

An Improved Finite Element-Based Model for Reliability Assessment of a Profile-Type Automotive Body Experiencing Uncertain Loading Conditions and Material Properties

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M. Shariyat
Professor, K.N. Toosi University of Tech

Mahboobeh Rajabi Ghahnavieh
Corresponding author, M. SC., K.N. Toosi

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ABSTRACT

Beam-type structural elements are generally utilized in construction of majority of the automotive structures, e.g. the buses, trailers, and solid axles. These components are usually subjected to spatially-random or uncertain load conditions during their service lives. Moreover, material properties of the beams-type structural elements may vary from a sample to another in a random manner. The situation will be more complex when both material properties and load conditions exhibit random natures in the spatial domain. In the present paper, an algorithm is presented to assess the probabilistic behavior of the beam-type vehicle's components in relation with the strength and deflection requirements. A consistent finite element reliability model that may be employed for beams with arbitrary inclinations under simultaneous spatially-random loading conditions and random material properties is introduced. The stochastic method is proposed based on the autocorrelation, power spectral density, and variability concepts. The stress stochastic formulations are proposed in the present paper, for the first time.

INTRODUCTION

Beam-type components are extensively used in automotive, aerospace, structural, offshore, and marine industries. Understructures of some commercial vehicles are constructed from frames composed of beam-type steel tubes or extrusion profiles that provide the displacement and strength requirements. Deviation of the mechanical properties due to

fluctuations in the manufacturing process, changes or deviations in the environment temperature, and some other factors is a common phenomenon. Furthermore, loading conditions of the passenger and commercial vehicles may vary from the curb vehicle weight (CVW) conditions to even beyond the design gross vehicle weight (GVW) conditions. Moreover, the load distribution, e.g. in urban buses or trailers, may have local deviations in the longitudinal direction of the vehicle. Therefore, in general, both the material properties and the loading conditions may experience uncertainties. In the present analysis, it is assumed that the dynamic loads of the vehicle may be substituted by their equivalent static loads.

The most widely used stochastic finite element researches have been proposed based on mean-centered perturbation techniques that generally ignore the cross correlation of the stochastic parameters [1, 2, 3, 4, 5]. This method has been developed by many researches such as Liu et al. [6,8]. Unfortunately, perturbation techniques have two main disadvantages: (i) their results are only reliable for small variations of the stochastic parameters, and (ii) it has been proved that results of the mentioned technique are insufficient to deal with the time-history responses due to predicting uncertain natural frequencies [9].

The stochastic finite element method (SFEM) may be employed for analyzing structures in which the uncertainties in the material properties or loads are distributed spatially [10]. Deodatis [11,12] developed a methodology to evaluate the spectral distribution-free upper bound of response

variability of stochastic finite element systems characterized by statistical dependencies among the stochastic fields of the different elements. He applied the method for beam-columns and trusses with random material properties. Impollonia and Elishakoff [13] derived some exact solutions for shear beams with stochastic flexibility, when these beams are acted upon by random loads. Singh and Zheng [14] proposed a finite element procedure for evaluation of variability of the response of beams having random material properties described by random functions along the length of the beams. The solution was obtained using the Taylor series expansion including the inter-element correlation. Stefanou [15] has reviewed the stochastic finite element methods proposed so far.

Review of the researches accomplished in the field of random analysis of the beam-type structures in literature reveals that almost all these researches have been performed for beams with either random material properties or random loads. Besides, only response variability of the displacement parameter has been investigated in the mentioned works. In the present paper, the previous researches are extended through considering conditions where both material properties and loads vary in a random manner. Moreover, the stress response variability is also considered in the present research.

THE GOVERNING EQUATIONS

1. THE RESPONSE VARIABILITY OF THE LATERAL DISPLACEMENT

1.1. Finite Element Formulation of Beams with Random Material Properties

To guarantee slope and subsequently stress continuity at the mutual nodes of the elements, Hermitian elements are adopted to trace variations of the lateral displacements. Consider the beam element shown in Fig. 1. In contrast to some researches, the axial displacements are also considered so that in each grid point, three degrees of freedom are defined.

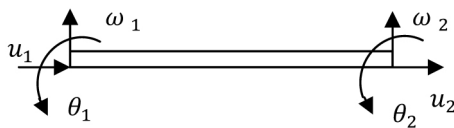


Figure 1. The considered two node beam element.

The modulus of elasticity of the beam is assumed to vary randomly along the beam length in the form of $E^{(e)}(x) = E_0^{(e)}[1 + f^{(e)}(x)]$ where $E_0^{(e)}$ is the mean value of the elastic modulus and $f^{(e)}(x)$ is a zero-mean homogeneous stochastic field. Due to the unknown stochastic field $f^{(e)}(x)$,

the element stiffness matrix consists of unknown parts $\Delta K_0^{(e)}$, $\Delta K_1^{(e)}$, $\Delta K_2^{(e)}$ which are related to the element stiffness matrix by the following form:

$$K^{(e)} = K_0^{(e)} + X_0^{(e)} \cdot \Delta K_0^{(e)} + X_1^{(e)} \cdot \Delta K_1^{(e)} + X_2^{(e)} \cdot \Delta K_2^{(e)} \quad (1)$$

where $X_0^{(e)}$, $X_1^{(e)}$, and $X_2^{(e)}$ are moments of orders 0, 1, 2, of the stochastic field [as Eq. (2) implies], respectively. It is obvious that the mean values of these random variables are equal to zero.

$$X_i = \int_0^{L^{(e)}} x^i f^{(e)}(x) dx \quad (2)$$

K_0 is the deterministic part of the element stiffness matrix and $\Delta K_0^{(e)}$, $\Delta K_1^{(e)}$, $\Delta K_2^{(e)}$ are the indeterministic parts which are given in the [appendix](#) section.

Following the standard finite element procedures, the element stiffness matrices and load vectors are assembled to make the global stiffness matrix and load vector of the entire structure. After introducing appropriate boundary conditions, the equilibrium equation may be written as:

$$KU = F \quad (3)$$

Since the global stiffness matrix involves random variable, the response of the beam to the deterministic load would be random. In order to obtain variance of the deflection, a specific procedure explained in the following paragraphs should be performed.

Consider as a first order approximation, Taylor's expansion of the displacement vector (U) around the mean values of the random variables:

$$U \cong U_0 + \sum_{e=1}^{N_e} \sum_{i=1}^{N_i} (X_{ei} - \bar{X}_{ei}) \left[\frac{\partial U}{\partial X_{ei}} \right]_E = [U_0] + X_0 \left[\frac{\partial U}{\partial X_0} \right] + X_1 \left[\frac{\partial U}{\partial X_1} \right] + X_2 \left[\frac{\partial U}{\partial X_2} \right] \quad (4)$$

$\left[\frac{\partial U}{\partial X_{ei}} \right]$ is obtained by differentiating Eq(3) with respect to X_{ei} .

$$\left[\frac{\partial U}{\partial X_{ei}} \right]_E = -K^{-1} \left[\frac{\partial K}{\partial X_{ei}} \right]_E U \quad (5)$$

$\left[\frac{\partial K}{\partial X_{ei}} \right]$ may be determined by differentiating Eq.(1) with respect to X_{ei} .

$$\left[\frac{\partial K}{\partial X_{ei}} \right]_E = \Delta K_{ei} \quad (6)$$

Substituting Eqs. (5) and (6) into Eq. (4) and evaluating the result at the mean values of X_{ei} yields:

$$U \cong U_0 - \sum_{e=1}^{N_e} \sum_{i=1}^{N_i} K_0^{-1} \Delta K_{ei} U_0 \cdot X_{ei} \quad (7)$$

Finally covariance of the matrix (u-u₀) will be :

$$Cov[U, U] = \epsilon[(U - U_0)(U - U_0)^T] = \sum_{e_1=1}^{N_e} \sum_{e_2=1}^{N_e} \sum_{i_1=1}^{N_i} \sum_{i_2=1}^{N_i} K_0^{-1} \Delta K_{ei} U_0 U_0^T \Delta K_{ei}^T (K_0^{-1})^T \cdot \epsilon\{X_{e_1 i_1} X_{e_2 i_2}\} \quad (8)$$

To derive the expected value appeared in Eq. (8), one may write:

$$\epsilon\{X_{e_1 i_1} X_{e_2 i_2}\} = \int_0^{L(e_1)} \int_0^{L(e_2)} \xi_1^{i_1-1} \xi_2^{i_2-1} \epsilon[f^{(e_1)}(\xi_1) f^{(e_2)}(\xi_2)] d\xi_1 d\xi_2 \quad (9)$$

The auto-correlation function of $f^{(e)}(x)$ which is denoted here by $R_{ff}(x)$ may be defined as:

$$R_{ff}(x) = \int_{-\infty}^{\infty} S_{ff}(k) e^{ikx} dk \quad (10)$$

where $S_{ff}(k)$ is the power spectral density. The inter-element correlation may be calculated by means of relating the local coordinate to the global one as:

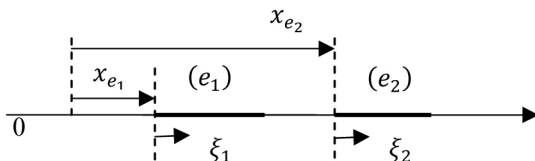


Figure 2. Configurations of elements (e₁) and (e₂)

$$\begin{aligned} \epsilon[f^{(e_1)}(\xi_1) f^{(e_2)}(\xi_2)] &= \epsilon[f(x_{e_1} + \xi_1) f(x_{e_2} + \xi_2)] = \\ R_{ff}(x_{e_2} + \xi_2 - x_{e_1} - \xi_1) &= R_{ff}(\Delta_{e_1 e_2} + \xi_2 - \xi_1) \end{aligned} \quad (11)$$

Therefore the expected value is obtained by substituting Eqs. (10) and (11) into Eq. (9):

$$\begin{aligned} \epsilon\{X_{e_1 i_1} X_{e_2 i_2}\} &= \int_{-\infty}^{\infty} S_{ff}(k) \int_0^{L(e_1)} \xi_1^{i_1-1} e^{-ik\xi_1} d\xi_1 \cdot \\ &\int_0^{L(e_2)} \xi_2^{i_2-1} e^{ik\xi_2} d\xi_2 e^{ik\Delta_{e_1 e_2}} dk \end{aligned} \quad (12)$$

Then variance of the displacement vector is:

$$Var[U] = \int_{-\infty}^{\infty} S_{ff}(k) \cdot V^E(k) dk \quad (13)$$

where

$$\begin{aligned} V^E(k) &= \sum_{e=1}^{N_e} \sum_{i_1=1}^{N_i} \sum_{i_2=1}^{N_i} diag(K_0^{-1} \Delta K_{e_1 i_1} U_0) K_0^{-1} \Delta K_{e_2 i_2} U_0 \cdot \\ &\int_0^{L(e_1)} \xi_1^{i_1-1} e^{-ik\xi_1} d\xi_1 \cdot \int_0^{L(e_2)} \xi_2^{i_2-1} e^{ik\xi_2} d\xi_2 e^{ik\Delta_{e_1 e_2}} \end{aligned} \quad (14)$$

where *diag* denotes the diagonal matrix corresponding to the argument vector. The superscript “E” in V^E shows that vector V is obtained from a random material properties analysis. The normalized response variability is achieved from the following expression [11,12]:

$$COV[u_i] = \frac{Std[u_i]}{|\epsilon\{u_i\}|} = \sigma_{ff} \cdot \frac{|V_i^E(k_i^*)|}{|\epsilon\{u_i\}|} \quad (15)$$

where σ_{ff} is the standard deviation of the stochastic field function $f(x)$.

1.2. Beams with Random Loading Conditions

In cases where the loads exerted to the beam have random natures, a procedure somewhat similar to that explained in the foregoing section may be used. Suppose now that the randomness is associated only with the applied load and the material and geometry properties are deterministic.

Let us express the uncertainty of the applied load by $q_0^{(e)}(1 + g^{(e)}(x))$ where $q_0^{(e)}$ is the mean value of the uniformly distributed load and $g^{(e)}(x)$ is a zero-mean stochastic field.

Employing the standard finite element method to construct the load vector of the element, one gets:

$$F^{(e)} = F_0^{(e)} + Y_0^{(e)} \cdot \Delta F_0^{(e)} + Y_1^{(e)} \cdot \Delta F_1^{(e)} + Y_2^{(e)} \cdot \Delta F_2^{(e)} + Y_3^{(e)} \cdot \Delta F_3^{(e)} \quad (16)$$

where F_0 is the stationary part of the load vector and $\Delta F_0^{(e)}$, $\Delta F_1^{(e)}$, $\Delta F_2^{(e)}$, $\Delta F_3^{(e)}$ are the spatially fluctuating parts which are given in the [appendix](#). $Y_0^{(e)}$, $Y_1^{(e)}$, $Y_2^{(e)}$ are random variables having the form:

$$Y_i = \int_0^{L^{(e)}} x^i g^{(e)}(x) dx \quad (17)$$

Using Taylor's first order expansion of the displacement vector (U) around the mean values of the random variables, yields:

$$U \cong U_0 + \sum_{e=1}^{N_e} \sum_{i=1}^{N_i} (Y_{ei} - \bar{Y}_{ei}) \left[\frac{\partial U}{\partial Y_{ei}} \right]_E \quad (18)$$

After differentiating [Eqs. \(3\)](#) and [\(16\)](#) with respect to Y_{ei} and computing the result at the mean value of random variables Y_{ei} , one has:

$$\left[\frac{\partial U}{\partial Y_{ei}} \right]_E = K_0^{-1} \left[\frac{\partial F}{\partial Y_{ei}} \right]_E \quad (19)$$

$$\left[\frac{\partial F}{\partial Y_{ei}} \right]_E = \Delta F_{ei} \quad (20)$$

Now, using [Eqs. \(18\)](#), [\(19\)](#), [\(20\)](#), covariance of ($u-u_0$) may be established as follows:

$$Cov[U, U] = \sum_{e_1=1}^{N_e} \sum_{e_2=1}^{N_e} \sum_{i_1=1}^{N_i} \sum_{i_2=1}^{N_i} K_0^{-1} \Delta F_{e_1 i_1} \Delta F_{e_2 i_2}^T (K_0^{-1})^T \cdot \epsilon\{Y_{e_1 i_1} Y_{e_2 i_2}\} \quad (21)$$

In order to calculate the autocorrelation function encountered in [Eq. \(21\)](#) the expression mentioned in [Eq. \(12\)](#) is implemented as:

$$\epsilon\{Y_{e_1 i_1} Y_{e_2 i_2}\} = \int_{-\infty}^{\infty} S_{gg}(k) \int_0^{L^{(e_1)}} \eta_1^{i_1-1} e^{ik\eta_1} d\eta_1 \cdot \int_0^{L^{(e_2)}} \eta_2^{i_2-1} e^{ik\eta_2} d\eta_2 e^{ik\Delta e_1 e_2} dk \quad (22)$$

Therefore, variance of the displacement vector can be written as:

$$Var[U] = \int_{-\infty}^{\infty} S_{gg}(k) \cdot V^F(k) dk \quad (23)$$

where:

$$V^F(k) = \sum_{e_1=1}^{N_e} \sum_{e_2=1}^{N_e} \sum_{i_1=1}^{N_i} \sum_{i_2=1}^{N_i} diag(K_0^{-1} \Delta F_{e_1 i_1}) \cdot K_0^{-1} \Delta F_{e_2 i_2} \cdot \int_0^{L^{(e_1)}} \eta_1^{i_1-1} e^{ik\eta_1} d\eta_1 \cdot \int_0^{L^{(e_2)}} \eta_2^{i_2-1} e^{ik\eta_2} d\eta_2 e^{ik\Delta e_1 e_2} \quad (24)$$

The normalized response variability is achieved from the following expression:

$$COV[u_i] = \frac{Std[u_i]}{|\epsilon\{u_i\}|} = \sigma_{gg} \cdot \frac{|V_i^F(k_i^*)|}{|\epsilon\{u_i\}|} \quad (25)$$

where σ_{gg} is the standard deviation of the stochastic field $g(x)$.

1.3. Beams with Both Random Loading Condition and Random Material Properties

For a beam with both random material properties and random loads, since random variables X_{ei} and Y_{ei} are independent, it is possible to express Taylor's first-order expansion of the displacement vector (U) around the mean values of each of the independent random variables simply by adding [Eqs. \(4\)](#) and [\(18\)](#) together:

$$U \cong U_0 + \sum_{e=1}^{N_e} \sum_{i=1}^{N_i} (X_{ei} - \bar{X}_{ei}) \left[\frac{\partial U}{\partial X_{ei}} \right]_E + \sum_{e=1}^{N_e} \sum_{i=1}^{N_i} (Y_{ei} - \bar{Y}_{ei}) \left[\frac{\partial U}{\partial Y_{ei}} \right]_E \quad (26)$$

Therefore, covariance of ($u-u_0$) may be readily calculated by adding the corresponding results of the previous sections together. Hence, the normalized response variability is achieved by adding [Eqs. \(15\)](#) and [\(25\)](#) as follows: