

Introduction of New Concept U^*_{sum} for Evaluation of Weight-Efficient Structure

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ABSTRACT

A new index U^*_{sum} for evaluating load path dispersion is proposed, using a structural load path analysis method based on the concept of U^* , which expresses the connection strength between a load point and an arbitrary point within the structure. U^*_{sum} enables the evaluation of the load path dispersion within the structure by statistical means such as histograms and standard deviations. Different loading conditions are applied to a body structure, and the similarity of the U^*_{sum} distributions is evaluated using the direction cosine and U^*_{sum} 2-dimensional correlation diagrams. It is shown as a result that body structures can be macroscopically grasped by using the U^*_{sum} distribution rather than using the stress distribution. In addition, as an example, the U^*_{sum} distribution of torsion loading condition is shown to comprehensively include characteristics of the U^*_{sum} distribution of other loading conditions.

INTRODUCTION

Recently, automobile weight reduction is increasingly demanded from the viewpoint of reducing the load on the global environment. Conventional concepts for reducing the weight of automotive body structures include stress dispersion and load dispersion. These approaches are aimed to reduce weight and increase stiffness and strength by broadly dispersing the load evenly throughout the structure. For this reason, it is important to understand whether the load is being dispersed within the structure. FEM methods that use stress and strain^[1,2] have already been presented for this

purpose. When applied to automotive body structures, these methods are affected by local structural characteristics such as joints, holes and bend sections, and it is not easy to macroscopically grasp load transfer paths and load dispersion states. In addition, a load transfer path analysis method that uses free-body diagrams^[3] has also been presented. This method enables evaluation of how loads are transferred between parts, but it is a challenge to understand the load paths of statistically indeterminate structure where components slightly deform out of plane, or structure which comprise of parts with complex curved surfaces, such as automotive bodies.

Another method for analyzing load transfer paths uses the index U^* ^[4]. U^* is an index that can numerically express the strength of the connection between a load point and an arbitrary point on the structure, based on the internal stiffness between the two points. This index enables not only comprehensive and visual understandings of the principal load paths, but also feasibility analysis of the detailed structure by tracing these load paths. However, load dispersing structures have countless load paths, which makes it a challenge to simply apply U^* , so a new index that can numerically evaluate the load dispersion state is needed.

Therefore, this paper defines a new index U^*_{sum} that is an expansion of U^* , and attempts to use this new index to quantitatively evaluate the load dispersion state of a structure. The load dispersion state of a structure is evaluated by statistically analyzing the uniformity of the U^*_{sum} values using the standard deviation and histogram of the structure's U^*_{sum} distribution. In addition, the U^*_{sum} distribution is



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calculated for multiple loading conditions. The similarity of the distributions under different loading conditions is evaluated using the direction cosine, and the higher similarity of U^*_{sum} compared to that of stress distribution is confirmed. Furthermore, U^*_{sum} 2-dimensional correlation diagram, which U^*_{sum} values of two loading conditions are plotted respectively on the vertical and horizontal axes, is proposed and used to verify the similarity of each loading condition in detail.

MATERIAL AND METHODS

LOAD PATH ANALYSIS INDEX U^*

First, the definition of the load transfer path analysis index U^* ^[4] used in this paper will be described. Figure 1(a) shows a model expression of a structure. Point A is the load point, point B is the support point, and point C is an arbitrary point within the structure. Equation (1) shows the mutual relationship of the internal stiffness between these three points.

$$\begin{Bmatrix} p_A \\ p_B \\ p_C \end{Bmatrix} = \begin{bmatrix} K_{AA} & K_{AB} & K_{AC} \\ K_{BA} & K_{BB} & K_{BC} \\ K_{CA} & K_{CB} & K_{CC} \end{bmatrix} \begin{Bmatrix} d_A \\ d_B \\ d_C \end{Bmatrix} \quad (1)$$

Here, p and d are 3-dimensional vectors that represent the load and displacement at each point and K is the tensor that represents the internal stiffness which corresponds to the spring constant between two points. Internal stiffness here indicates the mutual relationship between two arbitrary points within the structure, and expresses only the relationship between the external force and the displacement of each point, both having three degrees of freedom. Equation (1) expresses the behavior of the overall structure, and K is a unique value determined according to the structure, regardless of the loading condition. Since $d_B = 0$, the strain energy U applied to this structure can be expressed as follows.

$$\begin{aligned} U &= \frac{1}{2} p_A \cdot d_A \\ &= \frac{1}{2} (K_{AA} d_A + K_{AC} d_C) \cdot d_A \end{aligned} \quad (2)$$

On the other hand, Fig. 1(b) shows the same structure with the same displacement d_A applied as in Fig. 1(a), but constrained at an arbitrary point C. Since $d_B = d_C = 0$, the strain energy U' of this structure is expressed as follows.

$$\begin{aligned} U' &= \frac{1}{2} p'_A \cdot d_A \\ &= \frac{1}{2} (K_{AA} d_A + K_{AC} d_C) \cdot d_A \end{aligned} \quad (3)$$

Here, the index U^* is defined as follows.

$$U^* \equiv 1 - \frac{U}{U'} \quad (4)$$

From Eq. (2) and (3), this can be expressed as follows.

$$U^* = \left(1 - \frac{2U}{(K_{AC} d_C) \cdot d_A} \right)^{-1} \quad (5)$$

That is to say, the value of U^* depends only on the stiffness K_{AC} between the load point A and the arbitrary point C, and indicates the strength of the connection between the load point and the arbitrary point. U' is always larger than U , so U^* takes a value between 0 and 1, and is 1 at the load point, and 0 at the support point.

The force loaded on the structure is thought to be transferred to the support point through locations that have strong connection with the load point. In other words, the load path should stretch from the load point toward the support point passing through the locations with the highest U^* values, that is to say the ridgeline formed by the contour lines of the structure's U^* distribution obtained by numerical calculations. Therefore, the path that follows the ridgeline of U^* distribution to reach the support point can be defined as the load path. Figure 2(a) shows the U^* distribution of a flat plate with hole, with support points on one end and enforced displacement is applied to the other end in the tensile direction. Figure 2(b) shows the stress distribution for the same conditions. The stress distribution exhibits stress concentrations at the load points and around the periphery of the hole, which makes load path analysis difficult. In contrast, the U^* distribution has distribution ridgelines that skirt the hole, enabling the main load paths from the load points to be drawn.

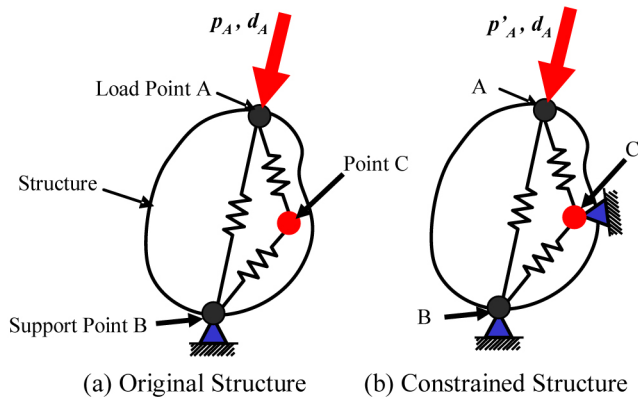


Fig. 1. Schematic Diagram of U^* Definition

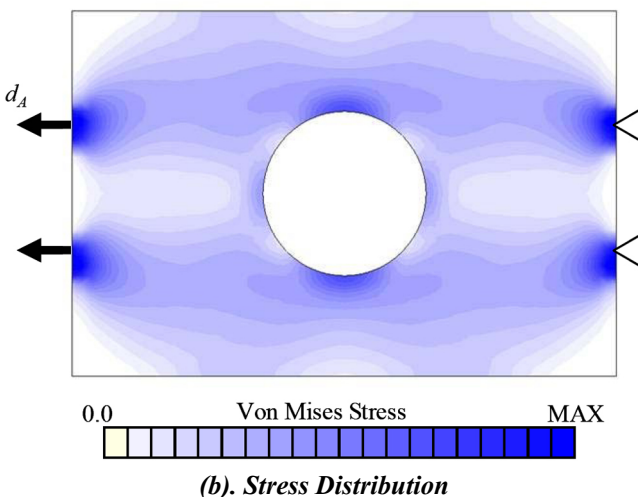
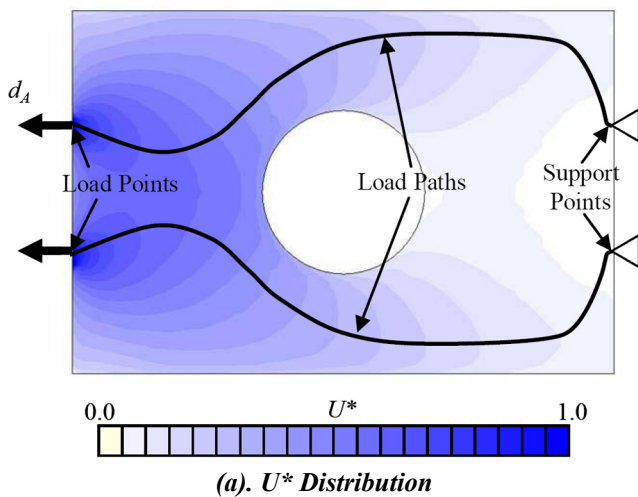


Fig. 2. U^* and Stress Distribution of Plate with Hole

INTRODUCTION OF U^* SUM

As mentioned in the introduction, automotive body structures have countless load paths of various degrees, and there are few cases where only a single load path is determined. This

makes it a challenge to evaluate load dispersing structures by simply using U^* , which can only visualize the principal load paths. Therefore, a new index is proposed and an attempt is made for the evaluation of a load dispersing structure. U^* is an index that only expresses the strength of connection with the load point, so the strength of connection with the support point is not considered. On the other hand, the strength of connection with the support point can be evaluated by switching the load point and support point and calculating U^* in the same manner. Locations that have strong connection with both the load point and the support point can be considered locations that function as load paths. Therefore, it is thought that whether a location functions as a load path can be determined by overlapping the U^* distribution from the load point and the U^* distribution from the support point. Figure 3(a) shows a schematic drawing of a simple 1-dimensional spring model and the new load path evaluation index U^*_{sum} . U^*_1 is the U^* value calculated with point A as the load point and point B as the support point. U^*_2 is similarly calculated with the load and support points exchanged. The U^*_{sum} is defined by the following equation.

$$U^*_{sum} = U^*_1 + U^*_2 \quad (6)$$

In case of a 1-dimensional spring, each point receives the force equally and the load path is distributed evenly over the entire structure, so U^*_{sum} would be a fixed value of 1.0 as shown in Fig. 3(a). Figure 3(b) shows the histogram for this U^*_{sum} .

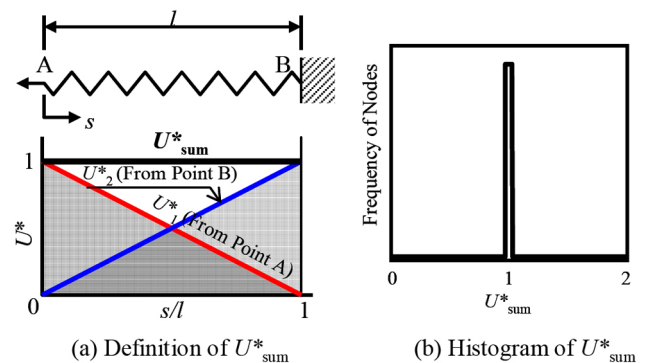


Fig. 3. Schematic Diagram of U^*_{sum}

INTRODUCTION OF U^* SUM 2-DIMENSIONAL CORRELATION DIAGRAM

Consider the case where the structure is given an external force with different loading conditions. If a similarity of load path distribution can be found under several different loading conditions, then the macroscopic characteristic of structure

can be grasped by examining load path distribution of fewer loading conditions. Furthermore, the location where load path needs modification will be similar between load cases, so the location to take measure can be easily specified. Therefore, an evaluation method that uses the U_{sum}^* 2-dimensional correlation diagram is proposed as shown in Fig. 4 to identify the similarity and difference in the amounts of load borne by each component of a structure under two different loading conditions. The horizontal and the vertical axes of the diagram each express the U_{sum}^* values for the two different loading conditions 1 and 2. The color shade represents the node distribution frequency. The correlation diagram is divided into a grid of fixed width, and the distribution frequency is defined as the value obtained by dividing the number of nodes n within each grid square by the total number of nodes N . In addition, the U_{sum}^* histogram of each load condition is shown next to the respective axis.

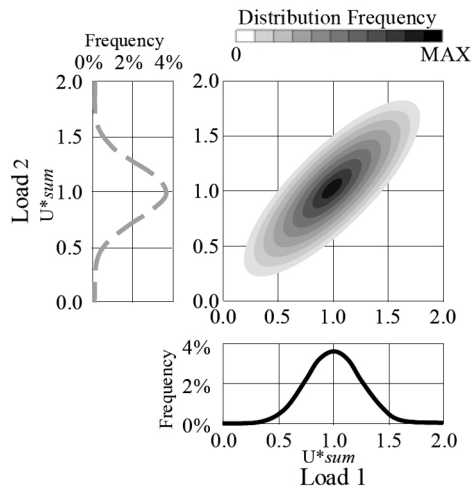


Fig. 4. U_{sum}^* 2-Dimensional Correlation Diagram and Histograms

Quantification based on the distribution shape of the 2-dimensional correlation diagram is necessary for the evaluation of the load dispersion state. Therefore, principal component analysis using the covariance tensor σ_{ij} shown in Eq. 7 is applied to quantitatively evaluate the distribution shape.

$$\sigma_{ij} = \frac{1}{N} \begin{bmatrix} \sum (U_{1n} - \mu_1)^2 & \sum (U_{1n} - \mu_1)(U_{2n} - \mu_2) \\ \sum (U_{2n} - \mu_2)(U_{1n} - \mu_1) & \sum (U_{2n} - \mu_2)^2 \end{bmatrix} \quad (7)$$

Here, subscripts 1 and 2 represent the loading conditions 1 and 2, U_{1n} and U_{2n} indicate the U_{sum}^* values at each node, μ_1 and μ_2 indicate the U_{sum}^* average values of all nodes, and N indicates the total number of nodes. σ_{ij} is the 2-dimensional

second rank symmetric tensor, so there are two eigenvalues λ and eigenvectors $\mathbf{x}$ that satisfy Eq. (8).

$$[\sigma]\{\mathbf{x}\} = \lambda\{\mathbf{x}\} \quad (8)$$

Labeling the two eigenvalues in Eq. (8) as λ_1 and λ_2 and making $\lambda_1 > \lambda_2$, λ_1 becomes the first principal component. Here, λ_1 represents the dispersion of U_{sum}^* distribution in the direction of the corresponding eigenvector $\mathbf{x}_1$. Figure 5 shows the U_{sum}^* 2-dimensional correlation diagram and the eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$. The eigenvector $\mathbf{x}_1$ indicates the direction of greatest dispersion of the distribution in the 2-dimensional correlation diagram, and represents the distribution trend. When the eigenvector $\mathbf{x}_1$ is oriented towards the upper right of the 2-dimensional correlation diagram as shown in Fig. 5(a), it indicates that locations with a large U_{sum}^* value under loading condition 1 also tend to have a large U_{sum}^* value under loading condition 2, hence showing similarity in the load path distributions within the structure for both loading conditions. This also means that locations that do not contribute much to the load path, i.e. where U_{sum}^* is small, are also shared, so it is thought that the load dispersion of the overall structure can be enhanced by implementing measures for those locations. On the other hand, when the eigenvector $\mathbf{x}_1$ of a structure is oriented towards the lower right of the 2-dimensional correlation diagram as shown in Fig. 5(b), this means that even when the U_{sum}^* value is high under one loading condition, it tends to be low under the other condition. That is to say, altering the loading condition greatly changes the load path distribution in the structure, and the components with a high degree of contribution to the load paths also change. This means that the structure is not in appropriate state to make effort in load dispersion under both loading conditions. In addition, when the eigenvector $\mathbf{x}_1$ is oriented in the horizontal or vertical direction as shown in Fig. 5(c), it indicates that there is little U_{sum}^* variance under one of the loading conditions, where the load is dispersed throughout the entire structure, but there is little load path dispersion under the other condition. These three conditions can be expressed quantitatively by the angle between the direction of eigenvector $\mathbf{x}_1$ and the horizontal axis.

The eigenvalues λ_1 and λ_2 represent the dispersion in the directions of the corresponding eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$. The relationship between these eigenvalues and their respective standard deviations s_1 and s_2 is given by the following equation.

$$s_i = \sqrt{\lambda_i} \quad (i=1,2)$$