

Backward-Looking Simulation of the Toyota Prius and General Motors Two-Mode Power-Split HEV Powertrains

2011-01-0948

Published
04/12/2011

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doi:10.4271/2011-01-0948

ABSTRACT

This paper presents a comparative analysis of two different power-split hybrid-electric vehicle (HEV) powertrains using backward-looking simulations. Compared are the front-wheel drive (FWD) Toyota Hybrid System II (THS-II) and the FWD General Motors Allison Hybrid System II (GM AHS-II). The Toyota system employs a one-mode electrically variable transmission (EVT), while the GM system employs a two-mode EVT. Both powertrains are modeled with the same assumed mid-size sedan chassis parameters. Each design employs their native internal combustion (IC) engine because the transmission's characteristic ratios are designed for the respective brake specific fuel consumption (BSFC) maps. Due to the similarities (*e.g.*, power, torque, displacement, and thermal efficiency) between the two IC engines, their fuel consumption and performance differences are neglected in this comparison. The road-load parameters defining each system are used to calculate the required mechanical power at the driven wheels necessary to follow a given drive-cycle. Admissible engine operating states are sought based on component performance limitations and the required mechanical power at the driven wheels. Each IC engine operating point defines an accompanying battery power consistent with the constraints of the electric machines. The design approach is to exhaustively search all admissible states and minimize an instantaneous cost function based on engine power and battery power, at each time instant of the drive-cycle. Two cost functions are considered which weight battery power usage using either a linear, or an inverse-tangent, function of the current battery state-of-charge (SOC). Selected operational states are then compared against each other based on the flexibility and power delivery capabilities of the powertrains. Fuel minimizing cost functions are

determined with the assistance of a charge sustaining index introduced by this paper. Finally, the most fuel efficient choices are used to determine the expected efficiency of both powertrains considered.

INTRODUCTION

The three fundamental HEV architectures in use today are series, parallel and power-split. The operation of the IC engine and electric machines (EMs) in each architecture ultimately dictates the powertrain's efficiency. Since IC engines are more variable in their efficiency than EMs, it suffices to consider the IC engine operation when qualitatively assessing overall powertrain efficiency. Parallel architectures employ a highly-efficient mechanical path to transmit input IC engine power to the driven wheels. Engine speed and efficiency is therefore constrained by a finite number of fixed-gear (FG) ratios or their equivalents. IC engine operation is best suited for steady power delivery (*e.g.*, highway driving at constant cruising speed), as opposed to dynamic power delivery (*e.g.*, urban or variable driving). For dynamic power delivery over a large range of vehicle speeds, a finite number of FG ratios can result in potentially inefficient engine operation. In series architectures, the IC engine operates independently of the road-load conditions providing steady, highly-efficient operation. Series electro-mechanical power delivery is best suited for urban driving with significant vehicle speed fluctuations. However, electro-mechanical power delivery is less efficient than mechanical due to energy conversion losses. Power-split designs combine the advantages of single path architectures, such as parallel and series, by providing two power paths between the IC engine and the driven wheels through gearing and EMs.

Since their introduction in the early 1970's [1], electrically variable transmissions (EVTs) have evolved into two distinct configurations found in common use today: namely the one-mode and the two-mode EVT. The historical development of power-split transmissions tracing this evolution can be found in [2]. One-mode EVTs have been extensively studied, specifically as they pertain to the THS-II input-split hybrid powertrain system. Detailed descriptions of their basic operation can be found in [3,4]. Due to energy conversion losses in the electrical energy path, engine operation on the engine optimal operation line may not minimize fuel consumption in power-split hybrids. Other studies have quantified their transmission efficiency, and in doing so, have shown that a critical value of a speed ratio can be defined which determines the onset of power circulation loops [5]. By considering transmission losses and engine specific fuel consumption, system operating points minimizing fuel consumption can be found [6]. These results give rise to the approach of analyzing planetary gear hybrid powertrain optimal operation based on multi-objective optimization, evaluating both fuel and electric energy consumption. By evaluating both fuel and electric energy consumption, minimization of an equivalent energy consumption index results in operation nearly as efficient as the theoretical maximum performance obtained from dynamic optimization, the globally optimal solution [7]. A number of optimization approaches have been applied to the supervisory control strategy of single-mode power-split hybrid electric vehicles [8]. It has also been shown that two-mode power-split transmissions show improvements in efficiency and dynamic performances in comparison to single-mode planetary transmissions [9, 10, 11, 12]. Although previous publications evaluate the rear-wheel drive (RWD) GM AHS-II powertrain, this paper presents an analysis of the FWD GM AHS-II powertrain.

THS-II ARCHITECTURE OVERVIEW

The THS-II architecture is an input-split EVT whereby the input engine power is effectively split between a mechanical and electromechanical path. The THS-II powertrain is composed of several key components: battery, IC engine, two inverters, two motor-generator EMs (M/GA and M/GB), and a planetary power-split device (PSD). [Figure 1](#) displays the FWD THS-II powertrain along with the positive power conventions assumed throughout this paper.

The PSD in the THS-II powertrain is a three axis simple planetary gearset with three pinion gears. The carrier shaft is directly linked to the engine and mechanically transmits the motive engine power to the outer ring gear and inner sun gear via pinion gears. EM M/GA typically generates electricity and is connected to the sun gear, giving rise to the serial power path. The ring gear is directly coupled to the larger propulsive EM M/GB and the front axle through fixed reduction gearing, giving rise to the parallel path. Focusing on the PSD, the IC

engine provides the input power and the ring and sun gear deliver output power. The rotational speed of the common M/GB -output shaft $\omega_{M/GB}$ is proportional to vehicle speed by final-drive ratio G , thus for any prescribed vehicle speed output angular velocity $\omega_o \equiv \omega_{M/GB}$ is known. Both EMs are capable of bi-directional energy flow by operating as either a motor or as a generator. The battery is an energy buffer that cannot be externally recharged, therefore battery SOC must be maintained within usable limits or the integrity of the battery is jeopardized. Diversion of engine power into the electro-mechanical power path in [Figure 1](#) is enabled by the PSD and will be detailed next.

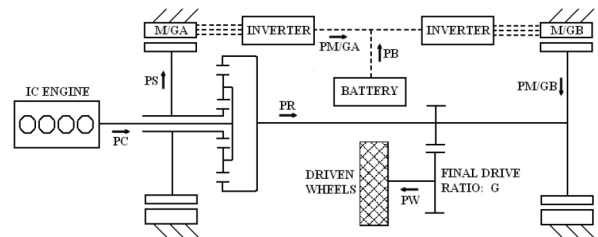


Figure 1. Cut-away of the THS-II powertrain

The PSD allows the IC engine power to be simultaneously transferred to M/GA and the wheels. A three axis simple planetary gearset with three pinion gears is shown in [Figure 2](#).

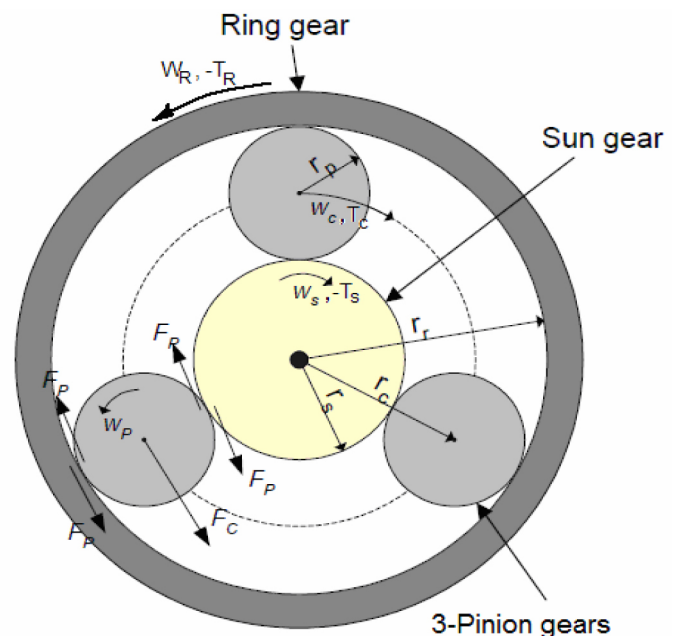


Figure 2. A simple planetary gearset is the PSD in the THS-II powertrain, reproduced from [13].

The characteristic ratio for the planetary gearset R is defined as,

$$R = \frac{r_{sun}}{r_{ring}} = \frac{N_{sun}}{N_{ring}}, \quad (1)$$

where r is the radius and N is the number of teeth on the respective gears. Torque constraints on the simple planetary are derived by imposing a power balance neglecting gearset accelerations and losses, reducing to the steady-state torque relationship,

$$T_C = (1+R)T_R = \frac{(1+R)}{R}T_S, \quad (2)$$

where T_C , T_R , and T_S are the carrier, ring, and sun gear torques respectively. Kinematic speed constraints based on equal speeds at points of contact where meshing occurs,

$$(1+R)\omega_C - \omega_R - R\omega_S = 0, \quad (3)$$

exist such that ω_C , ω_R , and ω_S are the carrier, ring, and sun gear rotational velocities respectively. From (2) and (3), it can be noted there are two independent speeds and one independent torque constraining the power delivery through all branches of the THS-II PSD. In what follows we return to using angular velocities ω_e , ω_o , and $\omega_{M/GA}$ for ω_C , ω_R , and ω_S , respectively, where ω_e denotes the IC engine angular velocity and $\omega_{M/GA}$ denotes the angular velocity of M/GA .

The speed ratio is an important measure that dictates the direction of power flow through M/GA by changing the rotational direction of the sun gear and ultimately the machine's functionality, motoring versus generating. The speed ratio of the transmission,

$$SR = \frac{\omega_e}{\omega_o} = \frac{1}{1+R} + \frac{R}{(1+R)} \frac{\omega_{M/GA}}{\omega_o}, \quad (4)$$

is the ratio of the input engine speed to the output shaft speed. Operation where the transmission is rotating but M/GA is stationary occurs at the "mechanical point" speed ratio where all of the input engine power is mechanically delivered to the common ring gear- M/GB output shaft. The "mechanical point" tends to be the most efficient ratio for power flow through the transmission since none of the transmitted mechanical power is subject to M/GA energy conversion losses. The "mechanical point" speed ratio is then,

$$SR_{mech} = \frac{1}{1+R}. \quad (5)$$

The smaller M/GA is controlled such that the machine is capable of dictating the speed ratio through the transmission using the sun gear, while the larger M/GB does not affect the speed ratio as it is connected directly to the wheels. Two relevant facts regarding planetary gearsets are: i) when any two terminals are connected together, all three terminals rotate at the same speed in the same direction *i.e.* the gearset becomes locked-up ii) when any terminal is stationary, no power flows through that terminal *i.e.* the gearset reduces to a conventional two-terminal gearset.

Power flow through each branch of the PSD is modeled with component efficiencies and constraining speed and torque equations. The IC engine is characterized by its thermal efficiency and both EMs are characterized by their efficiency maps, inverter energy conversion losses included. The required wheel power,

$$P_W = \eta_{FD}[P_o + P_{M/GB}], \quad (6)$$

is delivered from a combination of output ring gear power P_o and M/GB propulsive power $P_{M/GB}$ through the final-drive gearing as shown in Figure 1. In (6), P_W is the mechanical power delivered to the wheels and η_{FD} is the efficiency of the final-drive gearing. For the final-drive gearing, a constant efficiency 0.95 will be used in this analysis since the efficiency is known to be nearly independent of the road-load conditions [14]. The direction and type of power, either generating or motoring, delivered by M/GB depends on the direction of the applied torque T_o . The output ring gear power is mechanically transmitted through the planetary gearset. From the conservation of power, the output ring gear power is,

$$P_o = \eta_P[P_e - P_{M/GA}], \quad (7)$$

where P_e is the input engine power into the carrier gear, $P_{M/GA}$ is the power delivered to M/GA , and η_P is the planetary gear efficiency in transferring engine power through the planetary gearset. For the planetary gearset, a constant efficiency of 0.98 will be used in this analysis because the efficiency is known to be nearly independent of the road-load conditions [14]. The IC engine is continuously connected to the carrier gear $T_e = T_C$, therefore from (2), the output ring and torque delivered to M/GA with losses are,

$$T_o = \eta_P \frac{1}{1+R} T_e, \quad (8)$$

$$T_{M/GA} = \eta_P \frac{R}{1+R} T_e, \quad (9)$$

where η_P reduces the torque delivered by the engine. The ring and sun gear output powers are,

$$P_o = \eta_P \frac{1}{1+R} T_e \omega_o, \quad (10)$$

$$P_{M/GA} = \eta_P \frac{R}{1+R} T_e \omega_{M/GA}, \quad (11)$$

where the rotational speed of M/GA on the sun gear can be calculated using (3) and the definition of the speed ratio,

$$\omega_{M/GA} = \frac{(1+R)\omega_e - \omega_o}{R} = \frac{(1+R-1/SR)}{R} \omega_e. \quad (12)$$

Battery power is dependent on the functionality of M/GA and M/GB . Throughout this analysis the positive battery power convention is out of the battery and battery turn-around losses are neglected; *i.e.* $\eta_B = 1$. This has no bearing on the comparison of the two architectures considered herein. When rotating in the positive direction, M/GA acts as a generator by absorbing mechanical power through the sun gear and converting it into electrical power before final transmission to the electrical bus. For $SR > SR_{mech}$, the battery power expression,

$$P_B = \begin{cases} \frac{(T_{M/GB}\omega_o)}{\eta_{M/GB}} - \eta_{M/GA}\eta_P \frac{R}{1+R} T_e \omega_{M/GA} & , T_{M/GB} > 0 \\ \eta_{M/GB}(T_{M/GB}\omega_o) - \eta_{M/GA}\eta_P \frac{R}{1+R} T_e \omega_{M/GA} & , T_{M/GB} < 0 \end{cases}, \quad (13)$$

depends on M/GB functionality. When rotating in the negative direction, M/GA acts as a motor by consuming battery power and converting it into mechanical power, delivering it to the PSD and assisting in propelling the vehicle. The ensuing battery power for $SR < SR_{mech}$ operation,

$$P_B = \begin{cases} \frac{(T_{M/GB}\omega_o)}{\eta_{M/GB}} - \frac{(\frac{R}{1+R} T_e \omega_{M/GA})}{\eta_{M/GA}} & , T_{M/GB} > 0 \\ \eta_{M/GB}(T_{M/GB}\omega_o) - \frac{(\frac{R}{1+R} T_e \omega_{M/GA})}{\eta_{M/GA}} & , T_{M/GB} < 0 \end{cases}, \quad (14)$$

can be calculated. At the “mechanical point” speed ratio, the power from the engine is purely transmitted to the ring gear resulting in no M/GA power flow. For $SR = SR_{mech}$, battery power,

$$P_B = \begin{cases} \frac{(T_{M/GB}\omega_o)}{\eta_{M/GB}} & , T_{M/GB} > 0 \\ \eta_{M/GB}(T_{M/GB}\omega_o) & , T_{M/GB} < 0 \end{cases}, \quad (15)$$

is defined based on M/GB functionality.

GM AHS-II ARCHITECTURE OVERVIEW

The GM AHS-II powertrain is capable of operating in input-split or compound-split EVT modes as well as four FG configurations. Multi-mode operation is enabled by using multiple planetary gearsets and mode changing clutches. The GM AHS-II powertrain is composed of several key components: battery, IC engine, two braking clutches, two rotating transfer clutches, two inverters, two motor-generator EMs, and two planetary PSDs ($PG1$ and $PG2$). The key components of the FWD GM AHS-II powertrain and positive power conventions assumed throughout this analysis are displayed in Figure 3.

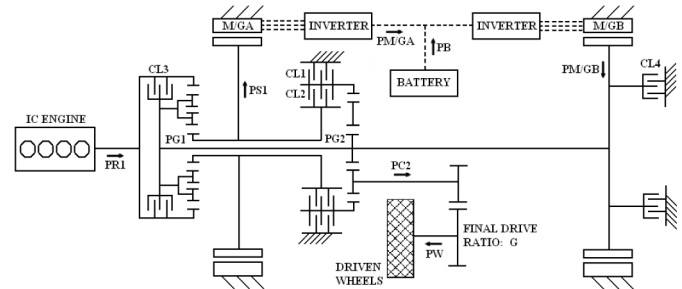


Figure 3. Cut-away of the GM AHS-II powertrain

The first PSD, $PG1$, is a compound planetary gearset with three sets of two pinion gears mounted on a single carrier. The second PSD, $PG2$, is a simple planetary gearset with three single pinion gears between the sun and ring gears as used in THS-II transmission. Power is input from the IC engine which is continuously connected to the ring gear of $PG1$. Power is transmitted from the carrier gear of $PG1$ to the sun gear of $PG2$ through a continuous connection. M/GA is continuously connected to the sun gear of $PG1$ and M/GB is continuously connected to both the $PG1$ carrier gear and $PG2$ sun gear. The rotational speed of $PG2$ carrier gear is proportional to vehicle speed by the final-drive ratio G . Thus for any prescribed vehicle speed, the rotational speed of the $PG2$ carrier $\omega_o \equiv \omega_{C2}$ is known. The four clutches displayed in Figure 3 allow additional connections required to implement any of the six operating modes. Table 1 displays the operating modes and the respective clutches that enable each.

Table 1. Clutches associated with mode selection

Mode	CL1	CL2	CL3	CL4
EVT-1	Engaged			
EVT-2		Engaged		
FG-1	Engaged		Engaged	
FG-2	Engaged	Engaged		
FG-3		Engaged	Engaged	
FG-4		Engaged		Engaged

Like the THS-II powertrain, both EMs are capable of bi-directional energy flow and the battery is an energy buffer that cannot be externally recharged. Therefore, SOC must be maintained within operational limits or the integrity of the battery is jeopardized.

The addition of the three axis compound planetary gearset with three sets of two pinion gears mounted on a single carrier gear in [Figure 4](#) enables the second EVT mode of the GM AHS-II powertrain.

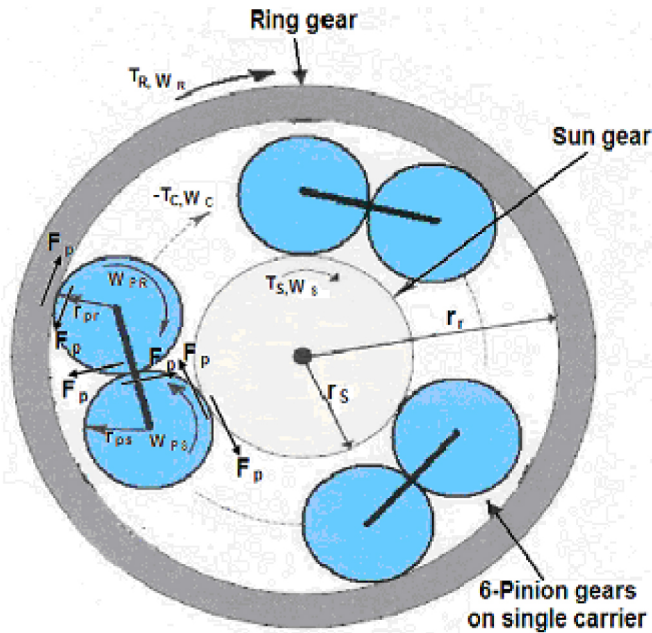


Figure 4. PG1 in the GM AHS-II powertrain is a compound planetary gearset, reproduced from [13].

The second PSD enables additional degrees of freedom and advantageous operating modes. The characteristic ratio for the compound planetary gearset R_1 and the simple planetary gearset R_2 in [Figure 3](#) are defined by,

$$R_1 = \frac{r_{1,sun}}{r_{1,ring}} = \frac{N_{1,sun}}{N_{1,ring}}, \quad (16)$$

$$R_2 = \frac{r_{2,sun}}{r_{2,ring}} = \frac{N_{2,sun}}{N_{2,ring}}, \quad (17)$$

where r is the radius and N is the number of teeth on the respective gears. Subscripts 1 and 2 denote the compound and simple planetary gearsets. Torque constraints on the planetaries are derived by imposing a power balance neglecting gearset accelerations, reducing to the steady-state torque relationships,

$$T_{C1} = -(1 - R_1)T_{R1} = \frac{(1 - R_1)}{R_1}T_{S1}, \quad (18)$$

$$T_{C2} = -(1 + R_2)T_{R2} = -\frac{(1 + R_2)}{R_2}T_{S2}, \quad (19)$$

where T_C , T_R , and T_S are the carrier, ring, and sun gear torques for the respective gearsets. Kinematic speed constraints based on equal speeds at points of contact where meshing occurs exist,

$$\omega_{C1} = \frac{1}{1 - R_1}\omega_{R1} - \frac{R_1}{1 - R_1}\omega_{S1}, \quad (20)$$

$$\omega_{C2} = \frac{1}{1 + R_2}\omega_{R2} + \frac{R_2}{1 + R_2}\omega_{S2}, \quad (21)$$

such that ω_C , ω_R , and ω_S are the carrier, ring, and sun gear rotational velocities of the respective gearsets. From the torque and speed constraints of each gearset, it can be noted there is one independent torque and two independent speeds constraining the power delivery through the three branches of each PSD. The relevant facts discussed in *THS-II Architecture Overview* regarding simple planetary gearsets also apply to compound planetary gearsets.

MODES OF OPERATION

Power flow through each branch of the PSDs is modeled by component efficiencies and constraining speed and torque equations. The IC engine is characterized by its thermal efficiency and the EMs are characterized by their efficiency maps, inverter losses included. The mode of operation dictates the constraining speed and torque equations of the EMs (motoring vice generating). The constraining component speed and torque equations are derived in terms of the IC engine input and PG2 carrier output for use in simulation. Engine power is a control strategy degree of freedom and PG2 carrier gear power is dependent on the drive-cycle.

EVT-1 corresponds to input-split operation and is very similar to THS-II operation. *PG1* splits input engine power between the mechanical and electro-mechanical power path. This dual-mode hybrid powertrain is designed to shift modes according to changes in the driving environment at a synchronous shift speed ratio between the two EVT modes. As vehicle speed increases for a given engine speed, energy flow through the electro-mechanical path reverses undesirably. Undesirable reversal of energy flow through the electro-mechanical path is prevented by switching to *EVT-2*. *EVT-2* corresponds to compound-split operation in which *PG1* splits input engine power and *PG2* combines the split mechanical and electro-mechanical power paths before the final-drive gearing. The GM AHS-II transmission also has four FG ratios that provide a purely mechanical power path from the IC engine to the driven wheels without the need for electro-mechanical power. FG operation also has the flexibility of utilizing one or both EMs to contribute to the tractive effort by motoring or generating. FG operation improves towing, climbing and maximum acceleration performance but constrains engine speed to one of four values for a given vehicle speed. Regardless of the transmission speed ratio, *FG-2* and *FG-4* correlate to “mechanical point” speed ratios such that *M/GA* and *M/GB* respectively do not rotate. By not transmitting power, the EMs are not subject to energy conversion losses. By holding *M/GA* stationary in *FG-2*, *FG-2* enables a seamless transition amongst EVT modes.

EVT-1

Input-split operation is achieved by locking the ring gear of *PG2* with *CL1*. The input engine power is split by *PG1* while *PG2* provides torque multiplication by serving as a conventional two terminal gearset. While operating in *EVT-1*, the constraining speed equations are,

$$\omega_{M/GA} = \frac{1}{R_1} \omega_e - \frac{(1-R_1)(1+R_2)}{R_1 R_2} \omega_o, \quad (22)$$

$$\omega_{M/GB} = \frac{1+R_2}{R_2} \omega_o, \quad (23)$$

and the torque constraints are,

$$T_{M/GA} = -R_1 T_e, \quad (24)$$

$$T_{M/GB} = -(1-R_1)T_e + \frac{R_2}{1+R_2} T_o. \quad (25)$$

EVT-2

Compound-split operation is achieved by locking the ring gear of *PG2* to the sun gear of *PG1* with *CL2*. The power is

split by *PG1* at the input and combined by *PG2* at the output. While operating in *EVT-2*, the constraining speed equations are,

$$\omega_{M/GA} = -\frac{R_2}{1-R_1-R_1 R_2} \omega_e + \frac{(1-R_1)(1+R_2)}{1-R_1-R_1 R_2} \omega_o, \quad (26)$$

$$\omega_{M/GB} = \frac{1}{1-R_1-R_1 R_2} \omega_e - \frac{R_1(1+R_2)}{1-R_1-R_1 R_2} \omega_o, \quad (27)$$

while the torque constraints are,

$$T_{M/GA} = -R_1 T_e + \frac{1}{1+R_2} T_o, \quad (28)$$

$$T_{M/GB} = -(1-R_1)T_e + \frac{R_2}{1+R_2} T_o. \quad (29)$$

The preferred direction of power flow though the EMs is reversed when operating in *EVT-2* relative to *EVT-1*.

FG-1

PG1 is locked by *CL3* allowing direct drive and the ring gear of *PG2* is held stationary by *CL1*, providing speed reduction. *FG1* enables the greatest FG ratio and is best utilized for maximum FG acceleration. Derived from the planetary gearsets constraining equations, *FG-1* angular velocities are,

$$\omega_e = \omega_{M/GA} = \omega_{M/GB} = \frac{1+R_2}{R_2} \omega_o, \quad (30)$$

and the torque constraint is,

$$T_o = \frac{1+R_2}{R_2} (T_e + T_{M/GA} + T_{M/GB}). \quad (31)$$

FG-2

Through the use of *CL1* and *CL2*, *FG-2* is the synchronous shift mode allowing a transition between *EVT-1* and *EVT-2* since *M/GA* is stationary. From *FG-2*, disengaging either *CL1* or *CL2* enables one of the EVT modes. Derived from the planetary gearsets constraining equations, *FG-2* angular velocities are constrained such that,

$$\omega_e = \frac{(1-R_1)(1+R_2)}{R_2} \omega_o, \omega_{M/GA} = 0, \omega_{M/GB} = \frac{1+R_2}{R_2} \omega_o, \quad (32)$$