

Adaptive EKF-Based Estimator of Sideslip Angle Using Fusion of Inertial Sensors and GPS

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ABSTRACT

This paper presents an adaptive extended Kalman filter (EKF)-based sideslip angle estimator, which utilizes a sensor fusion concept that combines the high-rate inertial sensors measurements with the low-rate GPS velocity measurements. The sideslip angle estimation is based on a vehicle kinematic model relying on the lateral accelerometer and yaw rate gyro measurements. The vehicle velocity measurements from low-cost, single antenna GPS receiver are used for compensation of potentially large drift-like estimation errors caused by inertial sensors offsets. Adaptation of EKF state covariance matrix ensures a fast convergence of inertial sensors offsets estimates, and consequently a more accurate sideslip angle estimate. By using a detailed simulation analysis, it is found out that the main sources of estimation errors include inaccuracies of pre-estimated vehicle longitudinal velocity obtained from nondriven wheel speed sensors, the GPS velocity signal latency, and the road bank-related disturbances. Several compensation methods are proposed to suppress the influence of these errors.

INTRODUCTION

Existence of accurate sideslip angle information allows for implementation of combined sideslip and yaw rate feedback control for improved robustness and responsiveness of the overall vehicle stability control system [1,2]. The main obstacle for a successful implementation of such a state-space controller for mass production vehicles relates to difficulties with sideslip measurement/estimation. Direct measurement is impractical due to a high cost of sideslip angle sensors. On the other hand, a robust sideslip angle estimator that should

ensure a high level of accuracy for various driving maneuvers and road conditions is difficult to design. Therefore, significant research efforts have been undertaken in the last twenty years in order to develop reliable sideslip angle estimators.

Traditionally, there are two main types of sideslip angle estimators: (i) vehicle dynamics model-based estimators [2,3,4,5] and (ii) kinematic model-based estimators [6,7]. The main problem with the vehicle dynamics model approach is that the uncertainty of model parameters such as the tires cornering stiffness and the vehicle mass can significantly affect the accuracy of estimation. Moreover, frequently used, simplified, linearized vehicle/tire models cannot accurately describe the vehicle motion at the limit of adhesion when the vehicle stability is compromised. On the other hand, the inertial sensor offsets, as well as the road bank, significantly affect the estimation accuracy when using the kinematic approach. Several concepts of combined estimation using both the dynamic and kinematic models have been proposed [1,8,9], in order to benefit from the complementary advantages of the two approaches.

Recent introduction of new vehicle dynamics sensors such as Global Positioning System (GPS) [10,11] or 6DOF inertial measurement units [12] opens some significant new possibilities towards viable sideslip estimation based on advanced sensor fusion concepts. Several fusion concepts of GPS and inertial sensors have been documented in literature. For instance, the single antenna GPS receiver (low update rate 1Hz) and yaw rate gyro have been combined within a Kalman filter (KF) [10], in order to compensate for the gyro offset and enable drift-less sideslip angle estimation.

However, this approach enables estimation only during abrupt transients, while under the steady-state conditions the vehicle heading and sideslip angles become unobservable. Another limitation is related to the assumption of constant sideslip angle between the two consecutive GPS measurements used for gyro offset compensation. In [11,19] the two-antenna GPS receiver, giving the vehicle velocities and course with the update rate of 5Hz, has been combined with inertial sensors measurements within a kinematic estimator of vehicle sideslip angle. In particular, the coarse measurement has been used for compensation of the gyro offset-related errors in vehicle heading and sideslip angle estimation. Moreover, the GPS velocity and accelerometers measurements are combined for the kinematic estimation of vehicle velocity and compensation of the accelerometer offset-related drift-like errors, w/o using the wheel speed measurements. The main shortcoming of this approach is a high cost of the two antenna GPS receiver which prevents this concept to be implemented in compact class vehicles.

This paper deals with sideslip estimation using fusion of inertial sensors (lateral accelerometer and yaw gyroscope) and a standard one-antenna GPS receiver. Inertial sensor approach can provide fast sideslip estimate response, but it suffers from estimate drift due to inertial sensor offsets, and it is sensitive to modeling errors (e.g. road bank influence). On the other hand, the GPS can provide accurate velocity estimates, but its sampling rate (typically 1 Hz) is too low for vehicle dynamics control applications and it has a limited availability (e.g. in urban canyon regions). The sensor fusion has a good potential to benefit from the complementary advantages of the two individual approaches. The sensor fusion is conducted through the adaptive Extended Kalman Filtering (EKF) approach [13,14,15]. The estimator state-space model has been designed based on an appropriate vehicle kinematic model extended with random walk processes for accelerometer and gyro offsets, while the GPS-based velocity measurements constitute the process output equation. The estimator performance has been verified by computer simulations using a 10DOF vehicle dynamics model [16].

KINEMATIC MODEL

The vehicle body motion and related inertial sensors measurements can be described by a set of kinematics equations. In order to formulate these equations, an appropriate coordinate frames need to be defined (Fig. 1). The inertial coordinate frame ($X Y Z$) is used as a reference frame, while the moving frame ($X_b Y_b Z_b$) is fixed to the vehicle body (Fig. 1a). Within the inertial frame the vehicle heading/yaw angle ψ , vehicle track angle/course ϕ , and vehicle velocity V are defined. The vehicle body-fixed frame origin resides in the vehicle body center of gravity (CoG) and its axes point in the forward, lateral, and upward direction. Within this moving frame, the vehicle velocities (u -

longitudinal, v - lateral, and w - vertical) and angular speeds (ω_x - roll rate, ω_y - pitch rate, and ω_z - yaw rate) are defined as illustrated in Fig. 1b.

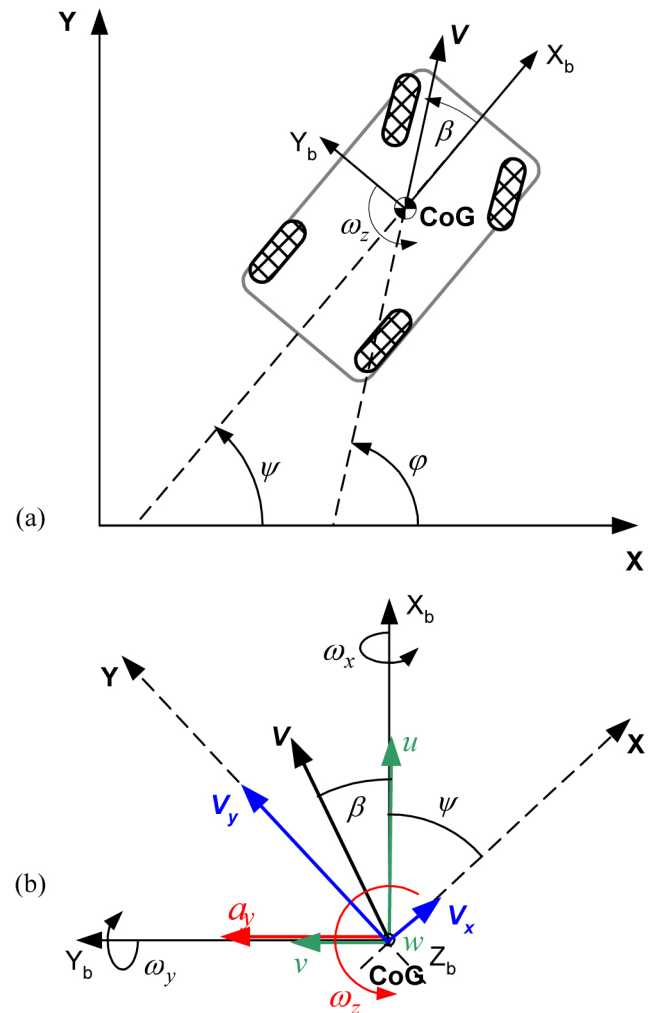


Figure 1. Definition of kinematic model coordinate frames (a) and kinematic variables (b).

The vehicle sideslip angle β can be defined within the inertial coordinate frame (Fig. 1a), as a difference between the vehicle course and heading angles, Eq. (1), or alternatively it can be derived according to Eq. (2) from the vehicle velocity components defined within the vehicle body-fixed coordinate frame (Fig. 1b):

$$\beta = \phi - \psi \quad (1)$$

$$\beta = \text{atan}\left(\frac{v}{u}\right) = \text{asin}\left(\frac{v}{V}\right) \quad (2)$$

The velocities defined in the inertial frame (V_x, V_y) and the vehicle body frame (u, v) are connected through equations:

$$V_y = u \sin \psi + v \cos \psi \quad (3)$$

$$V_y = u \sin \psi + v \cos \psi \quad (4)$$

The considered vehicle motion kinematic model and related state variable estimator rely on the measurements of lateral accelerometer ($a_{y,m}$) and yaw rate gyro ($\omega_{z,m}$), which are placed in the vehicle CoG and represent the **inertial sensors** typically used within VDC systems. These measurements are described by the following expressions (see [22] and references therein):

$$a_{y,m} = \dot{v} + \omega_z u - \omega_x w + g \sin \phi \cos \theta + a_{y,off} + v_a \quad (5)$$

$$\omega_{z,m} = \omega_z + \omega_{z,off} + v_\omega \quad (6)$$

where $a_{y,off}$ and $\omega_{z,off}$, and v_a and v_ω are accelerometer and gyro offsets and measurement noise, respectively, while ϕ (suspension roll and road bank) and θ (suspension pitch and road grade) are total roll and pitch Euler angles, respectively. For most applications the third right-hand side term in Eq. (5) can be neglected, because the vertical velocity component w is small and it may be conveniently modeled as an additional noise component in v_a . Furthermore, the approximation of trigonometric functions ($\sin \phi \approx \phi$ and $\cos \theta \approx 1$) can be applied for the small-angle assumption. Thus, the following approximate expression of lateral acceleration measurement holds:

$$a_{y,m} \approx \dot{v} + \omega_z u + g \phi + a_{y,off} + v_a \quad (7)$$

The offsets of inertial sensors measurements $a_{y,off}$ and $\omega_{z,off}$ represent a dominant source of errors in estimating the lateral velocity v and the heading angle ψ . Namely, since these estimates are obtained by direct integration of inertial measurements, they are prone to drift in the presence of the sensor offsets.

Apart from the inertial sensors, the **GPS receiver**, typically used for navigational purposes (i.e. vehicle position monitoring), can be utilized to obtain vehicle velocity, heading, and course [10,11]. Namely, the GPS receiver can provide three-dimensional vehicle velocity measurements in the inertial coordinate frame, V_{GPS}^N , V_{GPS}^E , and V_{GPS}^{up} , whereof the north and east velocity components are of the main interest for the automotive applications. These velocities are given by (see Fig. 1b and Eqs. (3) and (4)):

$$V_{GPS}^N \triangleq V_x + e_n = u \cos \psi - v \sin \psi + e_n \quad (8)$$

$$V_{GPS}^E \triangleq V_y + e_e = u \sin \psi + v \cos \psi + e_e \quad (9)$$

where e_n and e_e are measurement errors of north and east velocity components, respectively. The GPS-based velocity measurement errors change with satellite geometry or PDOP (Position Dilution of Precision) and/or multi-path errors. Typical velocity measurement accuracies (1σ confidence range) are 2-5 cm/s for the horizontal velocity components (i.e. north and east) and 4-10 cm/s for the vertical component [10].

Unlike the inertial sensors, the GPS receiver provides unbiased velocity measurements. However, the main drawbacks of the GPS-based measurements are related to occasional loss of signal and a low update rate (typically 1Hz for low cost receivers) that is insufficient for the vehicle dynamics control applications.

PROCESS MODEL FOR ESTIMATOR DESIGN

Sensor fusion concept is based on integration of the biased inertial sensors measurements $a_{y,m}$ and $\omega_{z,m}$, given by Eqs.

(6) and (7), with the GPS velocity measurements V_{GPS}^N and V_{GPS}^E , expressed by Eq. (8). The GPS velocity measurements are used along with the vehicle kinematic model in order to estimate the inertial sensors offsets. The state-space process model (10), used herein for offset estimation, is derived by initially neglecting the unknown roll/road bank-related gravity acceleration component from the accelerometer measurement (7) and by modeling the inertial sensors offsets as random walk states. The road bank angle ϕ , acting as a major disturbance, can be pre-estimated and the related acceleration component $g \sin \phi \approx g \phi$ is subtracted from the accelerometer measurement $a_{y,m}$ (see the compensation section below).

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{W}\mathbf{v} \quad (10)$$

where: $\mathbf{x} = [v \quad \psi \quad a_{y,off} \quad \omega_{z,off}]^T$, $\mathbf{u} = [a_{y,m} \quad \omega_{z,m}]^T$, $\mathbf{v} = [v_v \quad v_\omega \quad v_{y,off} \quad v_{\omega,off}]^T$, $\mathbf{W}$ is unit matrix, and

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -1 & \hat{u} \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & -\hat{u} \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Note that the above model is linear with respect to state variables (i.e. v , ψ , $a_{y,off}$, and $\omega_{z,off}$), but it is time-variant through vehicle longitudinal velocity $u = (\omega_{rl} r_{rl} + \omega_{rr} r_{rr})/2$ representing a model parameter. This velocity is not directly measured, but it can be reconstructed from the nondriven (rear) wheels speeds as follows:

$$\hat{u} = \frac{r_n}{2}(\omega_{rl} + \omega_{rr}) \quad (11)$$

where the nominal value of the tire effective radius r_n is used instead of the unknown true values r_{rl} and r_{rr} .

The GPS velocity measurements (8) represent the nonlinear output equation:

$$\begin{bmatrix} V_{GPS}^E \\ V_{GPS}^N \end{bmatrix} = \begin{bmatrix} \hat{u} \sin \psi + v \cos \psi \\ \hat{u} \cos \psi - v \sin \psi \end{bmatrix} + \begin{bmatrix} e_e \\ e_n \end{bmatrix} \quad \mathbf{y} = \mathbf{c}(\mathbf{x}) + \mathbf{e} \quad (12)$$

After linearizing the above output equation, the following linearized output model is obtained:

$$\begin{aligned} \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{e} \\ &= \begin{bmatrix} \cos \psi_0 & \hat{u} \cos \psi_0 - v_0 \sin \psi_0 & 0 & 0 \\ -\sin \psi_0 & -\hat{u} \sin \psi_0 - v_0 \cos \psi_0 & 0 & 0 \end{bmatrix} \mathbf{x} + \mathbf{e} \end{aligned} \quad (13)$$

where v_0 and ψ_0 denote the lateral velocity and heading angle operating point values.

EKF-BASED ESTIMATOR DESIGN

BASIC ESTIMATOR CONCEPT

For the EKF-based estimator design the discrete-time process model is needed, which is obtained from continuous-time model given by Eqs. (10) and (13), after applying Z-transform based upon the Zero-Order-Hold (ZOH) method:

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{F}(k-1)\mathbf{x}(k-1) + \mathbf{G}(k-1)\mathbf{u}(k-1) \\ &\quad + \mathbf{\Omega}(k-1)\mathbf{v}(k-1) \\ \mathbf{y}(k) &= \mathbf{H}(k)\mathbf{x}(k) + \mathbf{e}(k) \end{aligned} \quad (14)$$

with

$$\mathbf{F}(k) = e^{\mathbf{A}T_s} = \begin{bmatrix} 1 & 0 & -T_s & \hat{u}(k)T_s \\ 0 & 1 & 0 & -T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{G}(k) = \begin{bmatrix} T_s & -\hat{u}(k)T_s \\ 0 & T_s \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{\Omega}(k) = \mathbf{I}_{4 \times 4} T_s$$

$$\mathbf{H}(k) = \mathbf{C} = \begin{bmatrix} \cos \psi_0 & \hat{u} \cos \psi_0 - v_0 \sin \psi_0 & 0 & 0 \\ -\sin \psi_0 & -\hat{u} \sin \psi_0 - v_0 \cos \psi_0 & 0 & 0 \end{bmatrix}$$

where T_s denotes the sampling rate of the inertial sensors measurement signals. It should be noted that the GPS-based vehicle velocity measurements are sampled at a lower rate $1/T_g$, i.e. $T_g = nT_s$, $n \in \mathbb{N}$ (e.g. $T_s = 20\text{ms}$; $n = 50$; $T_g = 1\text{s}$).

The Extended Kalman Filter (EKF) equations are given as follows:

$$\hat{\mathbf{x}}(k | k-1) = \mathbf{F}(k-1)\hat{\mathbf{x}}(k-1 | k-1) + \mathbf{G}(k-1)\mathbf{u}(k-1) \quad (15)$$

$$\begin{aligned} \mathbf{P}(k | k-1) &= \mathbf{F}(k-1)\mathbf{P}(k-1 | k-1)\mathbf{F}^T(k-1) \\ &\quad + \mathbf{\Omega}\mathbf{Q}(k-1)\mathbf{\Omega}^T \end{aligned} \quad (16)$$

$$\hat{\mathbf{e}}(k | k-1) = \mathbf{y}(k) - \mathbf{c}(\hat{\mathbf{x}}(k | k-1)) \quad (17)$$

$$\mathbf{H}(k) = \begin{bmatrix} \cos \psi_0(k) & \hat{u}(k)\cos \psi_0(k) - v_0(k)\sin \psi_0(k) & 0 & 0 \\ -\sin \psi_0(k) & -\hat{u}(k)\sin \psi_0(k) - v_0(k)\cos \psi_0(k) & 0 & 0 \end{bmatrix} \quad (18)$$

$$\mathbf{K}(k) = \frac{\mathbf{P}(k | k-1)\mathbf{H}^T(k)}{\mathbf{H}^T(k)\mathbf{P}(k | k-1)\mathbf{H}^T(k) + \mathbf{\Psi}(k)\mathbf{R}(k)\mathbf{\Psi}^T(k)} \quad (19)$$

$$\hat{\mathbf{x}}(k | k) = \hat{\mathbf{x}}(k | k-1) + \mathbf{K}(k)\hat{\mathbf{e}}(k | k-1) \quad (20)$$

$$\mathbf{P}(k | k) = \mathbf{P}(k | k-1) - \mathbf{K}(k)\mathbf{H}(k)\mathbf{P}(k | k-1) \quad (21)$$