}) \quad (2)$$

where k_d is the stiffness of the clutch damper, β_d is its damping coefficient, θ_{sp} is the rotation angle of the input shaft of the splitter. Due to the very significant variation of the clutch damper stiffness as a function of its torsion angle, particular attention must be paid during the derivation of k_d , whose linearized value can be different as a function of the maneuver to be simulated, depending on the expected range of engine torques and clutch damper displacements. The differential speed can be expressed after incorporating (2) into (1) and applying the gear ratios of the splitter τ_{sp} , the gearbox τ_g and the differential τ_{diff} . The final engine equation is obtained in (3):

$$J_1 \ddot{\theta}_e = T_e - k_d (\theta_e + k_d \tau_{sp} \tau_g \tau_{diff} \theta_{diff}) - \beta_d (\dot{\theta}_e - \beta_d \tau_{sp} \tau_g \tau_{diff} \dot{\theta}_{diff}) \quad (3)$$

where θ_{diff} is the rotational displacement of the differential.

Splitter and gearbox

The second degree of freedom of the system includes the dynamics of the splitter shaft, the primary shaft and the secondary shaft of the gearbox. The following torque balance equation is written about the secondary shaft of the gearbox:

$$\left(J_s + \frac{J_{Drive}}{2} \right) \ddot{\theta}_s = T_p \eta_g \tau_g - T_{Drive} \quad (4)$$

The primary shaft output torque T_p (through the gear ratio τ_g and the efficiency η_g) and driveshaft torque T_{Drive} are applied to the secondary shaft. $\ddot{\theta}_s$ is the angular acceleration of the secondary shaft, J_s is its moment of inertia and J_{Drive} is the moment of inertia of the driveshaft. T_{Drive} is a function of driveshaft stiffness k_{Drive} and driveshaft damping coefficient β_{Drive} , [equation \(5\)](#). The driveshaft is modeled as a system of 2 inertias (each of them including half of the inertia of the driveshaft) connected through a torsion spring and damper. Half of the driveshaft rotates with the secondary shaft of the gearbox, the other half is geared with the differential case. The possible dynamic effects related to the dynamics of the driveshaft joints are neglected in the model, as they are not significant for the low frequency drivability analysis of the vehicle. The damping ratio of the driveshaft is very low due to the low internal damping of the material (steel), the same property is valid for the half-shafts.

The primary shaft output torque T_p can be calculated from [equation \(6\)](#), where τ_{sp} and η_{sp} are the splitter ratio and efficiency respectively, and T_{sp} is the splitter torque.

$$T_{Drive} = k_{Drive} (\theta_s - \tau_{diff} \theta_{diff}) + \beta_{Drive} (\dot{\theta}_s - \tau_{diff} \dot{\theta}_{diff}) \quad (5)$$

$$J_p \ddot{\theta}_p = \eta_{sp} \tau_{sp} T_{sp} - T_p \quad (6)$$

The torque balance equation of the splitter shaft is presented in [\(7\)](#), where J_{sp} is the splitter inertia and J_{c2} the inertia of the part of the clutch connected to the splitter. After incorporating the clutch torque ([equation \(2\)](#)) into [\(7\)](#) and the result into [\(6\)](#), the primary shaft torque is obtained in [\(8\)](#). By incorporating [\(8\)](#) and [\(5\)](#) into [\(4\)](#) and applying the ratios of the splitter and gearbox, the secondary shaft equation $\ddot{\theta}_s$ is obtained in [equation \(9\)](#).

$$(J_{sp} + J_{c2}) \ddot{\theta}_{sp} = T_c - T_{sp} \quad (7)$$

$$T_p = \tau_{sp} \eta_{sp} \left[k_d (\theta_e - \theta_{sp}) + \beta_d (\dot{\theta}_e - \dot{\theta}_{sp}) - \ddot{\theta}_{sp} (J_{sp} + J_{c2}) \right] - J_p \ddot{\theta}_p \quad (8)$$

$$\begin{aligned} \left[J_s + \frac{J_{Drive}}{2} + \eta_g \tau_g^2 \tau_{sp}^2 \eta_{sp} (J_{sp} + J_{c2}) + \eta_g \tau_g^2 J_p \right] \ddot{\theta}_s = \\ \eta_g \eta_{sp} \tau_g \tau_{sp} \beta_d \dot{\theta}_e + \eta_g \tau_g \tau_{sp} \eta_{sp} k_d \theta_e + \\ - (\eta_g \tau_g^2 \tau_{sp}^2 \eta_{sp} \beta_d + \beta_{Drive}) \dot{\theta}_s - (\eta_g \eta_{sp} \tau_g^2 \tau_{sp}^2 k_d + k_{Drive}) \theta_s + \\ \beta_{Drive} \tau_{diff} \dot{\theta}_{diff} + k_{Drive} \tau_{diff} \theta_{diff} \end{aligned} \quad (9)$$

Differential

The following equation is the torque balance for the half of the driveshaft which is geared with the differential casing:

$$\frac{J_{Drive}}{2} \ddot{\theta}_{Drive} = T_{Drive} - T_{Diff} \quad (10)$$

where T_{diff} is the differential torque (the torque the differential casing transmits to the gear on the driveshaft). [Equation \(11\)](#) is the torque balance equation of the differential casing, where T_{half} is the half-shaft torque, J_{diff} is the differential inertia and η_{diff} is the efficiency of the differential. After writing the half-shaft torque according to the torsion angle of the equivalent half-shaft (function of wheel angular displacement and differential angular displacement, respectively θ_w and θ_{diff}), [equation \(12\)](#) is obtained.

$$\eta_{diff} \tau_{diff} T_{diff} - T_{half} = J_{diff} \ddot{\theta}_{diff} \quad (11)$$

$$T_{half} = k_{hs} (\theta_{diff} - \theta_w) + \beta_{hs} (\dot{\theta}_{diff} - \dot{\theta}_w) \quad (12)$$

k_{hs} and β_{hs} are respectively half-shaft stiffness and damping coefficient. They are the sum of the coefficients for the left and right half-shafts, which work as springs and dampers in parallel. By incorporating [\(11\)](#) and [\(12\)](#) into [\(10\)](#) and considering the kinematic relationships, the final equation of the differential is achieved [\(13\)](#):

$$\begin{aligned} \left(J_{diff} + \eta_{diff} \tau_{diff}^2 \frac{J_{Drive}}{2} \right) \ddot{\theta}_{diff} &= \eta_{diff} \tau_{diff} k_{Drive} \theta_s + \eta_{diff} \tau_{diff} \beta_{Drive} \dot{\theta}_s - \\ & \left(\eta_{diff} \tau_{diff}^2 \beta_{Drive} + \beta_{hs} \right) \dot{\theta}_{diff} + \\ & - \left(k_{hs} + \eta_{diff} \tau_{diff}^2 k_{Drive} \right) \theta_{diff} - k_{hs} \theta_w - \beta_{hs} \dot{\theta}_w \end{aligned} \quad (13)$$

Wheel

The tire represents the main damping element within an automotive drivetrain (apart from the clutch damper or the torque converter [13]), due to the fact that the other torsionally flexible elements (driveshaft and half-shafts) of the transmission system are metal components, with a very low internal damping. The typical behavior of a tire subjected to a slip ratio and modeled through Pacejka Magic Formula is presented in Figure 2 and 3. Figure 2 shows the longitudinal force F_x as a function of the slip ratio SR, for different vertical loads F_N , and Figure 3 for different friction coefficients μ between the tire and the road. The rate of the linear part of the graph (measured around the origin) is the longitudinal slip stiffness C_s , and this rate changes as a function of the slip ratio (first non-linearity of the tire). Moreover, an increase of vertical load provokes a less than proportional increase of the force for an assigned slip ratio (second non-linearity of the tire). Finally, a variation of the friction coefficient between the tire and the road surface provokes a variation of the overall shape of the diagram (third non-linearity of the tire). All these phenomena are neglected by the tire model implemented within the linear vehicle model.

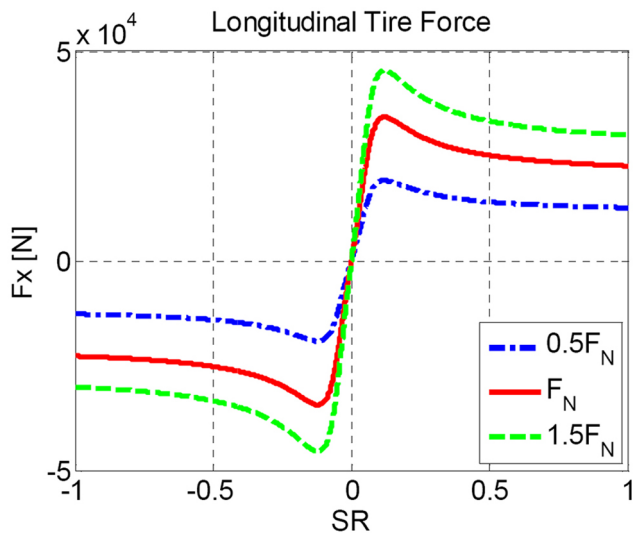


Figure 2. Tire longitudinal force as a function of slip ratio for different vertical loads

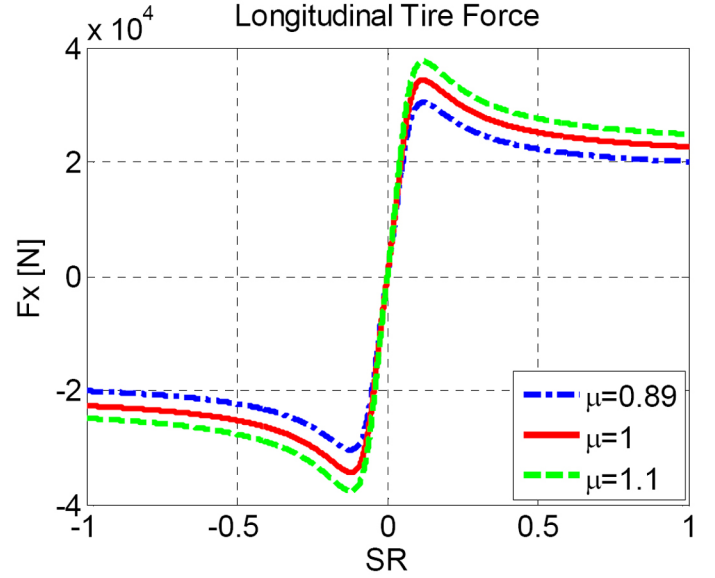


Figure 3. Tire longitudinal force as a function of slip ratio for different friction coefficients

In a very first approximation, if we neglect the effect of the load transfer (which modifies the rate of the linear section of the graph) and the non-linear shape for high values of slip ratio (they should be beyond the common working area of the tire for vehicle drivability analysis), it is possible to write that:

$$F_x = C_s SR \quad (14)$$

However, if we consider the torque on the wheel and we apply the definition of slip ratio, it is:

$$T_t = R_w F_x = R_w C_s \frac{\dot{\theta}_w - \dot{\theta}_v}{\dot{\theta}_v} \quad (15)$$

where $\dot{\theta}_v$ is the equivalent angular speed of the vehicle, given by the ratio between vehicle linear velocity and wheel radius R_w . The tire, within a linear vehicle model, can be represented by an equivalent torsion damping coefficient β_t :

$$\beta_t = R_w \frac{C_s}{\dot{\theta}_v} \quad (16)$$

As a consequence, the damping coefficient β_t is inversely proportional to vehicle longitudinal speed and the results of the frequency response analysis are related to the longitudinal speed of the vehicle, but also to the vertical load between the tires and the road surface (since C_s depends on the vertical

load), which represents another tire non-linearity. In the non-linear model, Pacejka Magic Formula is adopted for the computation of tire longitudinal force as a function of slip ratio and vertical load. In the linearized model, a constant value for β_t (for a given vehicle speed) is adopted, given the static load conditions of the vehicle and the predicted speed range during the maneuver.

The torque balance on the equivalent driven wheel of the vehicle is given by [equation \(17\)](#), where J_w is the wheel (or twin wheel) moment of inertia (on one side of the vehicle). For simplicity, in the linear model, the rolling resistance contributions of the tires have been included in the overall vehicle equation, and not in the driven wheel equations. The model also neglects the relaxation length of the tire.

$$2J_w \ddot{\theta}_w = k_{hs}(\theta_{diff} - \theta_w) + \beta_{hs}(\dot{\theta}_{diff} - \dot{\theta}_w) - \beta_t(\dot{\theta}_w - \dot{\theta}_v) \quad (17)$$

Vehicle

The vehicle is modeled as an equivalent inertia J_v , subjected to the total tire torque T_t , the torque due to the road gradient $T_{road_gradient}$, the torque due to the total tire rolling resistance $T_{rolling_resistance}$, and the aerodynamic drag torque T_{drag} . Each of these torques is characterized by a non-linear expression ([equation \(19\)](#)), which has been properly linearized in ([equation \(20\)](#)) within the implemented model.

$$J_v \ddot{\theta}_v = T_t - T_{road_gradient} - T_{rolling_resistance} - T_{drag} \quad (18)$$

$$\begin{aligned} (R_w^2 M_v + 2J_w) \ddot{\theta}_v &= \beta_t(\dot{\theta}_w - \dot{\theta}_v) - R_w M_v g \sin(\alpha) - \\ R_w M_v (F_0 + F_c \dot{\theta}_v^2 R_w^2) g \cos(\alpha) &- \frac{1}{2} \rho S C_d \dot{\theta}_v^2 R_w^3 \end{aligned} \quad (19)$$

$$\begin{aligned} (R_w^2 M_v + 2J_w) \ddot{\theta}_v &= \beta_t \dot{\theta}_w - (\beta_t + 2M_v g \cos(\alpha) F_c \dot{\theta}_{v0} R_w^3 + \rho S C_d \dot{\theta}_{v0} R_w^3) \dot{\theta}_v + \\ - R_w M_v g \sin(\alpha) - R_w M_v F_0 g \cos(\alpha) &- \dot{\theta}_{v0}^2 R_w^3 M_v F_0 g \cos(\alpha) + \\ \frac{1}{2} \rho S C_d \dot{\theta}_{v0}^2 R_w^3 + M_v g \cos(\alpha) F_c \dot{\theta}_{v0}^2 R_w^3 \end{aligned} \quad (20)$$

where M_v is the vehicle mass, F_0 and F_c are the rolling resistance coefficients of the vehicle tires, S is vehicle frontal area, α is the road gradient, g is the earth's gravity, $\dot{\theta}_{v0}$ is the vehicle speed about which the linearization is carried out, and ρ is the air density. The term $2J_w$ in [equation \(20\)](#) represents the moment of inertia of the undriven wheels (2 in this particular case).

State-space Formulation

The above derived linear equations are organized in a state-space formulation, according to [equations \(21\)](#) and [\(22\)](#):

$$\dot{Z} = [A][Z] + [B][U] \quad (21)$$

$$Y = [C][Z] + [D][U] \quad (22)$$

where $[Z]$ is the state space vector, $[A]$ is the state matrix, $[U]$ is the input vector (engine torque), $[B]$ is the input matrix, $[C]$ is the output matrix, $[D]$ is the feedthrough matrix and $[Y]$ is the output vector, which includes all the plotted parameters. The vectors and the matrices are attached in the [Appendix](#). The natural frequencies of the system are the eigenvalues of the state matrix $[A]$. The state-space formulation of the linear model has been implemented in Matlab, and a Simulink interface has been created for running the simulations in the time domain.

THE NON-LINEAR MODEL

The non-linear model ([Figure 4](#)) is based on the implementation of the dynamics of the individual components of the vehicle, the specific models are then connected to each other to form the complete model. As a consequence, the model has a modular design, which permits the simulation of different vehicle layouts and maneuvers, as functions of the specific case-study. In particular, the non-linear model:

- Permits to simulate the non-linearities of the clutch damper, in terms of non-linear torsion characteristic of the damper and in terms of hysteresis (variable with clutch damper torsion angle);
- Permits to simulate the non-linearities of the tire, through the adoption of Pacejka Magic Formula model and the relaxation length model;
- Simulates the load transfer within 2-axle and multi-axle vehicles, which affects the dynamic load on the driven wheels, and as a consequence the tire longitudinal slip stiffness and vehicle drivability during tip-in tests;
- Considers the non-linearity of the aerodynamic drag resistance and tire rolling resistance, which are applied to the single wheels of the vehicle;
- Allows the simulation of the gearshifts (even if it is not dealt with in this article), and as a consequence takes into account the engagement and disengagement process of the clutch and the synchronization dynamics. Friction phenomena within the clutch are simulated through a hyperbolic tangent model;



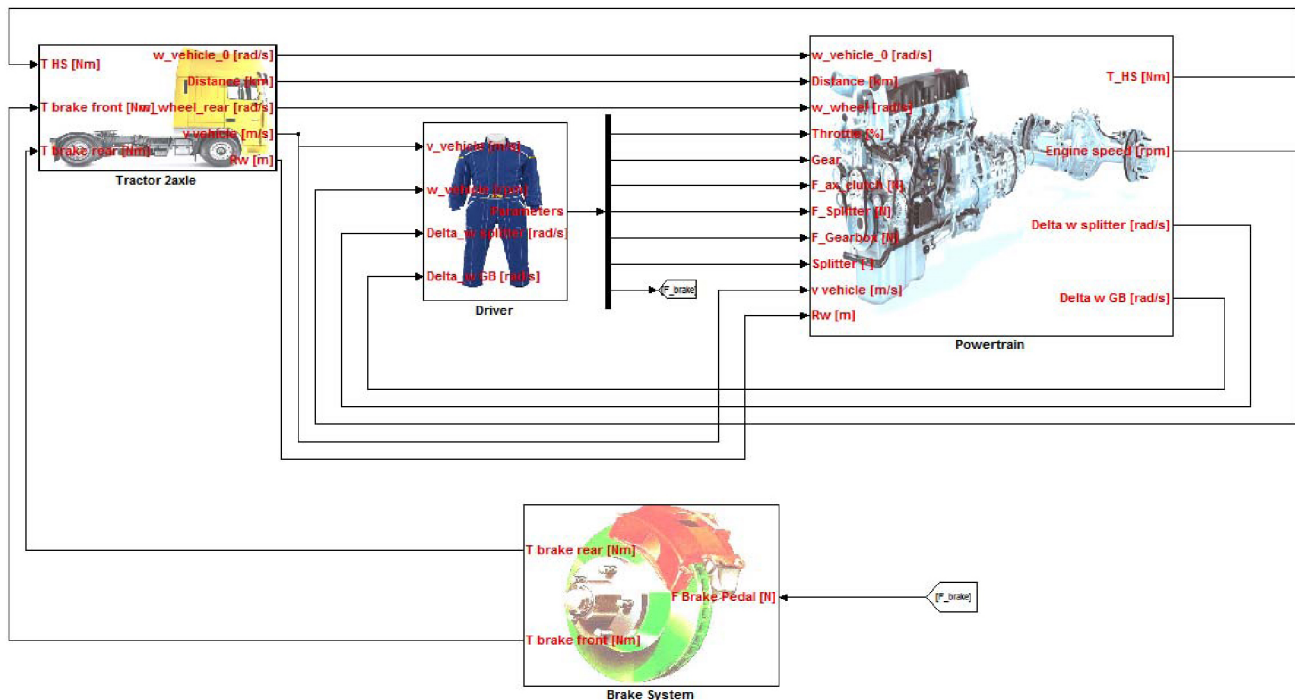


Figure 4. Non-linear model implementation in Simulink

- Includes a driver model capable of following an assigned velocity profile, and automatically performing the proper upshifts/downshifts.

SIMULATION RESULTS

Linear Model

The linear model has been adopted for the simulation of tip-in tests (a step input on the gas pedal), the analysis of the natural frequencies, and the Bode plots, for different values of the following parameters:

- Half-shaft stiffness;
- Driveshaft stiffness;
- Clutch damper stiffness;
- Engine inertia;
- Vehicle inertia.

The vehicle data of an existing 2-axle truck in unladen conditions have been adopted for the tests (unless differently specified, this is the reason for the high values of longitudinal acceleration). The vehicle is a 6 ton (in unladen conditions) truck capable of a maximum payload of 14 tons.

Figures 5,6,7,8 show vehicle response during a tip-in test in the lowest gear (1st gearbox ratio and 1st splitter ratio) and in the highest gear (4th gearbox ratio and 2nd splitter ratio). They will be called respectively gear 1 and gear 8 for simplicity within the paper. The results in Figures 5,6,7,8 are

plotted in terms of angular speeds and accelerations of the different components of the driveline, multiplied by the proper gear ratios, so that all results are in respect of the engine. In this way, the overlap between the graphs summarizes the effect due to the transmission dynamics, independently of the selected gear ratio. From Figures 5 and 6, it is evident that the highest gear is characterized by a higher frequency of the oscillations and a significant damping of the oscillations along the transmission. In particular, the amplitudes of engine oscillations are similar for the two gears, but for the higher gear the amplitude of vehicle oscillations is lower. From the figures, it is evident that a significant oscillatory dynamics between the differential and the wheels is induced by the torsion of the half-shafts. For this specific set of vehicle data, the dynamics of the driveshaft is not significant. The damping effect of the tire is particularly important for the highest gear (equivalent torsion dynamics between the wheel and the vehicle). Finally, the engine speed limiter is included in the engine model and in first gear the effect of the engine speed limiter is perceivable for a tip-in with torque demand varying from 0 to 100%.

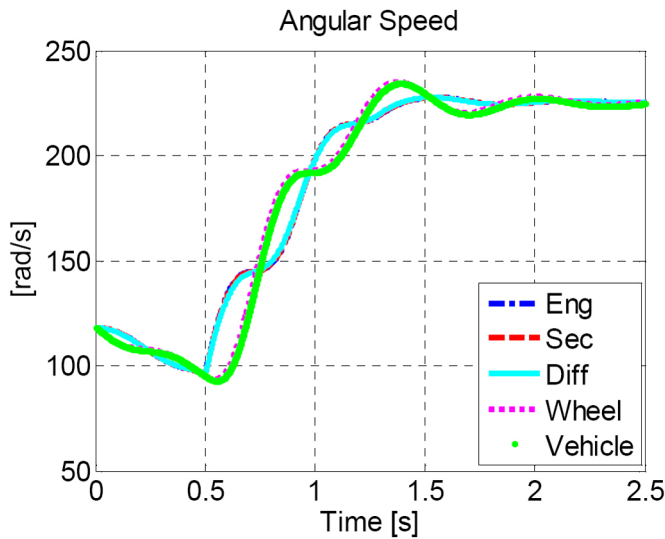


Figure 5. Angular speeds in gear 1 (tip-in, 0-100% of torque demand; Eng: engine speed, Sec: secondary shaft speed, Diff: differential speed, Wheel: wheel speed, Vehicle: vehicle speed)

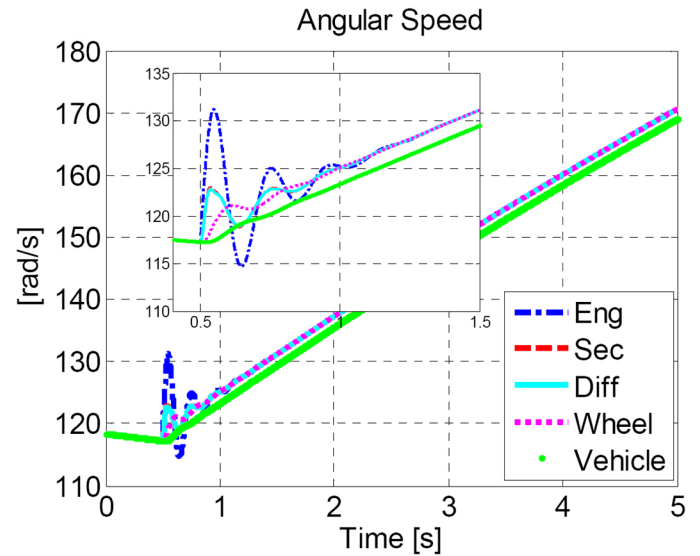


Figure 7. Angular speed in gear 8 (tip-in, 0-100% of torque demand; Eng: engine speed, Sec: secondary shaft speed, Diff: differential speed, Wheel: wheel speed, Vehicle: vehicle speed)

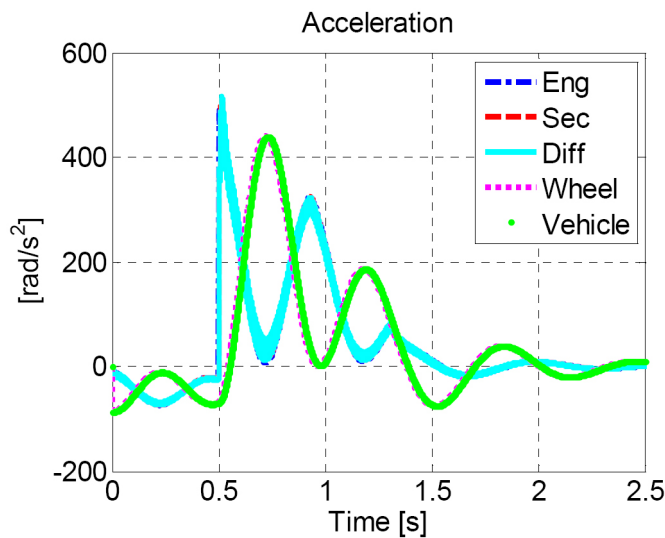


Figure 6. Angular accelerations in gear 1 (tip-in, 0-100% of torque demand; Eng: engine acceleration, Sec: secondary shaft acceleration, Diff: differential acceleration, Wheel: wheel acceleration, Vehicle: vehicle acceleration)

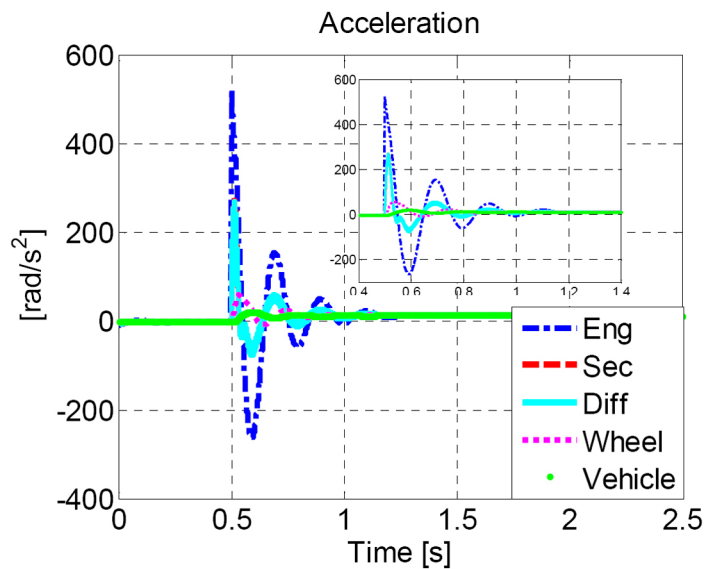


Figure 8. Angular acceleration in gear 8 (tip-in, 0-100% of torque demand; Eng: engine acceleration, Sec: secondary shaft acceleration, Diff: differential acceleration, Wheel: wheel acceleration, Vehicle: vehicle acceleration)

The following paragraphs contain an overview of the main simulation results of the sensitivity analysis carried out on the main vehicle and transmission parameters.

Half-shaft Stiffness

The effect of different values of half-shaft stiffness k_{hs} is shown in Figures 9, 10, 11, 12, 13, 14. Figures 9 and 10 show