

Statistically Precise and Energy Efficient Accelerated Life Testing

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ABSTRACT

The basic idea of accelerated life testing (ALT) is to expose test units of a product to harsher-than-normal operating conditions to expedite failures so that the failure time distribution of the product and the associated life-stress relationship can be determined in a short time period. However, ALT often consumes significant amounts of energy. To avoid waste of energy in a wide spectrum of product development processes, we explore a new experimental design methodology that improves the reliability estimation precision of an ALT experiment while minimizing the total energy consumption of the experiment. The resulting optimally designed ALT experiment depends not only on the reliability of the product to be tested but also on the characteristics of the test equipment and the capability of its controller. A numerical example is provided to demonstrate the use of the methodology in practice.

INTRODUCTION

The rapid growth of new products and global competition are placing an increasing demand on reliability testing to meet ever increasing expectations for high reliability and substantial reduction in product development time. For highly reliable products, it is often difficult, if not impossible, to observe failures under normal operating conditions in a short time period. When failure data is scarce, precise reliability estimation via statistical modeling becomes quite difficult. To overcome this challenge, ALT has been widely used to estimate the reliability of a product in a shorter time by applying harsher-than-normal operating conditions during tests (Nelson, 1990; Elsayed, 1996; Meeker and Escobar, 1998). By statistically determining an ALT model consisting of a failure time distribution and associated life-stress relationship based on the modest number of failure times or

degradation data due to acceleration, the product's long-term reliability under the normal operating conditions can be extrapolated with respect to both time and stresses. The precision of the reliability estimate depends on both ALT model and ALT plan. An appropriate ALT model provides a statistically precise fit to ALT data for extrapolating the product's reliability. Likewise, an optimal ALT plan, which determines relevant experimental variables, such as the stress loadings (constant-stress, ramp-stress, cyclic-stress, etc.), allocation of the test units at each stress level, can improve the estimation precision.

In the literature, extensive research has been conducted on the optimal design of ALT plans (Nelson, 2005) following the work done by Chernoff (1962). As for the constant-stress ALT, Nelson and Kielpinski (1976) obtained the optimal plans and the best traditional plans for estimating the medians of the normal and lognormal distributions respectively. Meeker and Hahn (1977) considered the optimal allocation of test units to accelerated stress conditions with the objective of minimizing the estimate of the product reliability under the design stress condition. Nelson and Meeker (1978) provided the optimal test plans to estimate the percentiles of the Weibull and extreme-value distributions at a specified design stress. Meeker (1984) compared the statistically optimal test plans to a more practical test plan involving three stress levels. Nelson (1990) reviewed statistically optimal and compromise plans for the single stress ALT planning problem. Meeter and Meeker (1994) extended the existing maximum likelihood theory for planning ALT to the non-constant scale parameter model and presented test plans for a large range of practical tests. Yang (1994) proposed an optimal design of 4-level constant-stress ALT plans considering different censoring times. Regarding the step-stress ALT, Miller and Nelson (1983) presented the cumulative exposure model and obtained the optimal test

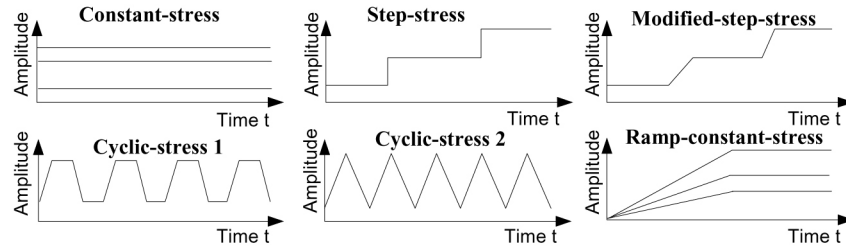


Fig. 1. Various stress loadings widely used in AT

plans. The optimal plans minimize the asymptotic variance of the maximum likelihood estimate (MLE) of the mean life at the design stress. [Bai et al. \(1989\)](#) extended the results to the case where a prescribed censoring time is involved. [Bai and Chun \(1991\)](#) obtained the optimal simple step-stress ALT with competing causes of failure. The optimal test plans minimize the sum, over all failure causes, of asymptotic variances of the MLEs of the log-mean lives at the use condition. [Chung and Bai \(1998\)](#) considered the optimal design of step-stress ALT in which a cumulative exposure model for the effect of changing stress is assumed. [Khamis and Higgins \(1996\)](#) presented 3-step step-stress plans assuming a linear or quadratic relationship between the life and the stress. [Xiong \(1998\)](#) addressed the statistical inferences on the parameters of a simple step-stress ALT model with Type II censoring. [Xiong and Milliken \(1999\)](#) studied the statistical models in step-stress ALT when the stress change times are random. [Xiong and Ji \(2004\)](#) studied the optimal design of a simple step-stress test plan involving grouped and censored data.

In the literature, most research has been focused on the optimal design of ALT plans for specific stress loadings. A question often overlooked is concerned with different energy consumption of ALT experiments using different stress loadings (e.g., see [Fig. 1](#)). It is quite common that several widely used stress loadings indeed show significant differences in this regard. This paper addresses a new methodology that improves the reliability estimation precision of an ALT experiment while minimizing the total energy consumption. The relevant statistical inference and energy consumption evaluation methods are discussed.

ENERGY CONSUMPTION OF AN ALT EXPERIMENT

To quantify the total energy consumption of an ALT experiment, let $\mathbb{P}(t; Z_i)$ be the instantaneous power consumption of the test equipment associated with stress loading Z_i , $i = 1, 2, 3, \dots, m$, at time t , and $\mathbb{EN}_i(t; Z_i)$ be the energy consumption attributed to Z_i up to time t . Then, the total energy consumption of the ALT experiment with duration of τ (singly Type I censored) can be obtained by integrating instantaneous power consumption:

$$\sum_{i=1}^m \mathbb{EN}_i(\tau; Z_i) = \sum_{i=1}^m \int_0^{\tau} \mathbb{P}(t; Z_i) dt. \quad (1)$$

Clearly, the energy consumption has a functional relationship with the parameters (e.g., high/low temperatures and ramp rate) of applied stress loadings and the test duration, which are among the most important decision variables in planning ALT experiments. Moreover, if it is flexible to choose control strategies (e.g., on/off or proportional-integral-derivative (PID) controllers), the controller design based on the model of the test equipment (e.g., transfer function) and the stress loadings will introduce another important set of decision variables. After all, different control strategies have different capabilities that impose certain constraints on the stress loadings. Let $u(t; Z_i)$ be the input (i.e., control signal) to the equipment at time t . Then, the total energy consumption can be expressed more generally as:

$$\sum_{i=1}^m \mathbb{EN}_i(\tau, \{u(t; Z_i), 0 \leq t \leq \tau\}) = \sum_{i=1}^m \int_0^{\tau} \mathbb{P}(u(t; Z_i)) dt, \quad (2)$$

where $\mathbb{P}(u(t; Z_i))$ is the corresponding instantaneous power consumption depending on the control signal.

OPTIMAL DESIGN OF ALT PLANS CONSIDERING ENERGY CONSUMPTION

Traditionally, most optimum ALT plans are focused only on the determination of stress loadings along with others, such as unit allocation to each stress loading, to minimize the uncertainty in reliability estimates. In the literature, a variety of optimization criteria have been used in planning ALT. Most ALT plans minimize the asymptotic variance of the MLE of a model parameter, or a function of parameter estimates such as a specified failure time quantile or reliability function under normal operating conditions. Let $G(\underline{\Theta}, Z_0)$ be the interested reliability index of a product under normal operating conditions Z_0 , where $\underline{\Theta}$ is the parameter vector of the underlying ALT model, and

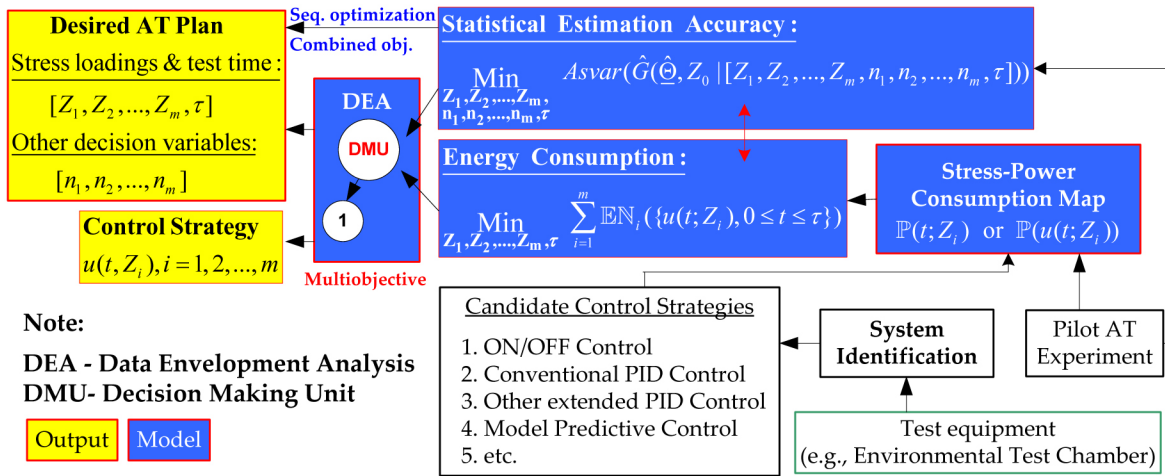


Fig. 2. Proposed methodology for design of statistically precise and energy efficient ALT

$\hat{G}(\hat{\Theta}, Z_0 | [Z_1, \dots, Z_m, n_1, \dots, n_m, \tau])$ be the estimate of $G(\Theta, Z_0)$, where n_i is the number of test units allocated to stress loading Z_i in ALT and $\hat{\Theta}$ is the maximum likelihood (ML) estimate of Θ . According to ML theory, $\hat{\Theta}$ asymptotically follows the multivariate normal distribution $MVN(\hat{\Theta}, \hat{\Sigma}_{\hat{\Theta}})$ with the variance-covariance matrix $\hat{\Sigma}_{\hat{\Theta}} = \left[E \left[-\frac{\partial^2 L(\text{data} | \Theta)}{\partial \Theta \partial \Theta^T} \right] \right]^{-1}$ evaluated at $\hat{\Theta}$, in which $E[\cdot]$ denotes expectation and $L(\text{Data} | \Theta)$ is the log-likelihood function of ALT data (Nelson, 1990). The statistical precision in estimated reliability index depends on the adopted ALT plan described by decision variables $[Z_1, \dots, Z_m, n_1, \dots, n_m, \tau]$. Mathematically, the optimum experiment design problem (PI) can be formulated as:

$$\begin{aligned}
 [Z_1^*, \dots, Z_m^*, n_1^*, \dots, n_m^*, \tau^*] = & \arg \min_{Z_1, Z_2, \dots, Z_m, n_1, n_2, \dots, n_m, \tau} \\
 & \text{Asvar}(\hat{G}(\hat{\Theta}, Z_0 | [Z_1, \dots, Z_m, n_1, \dots, n_m, \tau])) \\
 \text{subject to } & H_1(Z_1, \dots, Z_m, n_1, \dots, n_m, \tau) \leq 0, \\
 & H_2(Z_1, \dots, Z_m, n_1, \dots, n_m, \tau) = 0, \\
 & n_1, \dots, n_m \in \{1, 2, \dots\},
 \end{aligned} \quad (3)$$

where the asymptotic variance:

$$\text{Asvar}(\hat{G}(\hat{\Theta}, Z_0 | [Z_1, \dots, Z_m, n_1, \dots, n_m, \tau])) = \left[\frac{\partial \hat{G}}{\partial \Theta} \right]^T \hat{\Sigma}_{\hat{\Theta}} \left[\frac{\partial \hat{G}}{\partial \Theta} \right], \quad (4)$$

which quantifies the statistical estimation precision of the ALT plan is to be minimized; functions $H_1(Z_1, \dots, Z_m, n_1, \dots, n_m, \tau)$ and $H_2(Z_1, \dots, Z_m, n_1, \dots, n_m, \tau)$ consist of all inequality

and equality constraints, respectively, such as the ranges of stress amplitudes, the maximum ramp rate, and possible types of stress loadings (e.g., see Fig. 1). Usually, such optimum ALT plans may not perform well in terms of energy use. To the best of our knowledge, this gap has not been investigated in the statistics and operations research communities, but it has already become a barrier for technology innovation. To overcome this barrier, interfacing the optimum design of ALT plans with active energy reduction through state-of-the-art equipment control strategies is required.

Fig. 2 shows the proposed framework that interfaces the optimum design of ALT plans and active energy reduction. Prior to ALT planning, a pilot ALT experiment will be conducted to identify the underlying ALT model and estimate the baseline model parameters. Under this framework, different scenarios can be considered, e.g., the flexibility in determining control strategies and the flexibility in optimization (sequential optimization of statistical estimation precision followed by energy minimization or simultaneous optimization of statistical estimation precision and energy consumption).

In a particular scenario where the controller and test equipment are treated as “white boxes”, the physical model of the test equipment, such as the transfer function, can be obtained via a system identification tool often used in controls area. Meanwhile, the optimization design of ALT plan will be performed sequentially that determines an optimum ALT plan in terms of estimation precision followed by energy minimization by searching for a feasible controller that is capable of providing an energy efficient stress loading. The complexity of this problem is attributed to the flexibility in choosing and manipulating controllers such as on/off, PID, and model predictive controllers. In this scenario, instantaneous power consumption $\mathbb{P}(u(t; Z_i))$ given in Eq. (2) may be obtained based on the control signal $u(t; Z_i)$