

A Simulation and Optimization Methodology for Reliability of Vehicle Fleets

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ABSTRACT

Understanding reliability is critical in design, maintenance and durability analysis of engineering systems. A reliability simulation methodology is presented in this paper for vehicle fleets using limited data. The method can be used to estimate the reliability of non-repairable as well as repairable systems. It can optimally allocate, based on a target system reliability, individual component reliabilities using a multi-objective optimization algorithm. The algorithm establishes a Pareto front that can be used for optimal tradeoff between reliability and the associated cost. The method uses Monte Carlo simulation to estimate the system failure rate and reliability as a function of time. The probability density functions (PDF) of the time between failures for all components of the system are estimated using either limited data or a user-supplied MTBF (mean time between failures) and its coefficient of variation. The reliability-cost tradeoff analysis utilizes a user-supplied relationship between reliability and cost for each component. The tradeoff can be used to optimally determine the reliability of each component in order to maximize the system reliability and simultaneously, minimize the acquisition and repair cost for the system. A detailed example highlights the methodology and demonstrates its main features.

1. INTRODUCTION

System reliability analysis is critical to design, preventive maintenance, availability and maintainability through time [1]. Better reliability is usually achieved through better design, better materials or improved manufacturing methods and usually results in an increase in cost. The designer (or the

end-user) must take this tradeoff into consideration. In addition, system level reliability targets are usually provided and not the reliability targets of the individual components. This leads us to the problem of assigning component level reliability targets based on system targets. This problem is referred to as reliability allocation. It is obvious that many combinations of component level reliabilities (designs) can lead to the same system level reliability. How do we select a design among these that best satisfies the end user requirements? End users are usually concerned with multiple attributes which constrain the design space and help us reduce the number of designs that meet the system reliability targets. Multiple attributes however, increase the computational cost of evaluating designs for fitness to find the optimal one. Optimization methods should be carefully chosen or modified to handle this additional burden. Finally, the methodology for component level reliability allocation should incorporate tradeoff preferences of the end user. It should either incorporate the end user's utility function over the attributes *or* present him/her with a manageable number of designs from which the best one can be chosen based on their tradeoff behavior.

Reliability-cost tradeoffs and component reliability allocation are important in system acquisition and logistics as well as in evaluating new technologies to meet performance targets. The particular application we consider in this paper is reliability-cost study for vehicle fleets. Reliability methods in engineering have existed for some time. Analytical methods include the classical first-order and second-order reliability methods (FORM [2], SORM [3, 4]), and multipoint approximation methods [5]. Integrating reliability into a design problem has been studied under the umbrella term of

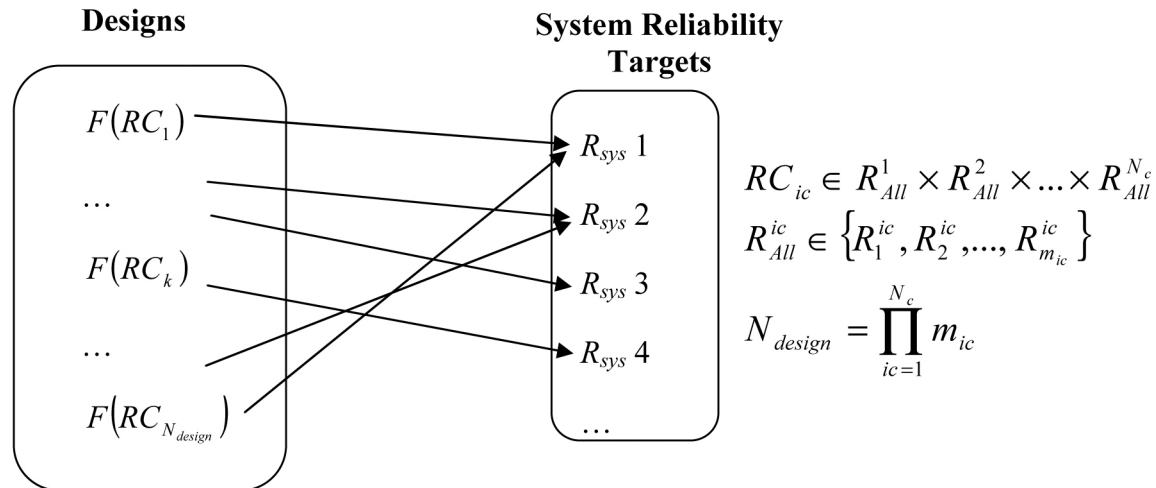


Figure 1. Mapping of component level reliabilities into system level reliability

reliability based design optimization (RBDO). The system is optimized subject to probabilistic constraints which ensure a small probability of violation [6, 7]. Optimization in RBDO is performed using either classical gradient based methods or heuristics like Genetic Algorithms [8]. Time dependence of reliability, is also critical to understanding the performance of real life engineering systems that exhibit degradation. Very few studies have investigated the interplay of reliability allocation in engineering design, time dependence of reliability and cost tradeoffs associated with increasing reliability.

In this paper we implement our methodology on a series system. It is equally applicable to repairable as well as non-repairable systems. For demonstration purposes, we assume that a component failure mileage¹ or time is a Beta distributed random variable [9]. This does not limit our discussion because the family of Beta distributions exhibits a lot of flexibility in modeling different types of distributions. For a given useful mileage, the mean value, standard deviation, failure counts and reliability of a component can be analytically obtained. However, the system reliability depends on the layout of the components, and may be generally estimated using Monte-Carlo Simulation (MCS). The unit cost of a component is assumed to exponentially depend on the mean time between failures (MTBF) while the coefficient of variation is assumed constant.

The main contributions of this research are: the development of a methodology for multi-objective optimization to enable reliability-cost tradeoffs and to decide target component reliability for a series system. The method can be extended to non-series system using fault tree analysis for example, to define system failure.

2. MODEL DESCRIPTION

2.1. Component Level Reliability Assignment

If the failure modes of a system are known, component level reliability maps directly to the system level reliability as shown in [Figure 1](#). The number of ways component level reliability can be chosen (designs) for a system composed of a large number of components is very large. As shown in the figure, for N_c components, each having m_{ic} variants (subscript i_c stands for component number), the set of all

designs is a Cartesian product of R_{All}^{ic} 's where $R_{All}^{ic} \in \{R_1^{ic}, R_2^{ic}, \dots, R_{m_{ic}}^{ic}\}$ and $R_{m_{ic}}^{ic}$ represents the reliability of the m_{ic} variant of the i_c^{th} component. Each of these designs can map to a system level reliability using the failure mode information embedded in the functional F , and the mapping is a surjection, i.e. for each feasible system level reliability target there is at least one design. However, more than one design can result in a given system reliability level. Therefore, the inverse problem of “given a system reliability target, allocate component level reliabilities” does not have a unique solution. As we will see later in the paper, the addition of the objective of cost helps us alleviate this issue.

2.2. System Description

Assume that there is a system of N_c components, which are numbered using $i_c = 1, 2, \dots, N_c$. We also assume that this system is a series repairable system so that any failure in a component results in a system failure. The failed component can be replaced using a brand new component of the same make, as shown in [Figure 2](#).

¹Mileage can be a better measure of “time” for systems that are not constantly in operation, such as a vehicle.

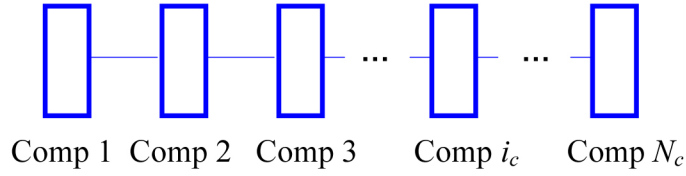


Figure 2. A series system layout

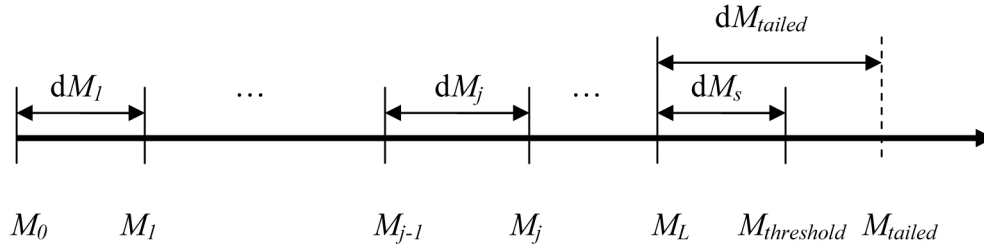


Figure 3. A sorted component failure odometer mileage readings

We further assume that the system begins service under similar operational conditions with all brand new components and the mileage M_0 set to zero. For a given system in service, when a component fails, we record the component ID i_c and

the odometer mileage reading $(M_j)_{i_c}$ for that component. The mileage between failures (TBF) is then equal to

$$d(M_j)_{i_c} = (M_j)_{i_c} - (M_{j-1})_{i_c}. \quad (1)$$

The failed component is then replaced with a brand new one and the system resumes its regular service. This procedure continues until the odometer mileage reading for the specific system reaches a given threshold value $M_{\text{threshold}}$. Data acquisition for this system is then stopped. Figure 3 shows our notation with the component ID dropped for simplicity.

In Equation (1) and Figure 3, $j = 1, 2, \dots, L$ is the failure sequence number for the given component. L is the total number of component failures before and at the threshold value $M_{\text{threshold}}$ for the specified component in a system. $M_0 = 0$ is the beginning odometer mileage reading and M_L is the maximum component failure mileage odometer or timer reading, before the censoring mileage, $M_{\text{threshold}}$. Obviously, the recorded component failure data are right-censored at the threshold mileage.

The $(L+1)^{\text{st}}$ component to replace the malfunctioned L^{th} component does not fail at or before $M_{\text{threshold}}$ (surviving component) but would fail at the odometer mileage M_{tailed} beyond the threshold value $M_{\text{threshold}}$ if the data were not

censored. The $(L+1)^{\text{st}}$ failure mileage of component i_c given by

$$d(M_{\text{tailed}})_{i_c} = d(M_{L+1})_{i_c} = (M_{\text{tailed}})_{i_c} - (M_L)_{i_c} \quad (2)$$

is *not* known from the recorded component failure data. Instead, the surviving component mileage can be retrieved as

$$d(M_s)_{i_c} = M_{\text{threshold}} - (M_L)_{i_c} \quad (3)$$

representing the used mileage/time of the $(L+1)^{\text{st}}$ component at the threshold mileage/time $M_{\text{threshold}}$.

Notice that there are cases that

$$d(M_s)_{i_c} = M_{\text{threshold}} \quad (4)$$

indicating that in the component data acquisition process, some components have no failure records at all. This missing information should be included in the surviving component data to make up a total of $N_{\text{Survi}} = N_C$ surviving components for a system. The first failure mileage of the system is defined and obtained as

$$M_{1f} = \min(M_1)_{i_c} = \min(dM_1)_{i_c} \quad (5)$$

for $i_c = 1, 2, \dots, N_C$.

This data acquisition procedure may be applied to a fleet of N_S systems of the same make and model, or the simulation may be repeated for the same system N_C times. Then the

corresponding records $(M_j)_{i_c}^{i_s}$, $(dM_j)_{i_c}^{i_s}$ for component i_c , are obtained and

$$(M_{1f})^{i_s} = \min(M_1)_{i_c}^{i_s} = \min(dM_1)_{i_c}^{i_s} \quad (6)$$

for $i_s = 1, 2, \dots, N_S$, $i_c = 1, 2, \dots, N_C$, and $i = 1, 2, \dots, (L)_{i_c}^{i_s}$. It should be noted that a complete set of system failure data samples can be obtained from actual data or simulations.

2.3. Data Sorting and Parameter Estimation

The originally recorded component failure data are raw data that are usually in a random order and cannot be used directly. Therefore, they need to be sorted according to the priority order of the system or simulation number $i_s = 1, 2, \dots, N_S = N_{sim}$, followed by the component serial number $i_c = 1, 2, \dots, N_C$, and then by the failure sequence $j = 1, 2, \dots, (L)_{i_c}^{i_s}$.

For a specified component of a specified system or simulation, the sorted component failure odometer mileage readings in ascending sequence, is given by

$$M_0 = 0 < M_1 < \dots < M_{j-1} < M_j < \dots < M_L < M_{threshold} \quad (7)$$

Note that the component failure odometer mileage reading M_j is related to the component mileage between failures (TBF) as

$$M_j = \sum_{k=1}^j dM_k \quad \text{for } j = 1, 2, \dots, L, L+1 \quad (8)$$

We assume that the mileage between failures (TBF) of all components is a Beta distributed random variable of the same parameters [9, 10, 11]. The Beta distribution is used because of its flexibility to model a wide variety of probability distributions. In this case,

$$dM_i = X_i \sim \beta(A, B, p, q), \quad (A \leq X_i \leq B, \text{ and } p > 0, q > 0) \quad (9)$$

The probability density function (PDF) of the Beta distribution is given by,

$$f(x, A, B, p, q) = \beta(p, q)^{-1} (x - A)^{p-1} (B - x)^{q-1} / (B - A)^{p+q-1} \quad (A \leq x \leq B, \text{ and } p > 0, q > 0) \quad (10)$$

and its cumulative distribution function (CDF) is

$$F(x) = P(X_i \leq x) = \int_{-\infty}^x f(t, A, B, p, q) dt \quad (11)$$

In Equation (10), the Beta function $\beta(p, q)$ is associated with the Gamma function Γ as,

$$\beta(p, q) = \Gamma(p)\Gamma(q)/\Gamma(p+q) \quad (12)$$

and appears as a normalization constant to ensure that the probability in Equation (11) integrates to unity when $x \rightarrow \infty$. The standard Beta distributed random variable Y_i is the special case of the general Beta distributed random variable X_i when $A = 0$ and $B = 1$; i.e.,

$$Y_i \sim \beta(0, 1, p, q), \quad (13)$$

where

$$X_i = A + (B - A)Y_i \quad (14)$$

In Equations (9, 10, 11, 14), A and B are the lower and upper bounds of the random variable X_i and p and q are the shape parameters. Figure 4 shows the probability density function (PDF) and the corresponding cumulative distribution function (CDF) of a component failure mileage for parameters $A = 0$, $B = 45,000$ miles, $p = 3$, and $q = 5$.

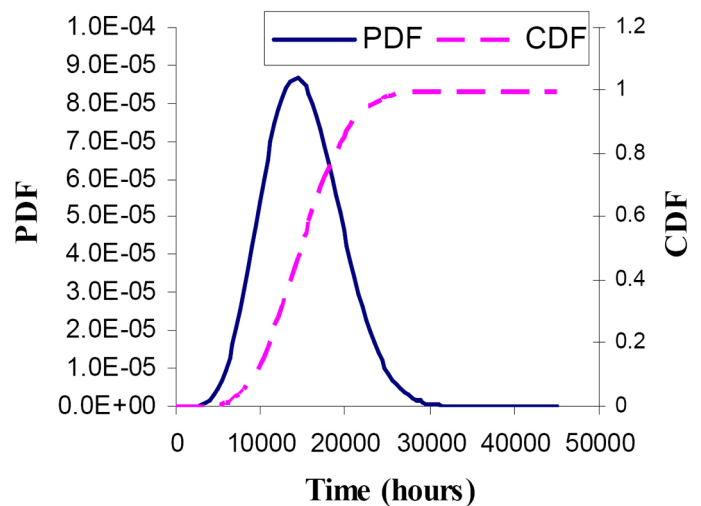


Figure 4. PDF and CDF of a Beta distribution

If we have a total of N_F failure mileage data points x_i , $i = 1, 2, \dots, N_F$ the sample mean and the sample standard deviation