

Free-Form Optimization Method for Designing Automotive Shell Structures

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Toyota Technological Institute

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ABSTRACT

In this paper, we present a parameter-free, or a node-based optimization method for finding the smooth optimal free-form of automotive shell structures, including global and local curvature distributions such as beads or embossed ribs. The design problems dealt with in this paper involve a stiffness problem. Stiffness is maximized using the compliance as an objective functional. The optimum design problem is formulated as a distributed-parameter, or non-parametric, shape optimization problem under the assumptions that the shell is varied in the normal direction to the surface and the thickness is constant. The shape gradient function and the optimality conditions are then theoretically derived. The optimum free-form, or optimal curvature distribution, is determined by applying the derived shape gradient function in the normal direction to the shell surface as pseudo external forces to vary the surface and to minimize the objective functional. With this method, the smooth optimal global free-form or the smooth optimal local beads of shell structures in automotive body and chassis parts is created without any shape design parameterization. The validity and practical utility of this method were verified through several design examples. We confirmed that the obtained structures were changed from the initial bending-carrying structures to membrane-carrying ones.

INTRODUCTION

Shell structures are widely used in automotive body, chassis and trim components, among other applications. Fabricated of steel, aluminum or various plastic materials, they can span large areas and have high load-carrying capacity in spite of their thinness. In the design of automotive shell structures, it is strongly required to find the optimal shape in order to make the structure light using the minimum amount of material,

while satisfying various mechanical characteristics and functions. Design variables that determine the shape of a shell structure include the in-plane shape variation that moves in the tangential direction to the surface and/or the out-of-plane shape variation that moves in the normal direction to the surface. In our previous works, we considered the in-plane shape variation in order to maintain the curvatures of the given initial shape, a problem that is often encountered in the final design stage [1][2]. In this paper, the out-of-plane direction is considered and a free-form optimization method is presented for designing the smooth optimal free-form, including local beads (i.e., optimal curvature distribution) of automotive shell structures, without any shape design parameterization. This is often encountered in the early design stage and is important in improving mechanical characteristics substantially.

Most of the literature regarding free-form shells has been based on parametric methods such as the basis vector method in which shapes must be parameterized in advance. Choi and Kim discuss various shape design parameterization methods in their monograph [3]. Ramm et al. used the Bezier function [4], as did Rao and Hinton for shape parameterization [5]. While shape parameterization has some advantages of reducing the number of design variables and of being compatible with computer-aided design (CAD) tools, the process itself is troublesome for designers (especially the basis vector method) and designers need parameterization knowledge and experience. Additionally, the optimal shape obtained is strongly influenced by the parameterization process. In contrast, only a limited number of papers seem to deal with the non-parametric, or the node-based shape optimization of free-form shell structures. The node-based method has needed to solve the problem of jagged boundaries. Bletzinger *et al.* proposed a parameter-free method with a regularization based on filtered sensitivities to

eliminate that problem [6]. Scherer *et al.* proposed a fictitious energy approach to constrain the shape change as a new regularization technique [7]. We have developed a gradient method in a Hilbert space as a non-parametric shape optimization method which does not require shape design parameterization and makes it possible to solve the problem of jagged boundaries. This method is called the traction method and has been applied to various shape optimization problems of solid and shell structures involving in-plane shape variation [1][2][8][9]. Recently, the author improved this method for out-of-plane shape variation and applied it to the basic free-form optimization problem of shell structures [10].

This paper presents the free-form optimization method that we have developed and applied to shell structures in automotive body and chassis parts. The design problems dealt with in this paper concern a stiffness problem. Using the compliance as an objective functional, stiffness is maximized under volume and state equation constraints. The optimum design problem is formulated as a non-parametric shape optimization problem under the assumptions that the shell is varied in the normal direction to the surface and the thickness is constant. The shape gradient function and the optimality conditions are then theoretically derived using the material derivative method, the Lagrange multiplier method and the adjoint variable method. The optimal free-form is determined by applying the derived shape gradient function to the improved traction method without any shape design parameterization.

In the following section, domain variation for free-form design and the governing equation of the shell structure will be described. Next, the formulation of the design problem and the derivation of the shape gradient function will be presented. After explaining the free-form optimization method in detail, the validity and practical utility of this method for finding the optimal free-form and creating optimal local beads of automotive shell structures will be verified through several design examples. It will be also demonstrated that the obtained structures have a membrane-carrying characteristic as expected

DOMAIN VARIATION FOR FREE-FORM DESIGN OF SHELL STRUCTURES

A technique for representing domain variation using the speed method will be discussed briefly before formulating the shape optimization problem. A detailed explanation of this technique may be found in reference [11] or [12].

As shown in Fig. 1-(a) and (b) and Eqs. (1), (2), (3), consider that a shell having an initial bounded domain $\Omega \subset \mathbb{R}^3$ (boundary of $\partial\Omega$), mid-area A (boundary of ∂A) and side

surface S undergoes domain variation in the out-of-plane direction to the surface such that its domain and boundary become Ω_s , A_s and S_s , respectively. The notation dA expresses a small area. The subscript s indicates the iteration history of domain variation. It is assumed that the plate thickness h is kept constant under the domain variation.

$$\Omega = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid (x_1, x_2) \in A \subset \mathbb{R}^2, x_3 \in (-h/2, h/2)\} \quad (1)$$

$$\Omega = A \times \left(-\frac{h}{2}, \frac{h}{2}\right), \quad S = \partial A \times \left(-\frac{h}{2}, \frac{h}{2}\right) \quad (2)(3)$$

It is assumed that the mapping of the local coordinate system $(x_1, x_2, 0)$, which gives the position of the mid-area of the plate, to the global coordinate system (X_1, X_2, X_3) , i.e., $\phi : (x_1, x_2, 0) \in \mathbb{R}^2 \mapsto (X_1, X_2, X_3) \in \mathbb{R}^3$, is piecewise smooth. The infinitesimal variation of the domain can be given by

$$T_{s+\Delta s}(\mathbf{X}) = T_s(\mathbf{X}) + \Delta s \mathbf{V} + O(|\Delta s|), \quad (4)$$

where the design velocity field (i.e., the amount of domain variation per iteration) $\mathbf{V}(x) = \partial T_s / \partial s(X)$, $x \in A_s$, $X \in A$ is given as a Euler derivative of $T_s(\mathbf{X})$. The optimal design velocity field $\mathbf{V}$ is determined by the traction method which will be explained later.

In order to derive the shape gradient function (i.e., shape sensitivity function), we use the following formula of the material derivative. When the objective functional J is given as a domain integral of the distributed function ϕ_s ,

$$J = \int_{\Omega_s} \phi_s d\Omega \quad (5)$$

The Euler derivative, or material derivative $\dot{J}$, is given by the following expression.

$$\dot{J} = \int_{\Omega} \phi' d\Omega + \int_{\Gamma} \phi V_n d\Gamma, \quad (6)$$

where $V_n = n_i V_i$. The vector $\mathbf{n}$ is an outward normal unit vector to the surface. $(\bullet)'$ indicates a shape derivative (Lagrange derivative). When the objective functional J is given as a boundary integral of the distributed function ϕ_s ,

$$J = \int_{\Gamma_s} \phi_s d\Gamma, \quad (7)$$

the Euler derivative $\dot{J}$ is given by



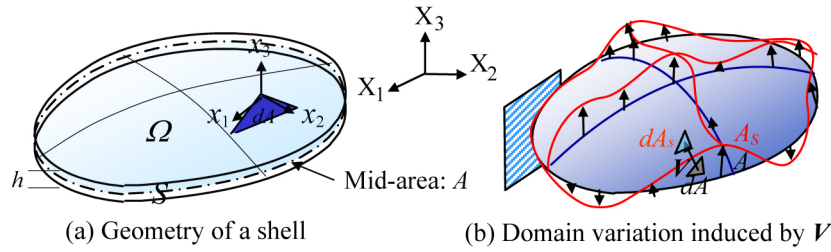


Figure 1. Shape variation in the out-of-plane direction to a shell surface

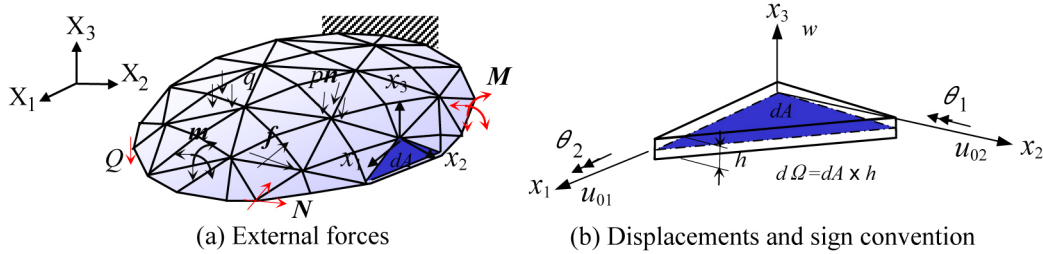


Figure 2. A shell as an assembly of planar shell elements

$$\dot{J} = \int_{\Gamma} \{ \phi' + (\phi_i n_i + \phi H) V_n \} d\Gamma, \quad (8)$$

where H expresses twice the mean curvature of Γ in $\mathbb{R}^3$.

GOVERNING EQUATION FOR A SHELL AS AN ASSEMBLY OF PLANAR ELEMENTS

It is assumed for simplicity that a shell structure occupying a bounded domain $\Omega \subset \mathbb{R}^3$ is a set of piecewise flat surfaces as shown in Fig. 2-(a).

Various finite elements have been developed for conducting a finite element analysis (TEA) of shell structures. While linear or quadrilateral triangular shell elements are often used to discretize free-form shell structures, planar triangular shell elements were used in this work. The Mindlin-Reissner plate theory is applied concerning plate bending, and coupling of the membrane stiffness and bending stiffness is ignored. Using the sign convention in Fig. 2-(b), the displacement expressed by the local coordinates $\mathbf{u} = \{u_i\}_{i=1,2,3}$ is considered by dividing it into the displacement in the in-plane direction $\{u_\alpha\}_{\alpha=1,2}$ and the displacement in the out-of-plane direction u_3 . The subscripts of the Greek letters are expressed as $\alpha = 1, 2$, and the tensor subscript notation with respect to $\alpha = 1, 2$ uses Einstein's summation convention and a partial differential notation for the spatial coordinates $(\cdot)_{,i} = \partial(\cdot)/\partial x_i$. The Mindlin-Reissner plate theory posits that

$$\sigma_{33} = 0, \quad (9)$$

$$u_\alpha(x_1, x_2, x_3) \equiv u_{0\alpha}(x_1, x_2) - x_3 \theta_\alpha(x_1, x_2), \quad (10)$$

$$u_3(x_1, x_2, x_3) \equiv w(x_1, x_2), \quad (11)$$

where $\{u_{0\alpha}\}_{\alpha=1,2}$, w and $\{\theta_\alpha\}_{\alpha=1,2}$ express the in-plane displacement, out-of-plane displacement and rotational angle of the mid-area of the plate, respectively. $\{\sigma_{ij}\}_{i,j=1,2,3}$ and $\{\varepsilon_{ij}\}_{i,j=1,2,3}$ are the stress and strain expressed by the local coordinates. The strain tensor is defined as

$$\varepsilon_{\alpha\beta} \equiv u_{(\alpha,\beta)} \equiv u_{0(\alpha,\beta)} - x_3 \theta_{(\alpha,\beta)}, \quad (12)$$

$$\varepsilon_{\alpha 3} \equiv u_{(\alpha,3)} \equiv \frac{-\theta_\alpha + w_{,\alpha}}{2}, \quad (13)$$

where $\theta_{(\alpha,\beta)}$ and $u_{0(\alpha,\beta)}$ express the curvatures and the in-plane strains $\varepsilon_{0\alpha\beta}$ which are defined by the following equations.

$$\kappa_{\alpha\beta} \equiv \theta_{(\alpha,\beta)} \equiv \frac{1}{2}(\theta_{\alpha,\beta} + \theta_{\beta,\alpha}), \quad (14)$$

$$\varepsilon_{0\alpha\beta} \equiv u_{0(\alpha,\beta)} \equiv \frac{1}{2}(u_{0\alpha,\beta} + u_{0\beta,\alpha}). \quad (15)$$

Then, the weak form of the equilibrium equation relative to $(\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}) \in U$ can be expressed as Eq. (16) by substituting Eqs. (9), (10), (11) into the variational equation (i.e., weak form) of the three-dimensional elastic theory, eliminating ε_{33}

and considering the relationship $\int_{\Omega} (\bullet) d\Omega = \int_A \int_{-h/2}^{h/2} (\bullet) dz dA$. As shown in Fig. 2(a), the external forces acting relative to the local coordinate system (x_1, x_2) on the domain A are defined as follows: an out-of-plane load q per unit area, an in-plane load $\mathbf{f} = \{f_\alpha\}_{\alpha=1,2}$ out-of-plane moment $\mathbf{m} = \{m_\alpha\}_{\alpha=1,2}$ per unit area and a pressure in the normal direction to the surface $p\mathbf{n}$. An in-plane load $\mathbf{N} = \{N_\alpha\}_{\alpha=1,2}$ per unit length, a shearing force $\mathbf{Q}$ per unit length and a bending moment $\mathbf{M} = \{M_\alpha\}_{\alpha=1,2}$ per unit length act relative to the local coordinate system (x_1, x_2) at the boundary $(\subset \partial A)$.

$$a((\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}), (\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}})) = l((\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}})), \quad \forall (\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}}) \in U, \quad (16)$$

where $(\bar{\bullet})$ expresses a variation. In addition, the bilinear symmetric form $a(\bullet, \bullet)$ and the linear form $l(\bullet)$ are respectively defined as shown below.

$$\begin{aligned} a((\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}), (\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}})) = & \int_{\Omega} \{C_{\alpha\beta\gamma\delta}(u_{0\alpha,\beta} - x_3\theta_{\alpha,\beta})(\bar{u}_{0\gamma,\delta} - x_3\bar{\theta}_{\gamma,\delta}) + C_{\alpha\beta}^S(w_{,\alpha} - \theta_{\alpha})(\bar{w}_{,\beta} - \bar{\theta}_{\beta})\} d\Omega \\ & + \int_A \{c_{\alpha\beta\gamma\delta}^B\theta_{(\gamma,\delta)}\bar{\theta}_{(\alpha,\beta)} + c_{\alpha\beta\gamma\delta}^M u_{0\gamma,\delta}\bar{u}_{0\alpha,\beta} + kc_{\alpha\beta}^S(w_{,\beta} - \theta_{\beta})(\bar{w}_{,\alpha} - \bar{\theta}_{\alpha})\} dA, \end{aligned} \quad (17)$$

$$l((\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}})) = \int_A (f_\alpha \bar{u}_{0\alpha} - m_\alpha \bar{\theta}_\alpha + q\bar{w} + p\bar{w}) dA + \int_{\partial A} (N_\alpha \bar{u}_{0\alpha} ds - M_\alpha \bar{\theta}_\alpha + Q\bar{w}) ds, \quad (18)$$

where $\{C_{\alpha\beta\gamma\delta}\}_{\alpha,\beta,\gamma,\delta=1,2}$ and $\{C_{\alpha\beta}^S\}_{\alpha,\beta=1,2}$ express an elastic tensor including bending and membrane components, and an elastic tensor with respect to the shearing component, respectively. $\{c_{\alpha\beta\gamma\delta}^B\}_{\alpha,\beta,\gamma,\delta=1,2}$, $\{c_{\alpha\beta}^S\}_{\alpha,\beta=1,2}$ and $\{c_{\alpha\beta\gamma\delta}^M\}_{\alpha,\beta,\gamma,\delta=1,2}$ express an elastic tensor with respect to bending, shearing and membrane stress, respectively. In addition, k denotes a shear correction factor (assuming $k=5/6$). It will be noted that U in Eq. (16) is given by the following equation.

$$U = \{(u_{01}, u_{02}, w, \theta_1, \theta_2) \in (H^1(A))^5 \mid \text{satisfying the restraint conditions}\}, \quad (19)$$

where H^1 is the Sobolev space of degree 1.

FORMULATION OF STIFFNESS DESIGN PROBLEM

With the aim of maximizing the stiffness of shell structures, we use the compliance as an index of stiffness. A compliance minimization problem subject to a volume constraint and the state equation (16) is formulated as a non-parametric, or a distributed-parameter shape optimization problem. The design variable to be determined is the design velocity field $\mathbf{V}$, or $A_s (=A + \Delta s \mathbf{V})$.

$$\text{Given } A \quad (20)$$

$$\text{find } \mathbf{V} \text{ (or } A_s) \quad (21)$$

$$\text{that minimize } l(\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}) \quad (22)$$

$$\text{subject to Eq.(16),} \quad (23)$$

$$M (= \int_A h dA) \leq \hat{M}, \quad (24)$$

where M and $\hat{M}$ denote the volume and its constraint value, respectively.

Letting $(\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}})$ and Λ denote the Lagrange multiplier for the state equation and the volume constraint, respectively, the Lagrangian functional L associated with this problem can be expressed as

$$\begin{aligned} L(\Omega, (\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}), (\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}}), \Lambda) = & l(\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}) + l(\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}}) - a((\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}), (\bar{\mathbf{u}}_0, \bar{\mathbf{w}}, \bar{\boldsymbol{\theta}})) + \Lambda(M - \hat{M}). \end{aligned} \quad (25)$$

For the sake of simplicity here, it is assumed that the sub-boundaries on which the non-zero boundary forces $\mathbf{N}$, $\mathbf{Q}$, $\mathbf{M}$ act do not vary (i.e., $\mathbf{V} = 0$) and the surface forces $\mathbf{f}$, $\mathbf{m}$, q , $p\mathbf{n}$ do not vary with regard to the space (i.e., $\mathbf{f}' = \mathbf{m}' = q' = p' = 0$). Then, using the design velocity field $\mathbf{V}$ to represent the amount of domain variation, the derivative $\dot{L}$ of the domain variation of the Lagrange functional L can be expressed as

$$\begin{aligned} \dot{L} = & l(\mathbf{u}'_0, \mathbf{w}', \boldsymbol{\theta}') + l(\bar{\mathbf{u}}'_0, \bar{\mathbf{w}}', \bar{\boldsymbol{\theta}}') - a((\mathbf{u}'_0, \mathbf{w}', \boldsymbol{\theta}'), (\bar{\mathbf{u}}'_0, \bar{\mathbf{w}}', \bar{\boldsymbol{\theta}}')) \\ & - a((\mathbf{u}_0, \mathbf{w}, \boldsymbol{\theta}), (\bar{\mathbf{u}}'_0, \bar{\mathbf{w}}', \bar{\boldsymbol{\theta}}')) + \Lambda'(M - \hat{M}) + l_G(\mathbf{V}), \quad \mathbf{V} \in C_\Theta \end{aligned} \quad (26)$$