

Stress Concentration Factor Solutions for Shoulder Filleted Shafts in Bending, including the Influence of Tapering

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ABSTRACT

An accurate set of solutions is presented for stress concentration factors in shoulder filleted shafts under bending, tension and torsion. Solutions were obtained from extensive elastic finite element analyses for both plain stepped shafts, and for shafts in bending with a variety of tapered shoulders.

For plain stepped shafts, conventional stress concentration factor graphs are presented, along with a convenient equation to facilitate numerical analyses. Sets of constants are provided for the equation to compute either the maximum principal stress or the maximum von Mises equivalent stress in the fillet, under each loading mode. This defines the complete state of biaxial stress at the root of the fillet. Furthermore, another convenient equation is provided to accurately compute the location of the maximum stress in the fillet, which varies with geometry. This information can be invaluable for experimental analysis or failure analysis.

With the newly established solutions for shoulder filleted shafts as a baseline, the influence of adding a conical taper to the shoulder was examined using a matrix of over 4000 FEA runs. It is demonstrated that tapering can reduce the stress concentration factors in the fillet significantly, but only over a small range of geometries. A sample calculation is used with notch strain analysis to demonstrate the fatigue life improvement that can be achieved with tapering. This example problem addresses some important factors for notch strain analysis, including the proper application of the Smith-

Watson-Topper parameter and the use of maximum principal stress versus von Mises equivalent stress in the notch root.

INTRODUCTION

The geometry of a shoulder fillet has long been of interest to shaft designers and structural analysts. For years engineers relied on solutions estimated by R.E. Peterson [1], as presented in Fig. 1, in the form of graphical design charts.

Previous work by Rolović [2,3] noted that the popular solutions from Peterson were actually only estimates, based on photoelastic solutions for the 2-dimensional geometry of filleted plates and 3-dimensional solutions for circumferential grooves. Thus, new, accurate solutions were published based on extensive FEA, which were validated with experimental results for a few geometries. The new solutions demonstrated that the Peterson approximations were in error by as much as 30%.

The Rolović solutions were generated over a wider range of geometries ($1.05 \leq D/d \leq 3$, and $0.002 \leq r/d \leq 0.3$) than Peterson's. In addition to conventional stress concentration factors in terms of maximum principal stress, the new solutions noted the multiaxial stress state in the notch root and included the von Mises equivalent stress. Furthermore, new solutions provided previously unavailable locations of the maximum stress in the fillet. The locations were defined in terms of three separate graphs and equations, one for each loading mode.

Subsequent to that work, FEA was used to analyze additional filleted shaft geometries. Shouldered shafts with conical

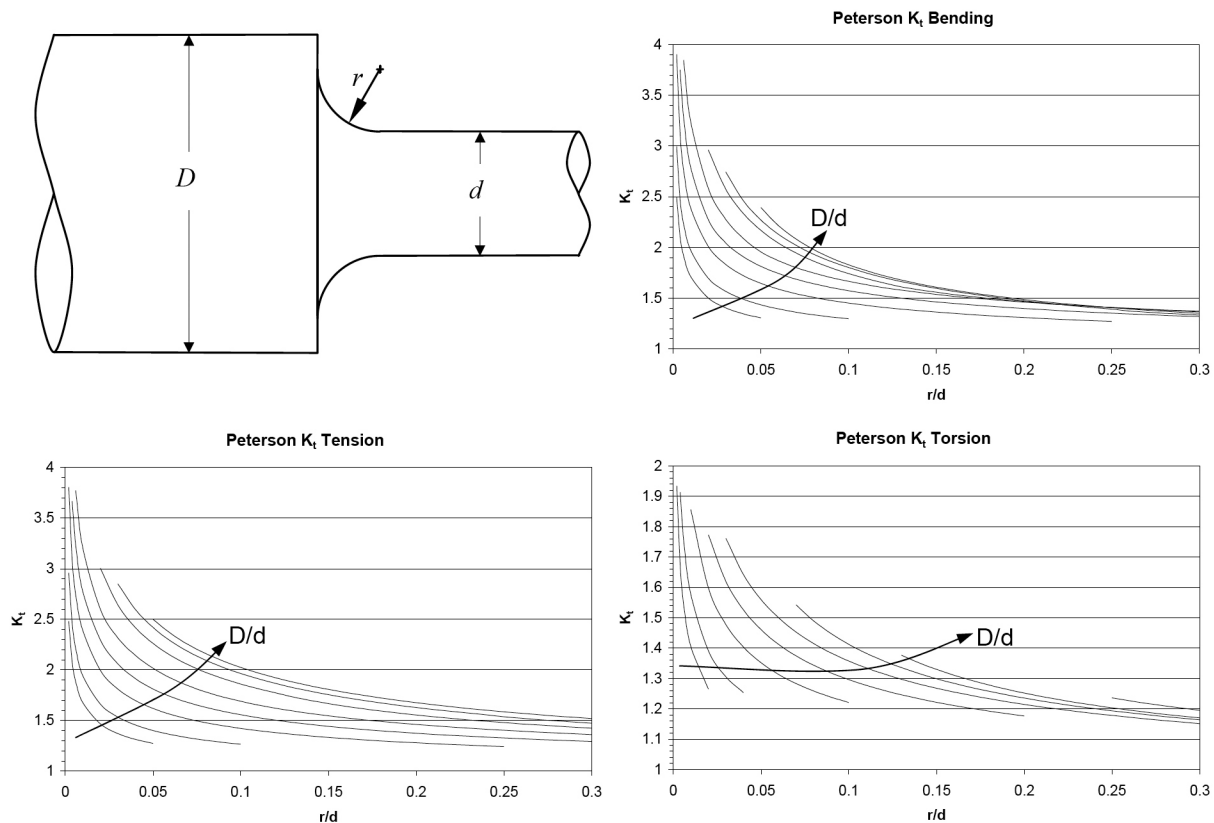


Figure 1. Stepped shaft geometry and Peterson original solutions for bending, tension, and torsion.

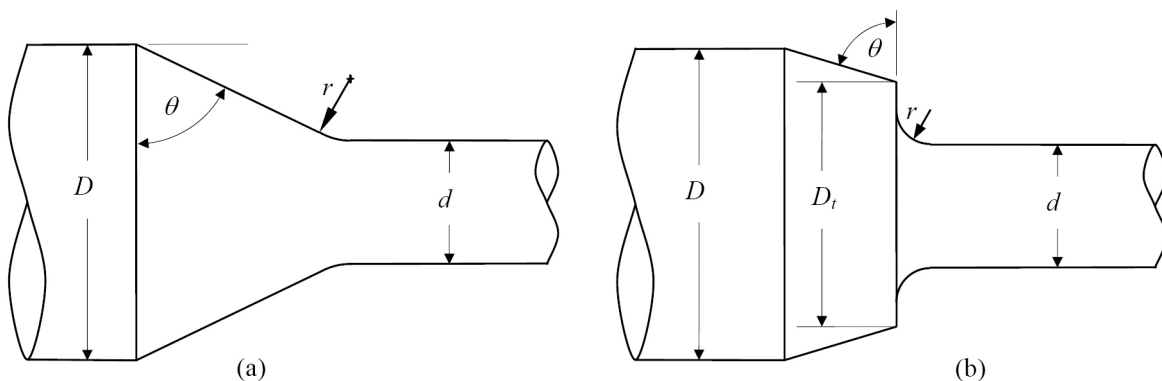


Fig. 2. (a) Shoulder with tapered conical section tangent to fillet radius, (b) shoulder geometry with a general tapered section.

tapered sections tangent to a fillet radius, as shown in Fig. 2 (a) were studied by Jallipalli [4]. Then, the more general case of a shoulder with a tapered section that is not necessarily tangent to the fillet, as shown in Fig. 2(b), was studied by Schmidt [5]. Relative to a stepped shaft, the tangent tapered geometry in Fig. 2(a) requires one additional parameter, θ , while the geometry in 2(b) requires two additional parameters, θ and D_t .

The geometry in Fig. 2(b) is the most general geometry for a shoulder fillet with a taper. In the limiting cases, this geometry can be identical to that in Fig. 1 (if $\theta = 0$) or Fig.

2(a) if $D_t = d + 2r(1 - \sin(\theta))$. Because of this, the results in Schmidt [5] replicated the results presented in the previous studies [2, 3, 4]. The newer study took advantage of greater computational power to utilize a finer meshsize, producing slightly refined results. The new stress concentration results agreed with the previous study to within a few percent. However, the finer meshsize in the fillet significantly improved the resolution of the location of the maximum stress. With the finer resolution, it was possible to fit a single equation, with a separate set of constants through the results from each load case. Revised solutions are presented in this paper, along with the new equation for maximum stress

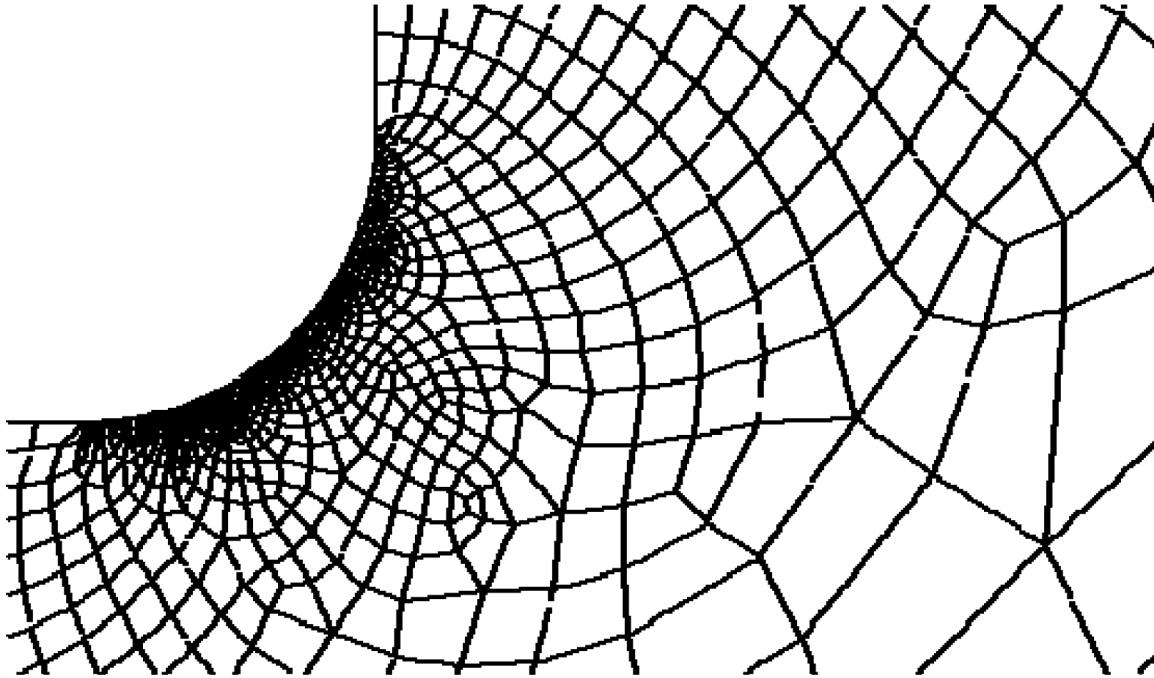


Figure 3. Typical mesh configuration for the plain shoulder filleted shaft geometry.

locations. With the stepped shaft results (Fig. 1 geometry) as a baseline, the influence of tapering (Fig. 2 geometry) is demonstrated based on the results of over 4000 FEA models per load case. The inclusion of a taper can reduce the stress concentration factor substantially in some instances. These are explained and demonstrated with an example problem.

SHOULDER FILLETED SHAFT SOLUTIONS

In this section, the results of the FEA analysis for the plain shoulder filleted geometry (Fig. 1) are presented. The ANSYS FEA package was used in this study, with linear (quadratic) elements. Loading was both axisymmetric loading (tension) and non-axisymmetric loading (bending and torsion). Mapped meshing was used in the fillet region in order to create a uniform distribution of elements along the surface and beneath the surface. A typical mesh is depicted in Fig. 3.

Mesheres were adjusted using automated ANSYS features for finer spacing and gradient sizing to assure convergence. Details about automatic mesh generation and intense convergence studies are presented in Ref. [5].

The stress concentration factor results for the plain stepped shaft are presented in Figs. 4, 5, 6, 7, 8 for bending, tension and torsion. In Figs. 4 and 6, the stress concentration factors for bending and tension, respectively, are defined in terms of principal (normal) stress in the loading direction, σ_1 , as

$$K_t = \frac{\sigma_1}{S_{nom}} \quad (1)$$

where S_{nom} = the nominal stress ($S_{nom} = \frac{32M}{\pi d^3}$ for bending and $S_{nom} = \frac{4F}{\pi d^2}$ for tension). The equivalent stress concentration factors for bending and tension are defined in Figs. 5 and 7, respectively, as

$$K_q = \frac{\sigma_{eq}}{S_{nom}} \quad (2)$$

where σ_{eq} = the maximum von Mises equivalent stress along the surface of the fillet. The equivalent stress is defined in terms of the principal stresses as

$$\sigma_{eq} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \quad (3)$$

since the stress normal to the surface, σ_3 , is equal to zero. The secondary principal stress, σ_2 , is the circumferential stress developed in the fillet due to notch root constraint.

With knowledge of both σ_1 and σ_{eq} in the fillet, Eq. 3 can be solved for the circumferential stress

$$\sigma_2 = \frac{1}{2} \left[\sigma_1 - \left(\frac{\sigma_1}{|\sigma_1|} \right) \sqrt{4\sigma_{eq}^2 - 3\sigma_1^2} \right] \quad (4)$$

fully defining the biaxial state of surface stress in the fillet.

The torsional results in Fig. 8 are presented in terms of the maximum shear stress in the fillet, according to

$$K_t = \frac{\tau_{\max}}{\tau_{\text{nom}}} \quad (5)$$

where the nominal torsional shear stress is given by

$\tau_{\text{nom}} = \frac{16T}{\pi d^3}$. For pure torsion, the magnitude of the maximum shear stress in the fillet, $\tau_{\max}$, is also equal to the magnitude of maximum and minimum principal stresses. The maximum von Mises equivalent stress in the fillet is given by

$$\sigma_{eq} = \sqrt{3}\tau_{\max} \quad (6)$$

The data in Figs. 4, 5, 6, 7, 8 are shown with curves generated using the equation presented in Table 1, which also presents sets of constants for each of the load cases corresponding to Figs. 4, 5, 6, 7, 8.

The angular location, ϕ , of the maximum stress for each loading case is given in Table 2. Figs. 9, 10, 11 show the FEA data along with the curves generated by the Table 2 equation.

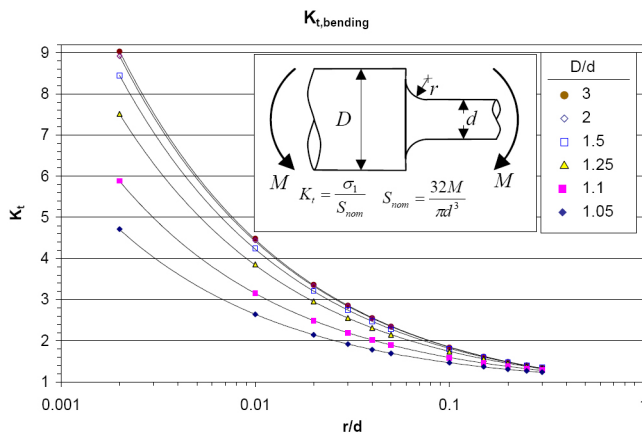


Figure 4. Stress concentration factors for shoulder filleted shafts in bending.

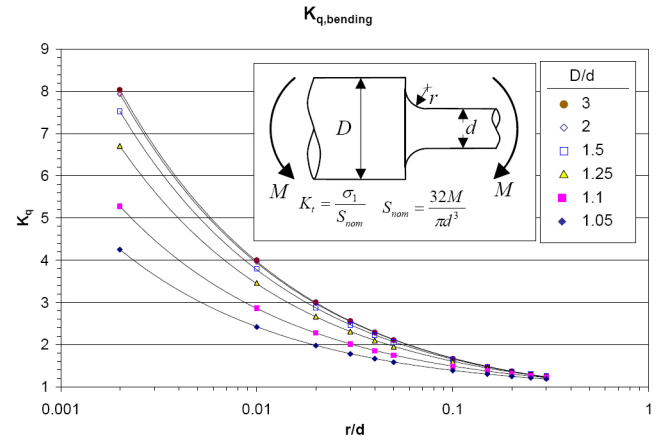


Figure 5. Equivalent stress concentration factors for shoulder filleted shafts in bending.

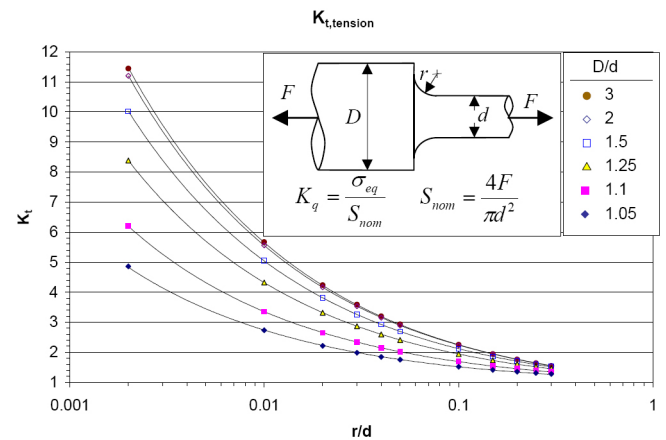


Figure 6. Stress concentration factors for shoulder filleted shafts in tension.

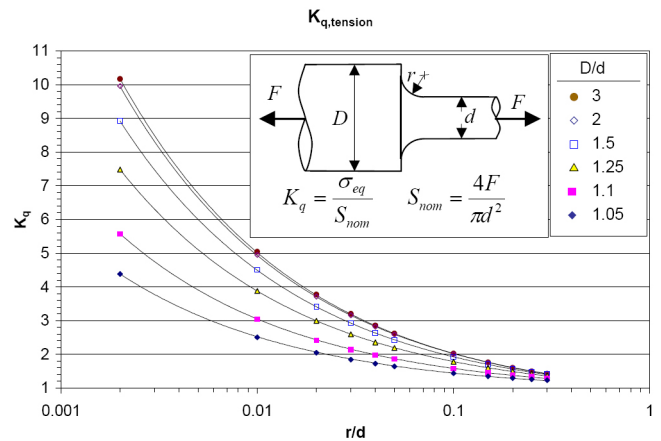


Figure 7. Equivalent stress concentration factors for shoulder filleted shafts in tension.