

Reliability Evaluation Based on Sequencing of Applied Stresses

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ABSTRACT

In probabilistic reliability analysis, it is common that the models relating time to failure with environmental stresses are insensitive to the sequence of the stress level application. For a step-stress model, this means that the probability of failure is a function of the exposure time at each stress and it is unaffected by the order of stress application. These models may lead to improper reliability evaluation since it has been shown in fatigue analysis and similar areas that the stress sequencing effect may be substantial. However there is limited discussion about this topic in the reliability field. The purpose of this paper is to bridge the relationship between cumulative damage concepts in fatigue analysis and modeling and prediction of reliability. We also discuss and present reliability models and conditions where the reliability estimation model is insensitive to the sequence of the applied stresses.

INTRODUCTION

Reliability estimation normally requires a model that describes the effect of environmental conditions on the lifetime of the units. Such reliability models usually consider the application of time varying stresses, such as in step-stress experiments. However it is quite common for these models to be insensitive to the effect of the stress sequence application. This concept is referred to as the stress sequencing or load history in fatigue analysis literature. The most widely used models in probabilistic reliability analysis are insensitive to this effect, as explained later. For a step-stress setting, this means that the prediction of the remaining lifetime is a function of the exposure time at each stress and it is unaffected by the order of the stress application (testing at high stress then stepping to a low stress or vice versa has the same effect on the reliability estimate). Indeed it has been shown that these insensitive models are not adequate for many applications, and significantly shorter lifetimes may be observed if stresses are applied in an order that causes the of the unit damage to build more rapidly [1].

Step-stress test plans beginning with a high stress followed by a low stress may yield shorter lifetimes when compared with a low stress followed by a high stress. In the latter case, the low stress may not initiate cracks, a common failure mode under fatigue loading, or some other preliminary damage process. However, it has also been shown for certain situations that the opposite may be true, i.e., a low stress followed by a high one results in a higher cumulative damage value. Van Paepegem and Degrieck [2] show that opinions of the fatigue analysis and modeling community are divided regarding the stress sequence effect on the damage accumulation. Some experiments support the opinion that a high-low sequence is more damaging than a low-high sequence while other experiments suggest the opposite. In either case, this effect is an important aspect that needs to be considered to accurately predict reliability. Its importance is exemplified by the two de Havilland Comet passenger jets that crashed in 1954 within a few months of each other as a result of overlooking this effect. Later investigation showed that different stress sequencings produced very different fatigue lives [3].

Accurate reliability estimation models should consider the cumulative damage effect due to the different stress types and levels as well as the sequence of stress application. Stresses may cause a quantifiable damage to occur which ultimately induces a failure. This concept is referred to as cumulative damage modeling and has its roots in fatigue analysis. Cumulative damage modeling involves the use of a damage curve which relates stress exposure time to a damage metric that has a value between zero and one (meaning undamaged and completely damaged respectively). The effect of stress sequencing has been extensively studied in the fatigue analysis literature but has not been adequately address in probabilistic reliability analysis literature. However, there are parallels between the two areas and damage accumulation concepts in fatigue analysis can be related to reliability analysis. The objective of this paper is to define this parallel relationship and to enable the probabilistic reliability models to capture the

effect of stress sequencing in order to provide a more accurate estimate of reliability. This paper is organized into three sections: the first section reviews the damage accumulation models in fatigue analysis and how these models capture the effect of stress sequencing; the second section discusses probabilistic reliability models and incorporates the stress sequencing concepts; and the third section concludes the paper with a numerical illustration followed by remarks and summary.

DAMAGE ACCUMULATION MODELS: FATIGUE ANALYSIS

Miner's linear damage accumulation model [5] is perhaps the most commonly used model for damage accumulation when units are subject to multiple stress levels. It is expressed by Eq. (1) as follows.

$$D = \sum \frac{n_i}{N_i} \quad (1)$$

This model defines the total damage D as a linear function of n_i and N_i which are the number of applied cycles and total number of cycles to failure for the i^{th} stress level respectively. The total number of cycles until failure is the average number of cycles to failure for a sample of units subjected to a constant stress level. The Stress-Number of Cycles (S-N) function relates different stress levels with the average number of cycles to failure. From the linear damage model, the average remaining number of cycles can be determined after a unit is subjected to different stress levels by equating Eq. (1) to 1 and solving for the unknown number of cycles. The principal drawback of the linear models is their insensitivity to the stress sequence application since the total damage is the sum of independent terms. Obviously the order of which these terms are added does not change the resulting total damage. The inability of Miner's model to predict failure of components sensitive to load order can be corrected by the use of a nonlinear model [1]. Models that can account for the effect of the stress sequence are nonlinear models, contrarily not all nonlinear models can account for the stress sequence. Marco-Starkey [4] damage model is an example of such nonlinear models, it is expressed by Eq. (2).

$$D_m = \left(\frac{n_m}{N_m} \right)^{\alpha_m} \quad (2)$$

Where D_m is the accumulated damage after the application of stress level m for n_m cycles, N_m is the average number of cycles to failure and the exponent α_m is a function of the stress level m . This model belongs to the nonlinear class of models; however it may or may not be sensitive to the loading order depending on how the *total* damage is defined. When the total

damage D_k after the application of k stress levels is a summation of terms as defined by Eq. (3), then the model is considered insensitive to the loading order. This model has applicability in some situations as shown in [2] where internal damage measurements are available. It is explained here only to illustrate that a nonlinear model is not necessarily sensitive to the stress loading order.

$$D_k = \sum_{i=1}^k \left(\frac{n_i}{N_i} \right)^{\alpha_i} \quad (3)$$

It should be noted that damage accumulation models can apply to circumstances where the internal degradation level of a testing unit is measured by some macroscopic material property. This is commonly referred to as degradation testing and in this setting a unit is considered in the failure state when the degradation level reaches a pre-defined threshold. For example, the failure of an LED unit occurs when the light intensity reaches 50% of its original value. Likewise, failure occurs when the crack length in a unit exceeds a critical length under fatigue stresses. In this paper, we only consider the actual failure times and no measurements of damage or degradation are made before failures. Therefore, the only information about the progression of damage over time are the times associated with a damage value of zero (nits have not been subjected yet to stresses) and the damage value of one (when the units fail after applying stresses). In terms of a regression model, this means that there are only two observed values of the dependent variable, damage. Therefore it is impossible to fit any function other than a straight line to this data set consisting of damage and time measurements, and even a linear model is questionable. Therefore the explicit forms for the damage curve are not attainable.

For most cases, the damage after exposure to different stress levels using the Marco-Starkey model is not computed as a summation. A different procedure to calculate damage is employed using the exposure concept that postulates at the instant the stress changes, the state of damage remains momentarily constant (iso-damage transition) as shown in [6]. The damage then follows the path defined for the current stress level beginning from the current level of damage. Analytically, this implies that the damage is computed iteratively, and the total damage D_k after k stress levels have been applied is given by Eq. (4).

$$D_k = \left(\frac{n_k}{N_k} + D_{k-1} \frac{1}{\alpha_k} \right)^{\alpha_k} \quad (4)$$

To illustrate the sensitivity of this model and its application to analysis of failure time data, consider two independent step-stress tests; low stress level to high stress level and high stress

level to low stress level. In each case, the first stress level is applied until the damage reaches D_A or D_B , equal damage values as shown in Figure 1, followed by the application of the second stress level until the damage value reaches 1. For the purposes of this illustration, a damage value of 1 indicates a failure. For each stress level the unit follows a Marco-Starkey damage path with $\alpha_H < 1$ for the high stress and $\alpha_L > 1$ for the low stress. (Note, these values are not meant to suggest that the exponent α should decrease with increasing stress, they are called low and high stresses only for reference.)

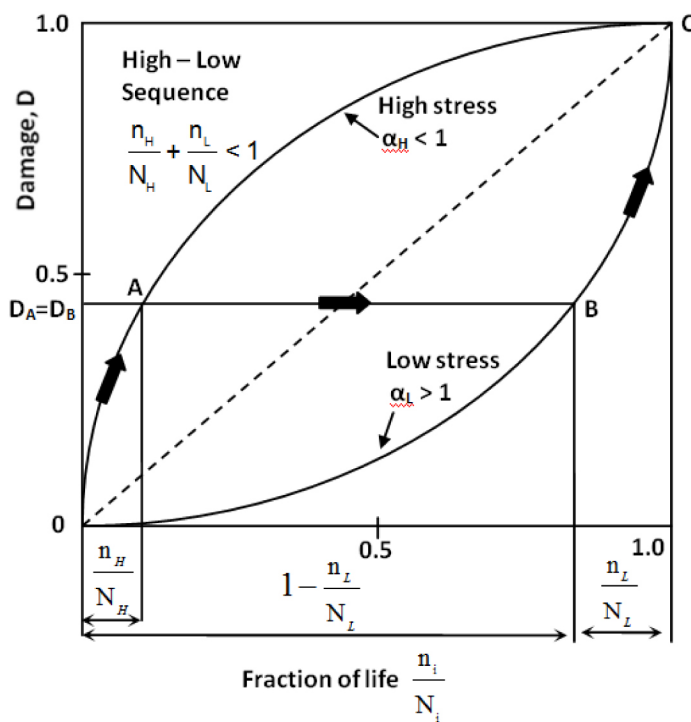
In Figure 1a, the high stress is applied until the damage equals D_A represented by point A, then the low stress is applied starting at the same damage value, point B, until the damage equals one at point C, implying failure of the test unit. In other words, the high stress is applied for a fixed amount of time, and then the low stress is applied until failure. The relationship between the fixed exposure time at the high stress and average time spent at the low stress until failure is established by equating the damage at points A and B as in Eq. (5).

Similarly in Figure 1b, the low stress is applied until the damage equals D_B represented by point B, then the high stress is applied starting at the same damage value, point A, until the damage equals 1 at point C. A relationship between the fixed exposure time at the low stress and average time spent at the high stress until failure is established by equating the damage at points B and A as in Eq. (6).

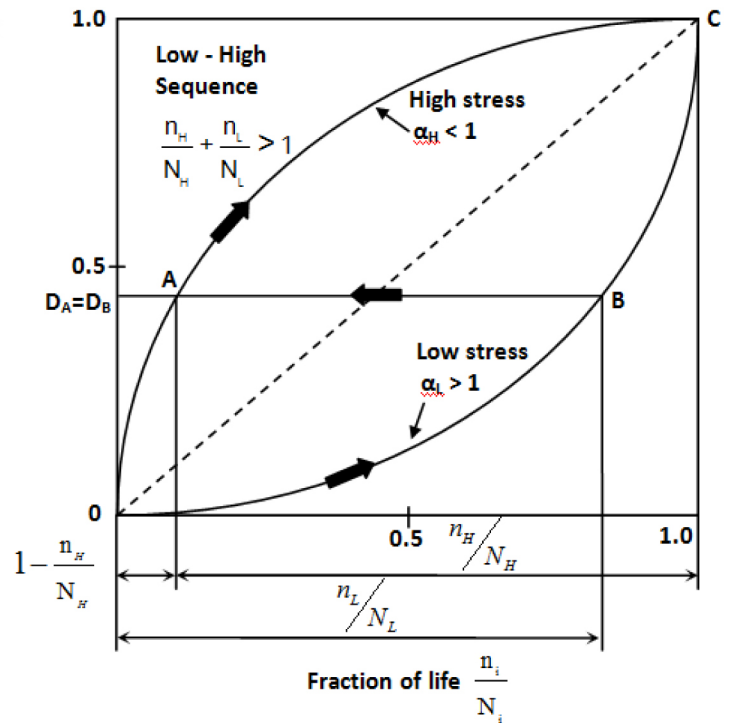
The estimation of the parameters of the model in Eq. (2) cannot be explicitly obtained using the failure times as described above. However, explicit values of these parameters are not required for the analysis of the failure data and it is sufficient to use the ratio of the exponents as shown in [7]. This concept is overlooked in some publications since estimates for the individual exponents are obtained when only failure times are recorded as in [8]. By taking the logarithm of Eq. (5), a linear relationship between $\log(n_L)$ and $\log(n_H)$ is obtained and can be readily used in a regression model. The ratio of exponents α_H/α_L is the only unknown term in Eq. (5) since N_H and N_L are the mean times to failure for the high and low stresses which are assumed to have prior estimates. In this regression, the independent variable is the fixed time at the first stress level, and the dependent variable is the failure time at the second stress level; random and may follow a failure time distribution. The slope of the linear relationship is the ratio of the two exponents.

$$D_A = D_B = \left(\frac{n_H}{N_H} \right)^{\alpha_H} = \left(1 - \frac{n_L}{N_L} \right)^{\alpha_L} \Leftrightarrow \frac{n_L}{N_L} = 1 - \left(\frac{n_H}{N_H} \right)^{\frac{\alpha_H}{\alpha_L}} \quad (5)$$

$$D_A = D_B = \left(\frac{n_L}{N_L} \right)^{\alpha_L} = \left(1 - \frac{n_H}{N_H} \right)^{\alpha_H} \Leftrightarrow \frac{n_H}{N_H} = 1 - \left(\frac{n_L}{N_L} \right)^{\frac{\alpha_L}{\alpha_H}} \quad (6)$$



a



b

Figure 1. Illustration of Load Order Sensitivity in the Marco-Starkey Damage Curve Model