

An Improved Slip-Based Model for Tire-Snow Interaction

2011-01-0188

Published
04/12/2011Jonah Lee
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doi:10.4271/2011-01-0188

ABSTRACT

The performance of a vehicle traversing over natural snowy terrain depends on the geometric and material properties of snow; these properties can change spatially and temporally depending on environmental conditions. The author's research group in recent years has been developing physically-based tire-snow interaction models that are snow-depth and slip dependent; these efforts are in contrast with previous models that are mostly empirical. An important element in tire-terrain interaction models is the pressure-sinkage relationship typically modeled empirically using plate indentation tests. Recently, a new mechanics-based indentation model for snow has been developed which significantly improves and simplifies the prediction of pressure-sinkage relationship. In addition, new vehicle-snow interaction slip-dependent test data have been obtained by the author's research group for the validation of vehicle-snow interaction models. In this paper, the new indentation model for snow is incorporated into a tire-snow interaction model; the results of the model are compared with the new test data. Parametric studies of the model indicate that the motion resistance and traction when normalized against the normal force on the tire decrease with the increase of normal force; normalized motion resistance and traction decrease with the decrease of snow depth. Softer snow leads to larger motion resistance. The minima and maxima of normalized drawbar pull and torque as measured by an instrumented vehicle are compared with the results from the improved model. The model with a larger coefficient of rolling resistance has reasonably good comparison with the test data; the slips associated with the minima of drawbar pull and torque also show good comparison with test data; on the other hand, slips associated with the maxima predicted by the model compare poorly with test data indicating that a better shear stress-shear displacement sub-model is needed. The range of tire sinkage

from the model compares well with the range of sinkage of test data obtained using a 3-D laser-based profilometer.

INTRODUCTION

Interfacial forces between tire and ground are known to vary with the longitudinal and lateral slip of the tire, whether for on-road or off-road applications. For off-road applications, the resistance due to the compaction of the terrain plays an important role in vehicle performance which depends on the complicated mechanical properties of the terrain. In recent years, the author and collaborators have been developing slip- and depth-dependent tire-snow interaction models ranging from deterministic high-fidelity finite element model [1-2], physics-based analytical model [3], as well as stochastic models [4-5] to take into consideration the large variations of material properties of snow. A recent review of relevant literature can be found in [2, 5]. Due to a lack of experimental data that quantify the effects of slip on all of the interfacial forces - traction, drawbar pull and motion resistance - the validation of these models has been limited. Recently, the author has developed a new validated mechanics-based indentation model for snow [6] to replace the empirical model in pressure-sinkage relationship [7-8] commonly used for vehicle-terrain interaction; the model [6] uses only a few material constants that represent the elastic-plastic deformation of the material which also simplifies significantly the modeling process. The model, however, has not been incorporated into the studies of tire-snow interaction. In addition, recently, the author's research group has been conducting experiments to obtain slip-based data towards the validation of tire-snow interaction models [9-10]. The purpose of this paper is to improve a semi-analytical tire-snow interaction model [3] by using the newly developed indentation model for snow [6], and compare model results with experimental data. Although the models developed in the past by the author's group are all for combined

longitudinal and lateral slip, this paper focuses only on the longitudinal aspect of tire-snow interaction.

The paper is organized as follows. The governing equations of the elastic-plastic properties of snow are first discussed followed by the presentation of the essential components of the semi-analytical tire-snow interaction model. Results of the interfacial forces, moment, and tire sinkage including the effects of vertical load, snow depth, snow hardness, and rolling resistance are presented next. Finally, we discuss comparison in the minima and maxima of normalized drawbar pull and normalized torque with associated slip, and tire sinkage between model and test results.

GOVERNING EQUATIONS

In this section, we first present the pressure-sensitive Drucker-Prager yield criterion with cap to model low-density dry snow with a special focus on its application to obtain the pressure-sinkage relationship [6] where additional references on the material model can be found. Tire-snow interaction equations, common for soft terrains such as soils and snow [8], are then discussed that include the pressure-sinkage relationship as well as shear stress-shear displacement relationship, the latter is directly related to the longitudinal slip.

DRUCKER-PRAGER YIELD CRITERION

The Drucker-Prager yield criterion can be described as

$$\bar{\sigma} - p \tan \beta - p_d = 0 \quad (1)$$

where p is the pressure ($p = -\frac{\sigma_{kk}}{3} = -\frac{\sigma_{ij}\delta_{ij}}{3}$), δ_{ij} is the Kronecker delta; $i, j = 1, 2, 3$; $\bar{\sigma}$ is the equivalent stress ($\bar{\sigma} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}}$) where σ'_{ij} is the deviatoric stress ($\sigma'_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3} \delta_{ij}$); β is the internal friction angle, p_d is the cohesion. The capped Drucker-Prager yield criterion is characterized by a limit in the hydrostatic pressure the material can sustain which can be approximated as follows

$$\bar{\sigma} - p_a \tan \beta - p_d = 0 \quad (2)$$

where p_a is considered as a material constant which governs the hardening of the material and is parameterized (in MPa) as [5]

$$\log_{10} p_a = c_1 - c_2 \exp(-\epsilon_v^p - c_3 (\epsilon_v^p)^3) \quad (3)$$

where ϵ^p is the volumetric plastic strain, $\epsilon^p = \epsilon - \epsilon^e$ is the axial plastic strain, ϵ is the total axial strain, ϵ^e is the axial elastic strain, C_1, C_2, C_3 are fitting constants.

Drucker-Prager yield criterion can be considered as a three-dimensional generalization of the Mohr-Coulomb yield criterion such that the cohesion (c) and internal friction angle (ϕ) of the Mohr-Coulomb yield criterion can be related to those of the Drucker-Prager yield criterion as

$$\tan \phi = \frac{\tan \beta}{3\sqrt{3}}, \quad c = \frac{p_d}{\sqrt{3}} \quad (4)$$

TIRE-SNOW INTERACTION EQUATIONS [3, 8]

Figure 1 shows relevant variables for the longitudinal motion of tire over a snow cover of depth h_s . The tire, of radius r , is subjected to a vertical force (F_z), and an applied torque (T_{app} or M_y); it rotates with an angular velocity ω , linear speed v ; the maximum sinkage of the tire is z_0 , sinkage at a typical point is z . The angle between the bottom of the tire and a typical point is θ , the maximum (exit) angle is θ_0 . z and θ are related by

$$z(\theta) = r(\cos \theta - \cos \theta_0) \quad (5)$$

At the interface between the tire and snow, there exists a normal stress σ_n and a shear stress τ . Force equilibrium in the vertical direction gives

$$F_z = br \left[\int_0^{\theta_0} \sigma_n(\theta) \cos \theta d\theta + \int_0^{\theta_0} \tau(\theta) \sin \theta d\theta \right] \quad (6)$$

where b is the width of the contact patch of the tire. The motion resistance is defined as

$$R_x = br \int_0^{\theta_0} \sigma_n \sin \theta d\theta \quad (7)$$

The traction (tractive force) is defined as

$$F_{tx} = br \int_0^{\theta_0} \tau \cos \theta d\theta + f_{rr} F_z \quad (8)$$

where f_{rr} is the rolling resistance coefficient. The drawbar pull is defined as

$$F_x = F_{tx} - R_x \quad (9)$$

The applied torque (M_y) is defined as

$$M_y = rb \int_0^{\theta_0} \tau(\theta) d\theta \quad (10)$$

The normal stress σ_n is a function of θ which is related to tire sinkage z via [equation \(4\)](#). Consequently, σ_n as a function of z , i.e., the pressure-sinkage relationship, is needed to evaluate the integrals of [equations \(5\) and \(6\)](#). For low-density dry snow studied here, the deformation of the tire can be ignored such that the tire can be modeled as rigid, this is reflected in the lower integration limit of $\theta=0$ in the above equations.

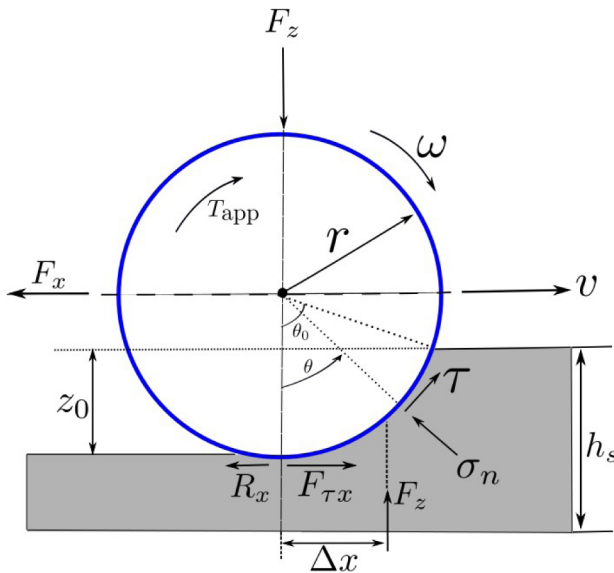


Figure 1. Schematic of tire-snow interaction.

Pressure-Sinkage Relationship

The pressure-sinkage model developed in [6] distinguishes three deformation zones during an indentation test. The first (zone I) is an elastic zone characterized by the elastic modulus, the zone ends when initial plastic deformation occurs. Plastic deformation with hardening then occurs afterward. During indentation, there is a pressure bulb underneath the indenter which propagates downward. If the pressure bulb reaches the bottom of the snow, then snow is considered as shallow (zone III); if not, the snow is considered as deep (zone II); whether snow is considered as shallow or deep depends on the depth and material properties of the snow, and the vertical load on the tire. The input parameters for this model are the Young's modulus and the Poisson's ratio of the tire and snow, the friction angle (β), cohesion (p_d), and strain hardening parameter (p_a) of snow;

width of the contact patch of the tire (b) is also needed. The model presents significant improvement over previous models in that only a few physically-meaningful material parameters are needed and the pressure-sinkage relationship is arrived at using mechanistic solutions for each of the three deformation zones.

Shear Stress-Shear Displacement Relationship

The shear stress τ is a function of θ which can be related to shear displacement as follows [3, 8]. We first define the longitudinal slip as

$$i_x = 1 - \frac{v}{r\omega} \quad (11)$$

The longitudinal shear displacement is defined as

$$j_x = r \left[\theta_0 - \theta - (1 - i_x) (\sin \theta_0 - \sin \theta) \right] \quad (12)$$

The shear stress-shear displacement relationship can be approximated as

$$\tau = (c + \sigma_n \tan \phi) \left[1 - \exp(-j_x / K_{shear}) \right] \quad (13)$$

where K_{shear} is a material constant. Depending on the coefficient of friction (μ) between the tire and snow, there are two friction zones: the sliding zone and the adhesion zone such that the maximum shear stress is limited to $\mu\sigma_n$ in the sliding zone, or,

$$\tau = \min(\tau, \mu\sigma_n) \quad (14)$$

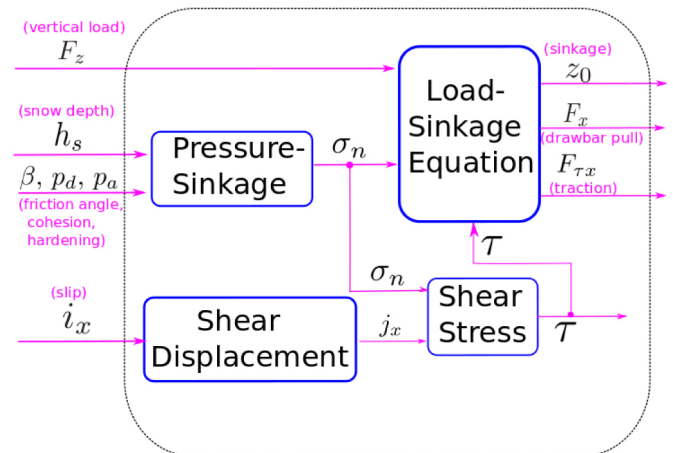


Figure 2. Flowchart of tire-snow interaction model

Figure 2 shows the flowchart of the solution using the current model. In addition to the inputs for the pressure-sinkage sub-model, the inputs are snow depth (h_s), vertical force on the tire (F_z), longitudinal slip (i_x), the shear deformation modulus (K_{shear}), the coefficient of friction (μ), the coefficient of rolling resistance (f_{rr}), and radius of the tire (r). An iterative procedure is used such that equilibrium in the vertical direction (equation (6)) is satisfied. The outputs are the maximum tire sinkage (z_0), the tractive force (F_{tx}), motion resistance (R_x), the torque (M_y), and the drawbar pull (F_x).

RESULTS

Most of the parameters chosen in the paper are the same as those used in previous models [3, 6] in order to assess their quality. The input parameters for snow and tire, unless specified otherwise, are given in Tables 1-2. Goodyear Wrangler HT LT235/75R15 tire without tread pattern was used in the model. In Table 1, we distinguish two snows, the hard snow has a larger strain-hardening modulus than the soft snow such that variations of snow properties are somewhat taken into account. Three vertical forces ($F_z=4,500$ N, 6,240 N, 7,500 N), and two snow depths ($h_s=0.1$ m, 0.2 m) were used.

We first discuss model results in the interfacial forces and moment versus the longitudinal slip; effects of the vertical force, snow depth, snow hardness, coefficient of rolling resistance are also assessed. Tire sinkage is discussed next. We present, when applicable, dimensional results as well as normalized results to better capture the intrinsic behavior of tire-snow interaction.

INTERFACIAL FORCES AND MOMENT

The interfacial forces versus longitudinal slip for snow of 20 cm depth, $F_z=4,500$ N and 7,500 N, are shown in Fig. 3. As expected, the higher the normal force, the larger the absolute value of motion resistance (plotted as negative); same trend is shown for the traction and the drawbar pull. It should be noted that the slip at which the tractions become zero for both vertical loads is about -0.27 , different from the on-road situation where zero traction corresponds approximately to zero slip. The drawbar pull for each vertical load is less than zero indicating, for a given traction, the large motion resistance predicted by the model. It is also instructive to look at the interfacial forces normalized by the vertical force shown in Fig. 4; the normalized traction and motion resistance decrease with increase of the vertical force, i.e., lower vertical force gives rise to higher normalized traction and normalized motion resistance. The trend for the normalized drawbar pull as a function of vertical force, however, is harder to predict since it depends on the difference between the normalized traction and normalized motion resistance. Fig. 4 shows that the normalized drawbar

pull increases with increase of the vertical force for slip > -0.27 but decreases for slip < -0.28 .

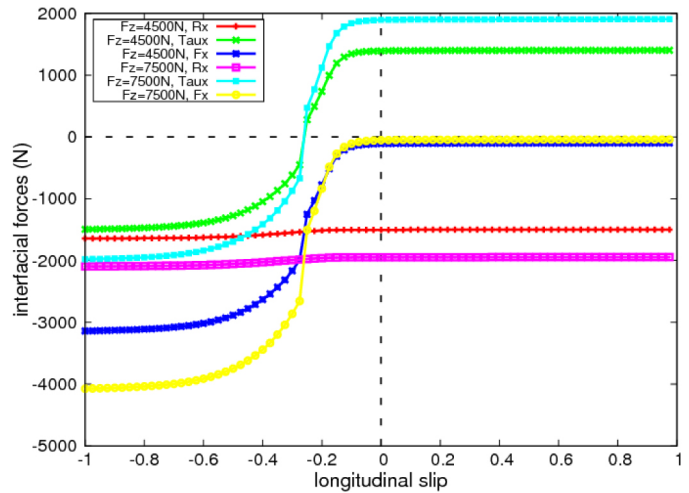


Figure 3. Interfacial forces versus longitudinal slip, snow depth=20 cm.

Figures 5,6,7 show the normalized interfacial forces as a function of longitudinal slip for three vertical forces and two snow depths. Fig. 5 shows that the traction for a given vertical force decreases slightly with decrease in snow depth; this is attributed to the smaller tire sinkage of shallower snow leading to smaller contact surface area and smaller interfacial shear stress. Fig. 6 shows that the motion resistance decreases with a decrease in snow depth due to a decrease in tire sinkage, as expected (cf. Fig. 11). Fig. 7 shows that the drawbar pull increases with a decrease of snow depth indicating that the decrease of motion resistance is large enough to compensate for the slight decrease of traction for shallower snow; the drawbar pull even becomes positive at a slip of -0.03 .

Figure 8 shows the applied torque (M_y) divided by the radius of the tire as a function of longitudinal slip; the trend is similar to that of the traction but the curves are shifted slightly to the left (to lower slips).

Figure 9 shows the interfacial forces as a function of longitudinal slip for two vertical forces and snow depth of 20 cm, but for a softer snow. Comparing the results with the harder snow in Fig. 3, the motion resistance for softer snow is slightly larger, as expected; the traction for softer snow is slightly lower resulting in slightly lower drawbar pull, but not significantly.

Figure 10 shows the effect of increasing rolling resistance coefficient (f_{rr}) from 0.06 (cf. Fig. 3) to 0.2 on the interfacial forces. All forces increase with higher f_{rr} , as expected; the drawbar pull also becomes positive at a slip $= -0.18$.