

Simulation Analysis of the Effect of Tire and Suspension Non-Linearities on the Handling Response of Rigid and Articulated Vehicles

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ABSTRACT

This paper presents a simulation based analysis of the effect of various non-linearities on vehicle handling of rigid and articulated vehicles. First a description of a non-linear vehicle model is presented and non-linearities, including tires, suspension elasto-kinematics, springing and damping are discussed. Later sections present the simulations results for the effects of each non-linearity. Simulation analyses are carried out for the case of step steering maneuvers, which permit to deduce the overall vehicle model response variation in steady-state and dynamic conditions.

INTRODUCTION

In the past, various authors have addressed the dependence of vehicle handling performance and roll dynamics, on various parameters and non-linearities included in their vehicle dynamics models, at different level of complexities. These correlations were investigated for various types of steering inputs and maneuvers.

Tire lateral force characteristics (linear and/or non-linear) play a significant role for the handling behavior of the vehicle and are responsible for the degree of vehicle oversteer/understeer and its ultimate stability in steady-state [1] and transient conditions. Xia [2] investigated the effect of tire cornering stiffness on vehicle linear handling performance using different evaluation methods, for example a method based on the phase lag between steering input and yaw rate response, and a method based on the combination of several vehicle measurables, in particular sideslip angle, yaw rate peak time, phase lag of yaw rate, lateral acceleration, yaw rate gain, yaw damping and natural frequency. The author concluded that: (a) higher tire cornering stiffness results in better vehicle linear handling performance (b) different

methods of evaluation give similar rating of the tires. Pei Lu [3] investigated the effect of tire camber on vehicle dynamics in terms of lateral force generation. Pei Lu chose extreme cornering conditions and used the UniTire model [4] for the research. The author identified that there was a significant effect of introducing the initial (static) camber on lateral force generation, hence on the vehicle handling as well. Handling characteristics studied in the research were lateral acceleration, body sideslip angle and roll dynamics, with a particular focus on rollover stability. Yaw dynamics were not considered in the study. Tateishi [5] also carried out a similar study and found that yaw rate response is also a function of camber variation for front and rear axles. Genta [6] showed that toe angle affects vehicle under/oversteer already in the linear operating region, however a quantitative evaluation of its optimal tuning and its actual effects in terms of vehicle response is missing.

Mittal [7] used a linear six DOF vehicle model, to identify the parameters affecting the rollover propensity of a vehicle. In particular, the sensitivity analysis was focused on center of gravity height, track width, wheelbase, tire cornering stiffness and front and rear suspension stiffness. To carry out a more detailed analysis on these parameters, ADAMS car was then used for the simulation of dynamic steering maneuvers. Mittal found that suspension stiffness, center of gravity height and track width are the most significant parameters. All of the parameters addressed in reference [7] have an influence on vehicle yaw rate response.

Vehicle suspension stiffness and damping can be characterized by important non-linearities, which are related to the handling characteristics and ride comfort properties. Moreover, the load conditions of the vehicle can heavily affect, especially on city cars and goods vehicles, the ratio between the mass distribution and suspension stiffness

distribution. Substantial material discusses the impact of suspension stiffness on ride comfort and vehicle dynamics (references [6], [8] and [9]), however, a clear identification of the effect of shock absorber non-linear tuning on vehicle dynamic response is missing.

The first section of this paper describes the non-linear and linear models of rigid and articulated vehicles implemented within this activity, with particular focus on non-linear tire characteristics (functions of toe angle, camber angle and vertical load), non-linear vehicle characteristics (damper properties) and vehicle parameters (center of gravity height) inducing non-linear response. Then general trends, regarding vehicle handling performance sensitivity to the various non-linearities, are derived and discussed in terms of variation of the yaw and roll dynamics for different values of steering angle input in step steer maneuvers. The same procedure is repeated for the case of articulated vehicles. The conclusion of the paper is the summary of the effect and the significance of each source of non-linearity on the overall vehicle response, for an advanced design of vehicle dynamic performance, handling and control.

VEHICLE MODELS

NON-LINEAR VEHICLE MODELS

Rigid Vehicle Model

Numerous models, from fairly simple to increasingly comprehensive, have been adopted for rigid vehicle dynamics analysis [10],[11],[12],[13],[14],[15],[16]. In this study, an eight Degree-Of-Freedom (DOF) non-linear vehicle model has been employed. This model is characterized by the maximum possible simplicity in relation to the non-linear effects it should evaluate. Figure 1 depicts the two-dimensional schematics of the rigid vehicle model. In this formulation, vehicle sprung mass has four DOFs, i.e. two translational (longitudinal and lateral displacement) and two rotational motions (yaw and roll). The other four DOFs are wheel rotations. It has been assumed that vehicle characteristics are symmetrical along the center line of vehicle.

$$m(\dot{u} - v\dot{\psi}) = \sum_{i=1}^n F_{x_i} \cos(\delta_i) - \sum_{i=1}^n F_{y_i} \sin(\delta_i) - \frac{1}{2} \rho S C_D u^2 \quad (1)$$

$$m(\dot{v} + u\dot{\psi}) = \sum_{i=1}^n F_{x_i} \sin(\delta_i) + \sum_{i=1}^n F_{y_i} \cos(\delta_i) \quad (2)$$

$$J_z \ddot{\psi} = \sum_{i=1}^n [F_{x_i} \sin(\delta_i) + F_{y_i} \cos(\delta_i)] x_i - \sum_{i=1}^n [F_{x_i} \cos(\delta_i) - F_{y_i} \sin(\delta_i)] y_i \quad (3)$$

$$J_x \ddot{\phi} = \sum_{i=1}^n [F_{x_i} \sin(\delta_i) + F_{y_i} \cos(\delta_i)] (h_{CG} - RCH_j) \\ \left\{ \cos(\phi) - \text{sign}(F_{y_i}) \frac{h_{CG} - RCH_j}{y_i} \sin(\phi) \right\} - \sum_{j=1}^l M_{K_j}(\phi) + \\ + \sum_{j=1}^l M_{d_j}(\dot{\phi}) + mg \sin(\phi) (h_{CG} - RCH_{CG}) - \text{sign}(\dot{\phi}) \sum DF \quad (4)$$

where $i = 1, 2, 3 \dots n$. n is the total number of wheels of the vehicle

The longitudinal and lateral tire forces, denoted by F_x and F_y respectively, are non-linear functions of tire slip angles, slip ratios, camber angles and vertical loads. As the effect of tire aligning torque on vehicle handling is negligible (minor) [2], only lateral and longitudinal forces are considered in this model. Vehicle suspension is assumed to have only roll motion. Non-linear suspension stiffness and damping characteristics, giving origin to the roll torques M_{K_j} and M_{d_j} , can be adopted within the model. For the dampers, various types of force vs. speed characteristics (properly converted into $M_{d_j}(\dot{\phi})$) have been tested in simulations for comparisons and analyses, including linear, non-linear progressive and non-linear regressive behaviors. The generic model described by the above equations considers all the axles to be steered, but for simplicity this analysis is assuming only the wheels of the front axle to be steered.

The lateral (transversal) load transfer and its distribution between the axles of the vehicle is one of the crucial components for the computation of non-linear lateral tire force. From Figure 1 the difference in the load shift from one side of an axle to the other side may be represented by the following equation:

$$\Delta F_{y_j} = \frac{\text{sign}(y_j)}{y_j} \left(\sum_{i=1}^{n_j} F_{y_i} RCH_j + M_{K_j}(\phi) + M_{d_j}(\dot{\phi}) + \text{sign}(\dot{\phi}) \Sigma DF \right) \quad (5)$$

where n_j is the total number of tires on the j^{th} axle of the vehicle, y_j is the track width of the j^{th} axle. The longitudinal load transfer, due to vehicle forward acceleration and aerodynamic drag (here applied to the vehicle center of gravity) is modeled using the following equation in case of a 2-axle vehicle.

$$\Delta F_{z_{x_j}} = - \frac{\text{sign}(x_{j_i})}{L n_j} \left(m \dot{u} + \frac{1}{2} \rho S C_D u^2 \right) h_{CG} \quad (6)$$

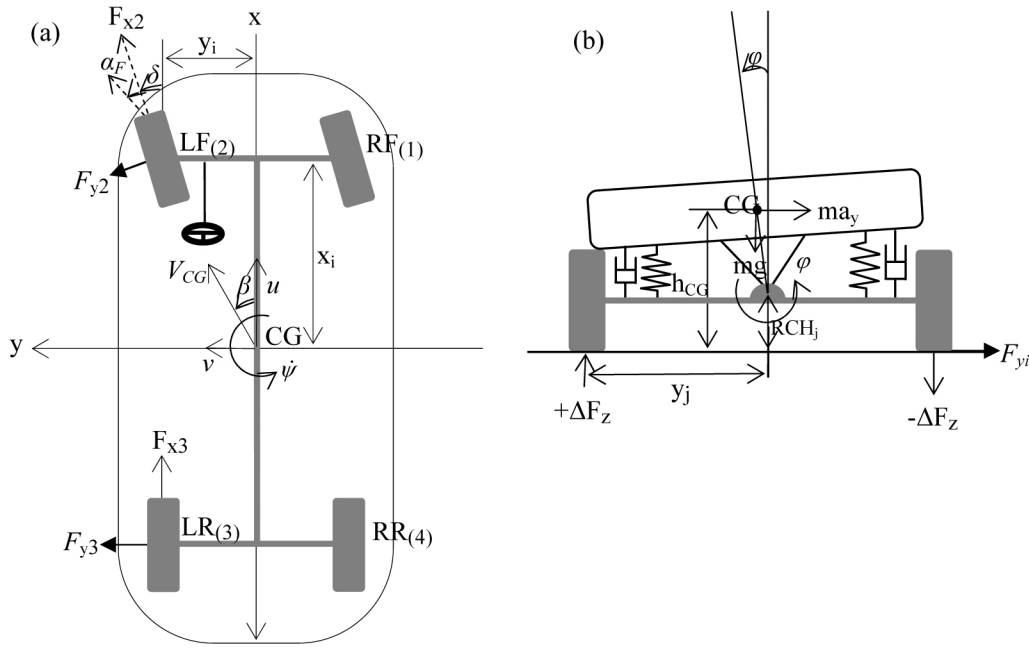


Figure 1. Schematics of the two-axle rigid vehicle model

where x_{ji} is the longitudinal distance of centre of the i^{th} tire of the j^{th} axle from CG, n_j is the total number of tires of the j^{th} axle and L is the wheelbase of the vehicle. For rigid vehicles characterized by more than 2 axles (or articulated vehicles), the equations of the longitudinal load transfer have to consider the suspension stiffness and damping distribution along the vehicle. From the vehicle geometry in Figure 1, a general non-linear tire slip angle equation is constructed as described in Equation (7), where the following contributions are incorporated:

- static toe angle;
- steering angle as imposed by steering system kinematics (different for each wheel);
- toe angle variation as a function of vehicle roll motion;
- toe angle variation induced by half-track variation as a function of vehicle roll motion;
- toe angle variation due to the suspension compliance induced by tire lateral force.

$$\alpha_i = \tan^{-1} \left(\frac{v + \text{sign}(y_i) HT_i(\phi) + x_i \dot{\psi}}{u - y_i \dot{\psi}} \right) - \delta_i + \text{sign}(y_i) \Delta \alpha_{\text{ToeStatic},j} + \text{sign}(y_i) \Delta \alpha_{\text{Toe},\phi_i}(\phi) + C_{\alpha,F_{y_i}} F_{y_i} \quad (7)$$

where $i = 1, 2, 3 \dots n$. n is the total number of wheels of the vehicle and the 'sign' function is used to distinguish the left

or right wheel depending on the y -coordinates (lateral) from the center line of the vehicle. When considering the actual dynamics of a real vehicle suspension, Equation (7) should contain more terms, especially those related to the variation of wheel longitudinal position induced by suspension kinematics and longitudinal force. However, given the nature of this simulation work, mainly focused on the handling response of the vehicle in extreme cornering conditions (and not on ride comfort), these additional contributions have been neglected. Camber angle calculation is carried out in the same way as slip angle calculation and considers static camber, the variations of camber due to roll motion, steering angle (effect of kingpin and castor) and suspension compliance induced by lateral force, as described in Equation (8):

$$\gamma_i = \text{sign}(y_i) \gamma_{i,\text{Static}} + \text{sign}(y_i) \Delta \gamma_{\phi_i}(\phi) + \text{sign}(y_i) \Delta \gamma_{\delta_i}(\delta_i) + C_{\gamma,F_{y_i}} F_{y_i} \quad (8)$$

Articulated Vehicle Model

A five (excluding wheel rotations) DOF non-linear model for an articulated vehicle has been developed using force and moment balance equations for the two rigid bodies - tractor and semi-trailer - connected by a vertical cylindrical hinge. Detailed Mathematical description of the model was presented in [14]. As the semitrailer axles are not steered, tire slip angle is described by Equation (9), where static toe angle, and toe variation due to roll and suspension compliance are included in the complete formula:

$$\alpha_{ST_i} = \tan^{-1} \left(\frac{u \sin(\theta) + v \cos(\theta) - (c + l_{ST_i})\dot{\psi} + l_{ST_i}\dot{\theta}}{u \cos(\theta) - v \sin(\theta) + (c - y_i)\dot{\psi} + y_i\dot{\theta}} \right) +$$

$$+ \text{sign}(y_i) \Delta \alpha_{Toe_{Static,i}} +$$

$$+ \text{sign}(y_i) \Delta \alpha_{Toe_i}(\varphi) + C_{\alpha, F y_i} F_{y_i}$$

(9)

A longitudinal load transfer model similar to what described in [17] is adopted for the articulated vehicle, whilst the lateral load transfer can be computed as presented in Equation (5).

LINEAR VEHICLE MODELS

During the activity, a comparison between the response of the non-linear model of the previous sections and a simplified linear model has been carried out. A linear rigid vehicle model may be obtained by considering constant forward speed and small tire slip angles. Constant cornering stiffness and a first order tire dynamic model are used within the linearized model [13] and [14], as discussed in the previous paragraph.

TIRE MODELS

Tire forces (especially cornering forces) play a key role in the simulation of vehicle dynamics. Therefore the choice of a tire model is decisive for realistic results. In this research two basic tire models have been considered. The first one is a linear tire model, for which the lateral tire force is a linear function of slip angle, with tire cornering stiffness being the constant of the linear proportionality. The second adopted tire model is Pacejka Magic Formula [1]. Pacejka Magic Formula takes into account tire slip angle, longitudinal slip ratio, camber angle, static load and dynamic load on the tire to define the non-linear lateral and longitudinal tire force generation. It also takes into account the interaction between lateral and longitudinal tire force (friction ellipse). To simulate the delay of tire dynamics, a first order model [18] (Relaxation Length Tire Model, RLTM) has been incorporated within the tire model, in order to calculate the delayed (according to the properties of the tire) values of slip angle, slip ratio and camber angle, to be adopted within both the linear tire model and Pacejka Magic Formula:

$$\frac{L}{\Omega R} \dot{\alpha}_D = \alpha_{TH} - \alpha_D$$

(10)

RIGID VEHICLE SIMULATIONS AND RESULTS

Matlab based simulations have been carried out for the comparison of the linear and non-linear vehicle models. The data used for vehicle geometry, kinematics, body, chassis and tires are available in the [Appendix](#). The data adopted for the simulation of the rigid vehicle (RV) are typical of an understeering saloon car. The simulations were carried out for three different values (i.e. 1°, 3° and 5° at the wheel internal to the bend, vehicle turning left or +ve yawing) of step steering angles input. Step steer was selected to analyze the handling response because it has the capability to excite all the system modes, and, if implemented for different values of steering angles, also permits to characterize vehicle steady-state response. In this research activity, the effects of the following parameters on vehicle handling response have been analyzed:

- Shape of the tire cornering force characteristic;
- Toe angle;
- Camber angle;
- Roll (suspension) stiffness;
- Shock absorber characteristics;
- Center of gravity height.

TIRES

By using Pacejka 96 Magic Formula, three different tire settings, represented by Tire 1, Tire 2 and Tire 3, have been adopted during the simulations of the same rigid vehicle, as shown in [Figures 2 and 3](#) (for the static values of vertical load of the axles of the case study vehicle):

- Tire 1, characterized by a peak of lateral force as a function of slip angle, followed by a reduction for high values of slip angle;
- Tire 2, characterized by the same non-linear cornering stiffness characteristic as Tire 1, but differing from Tire 1 for the lack of the reduction of tire force for high values of slip angle (tire force approximately constant for high slip angle values);
- Tire 3, having the same shape of the graph as Tire 2 and also the same cornering stiffness, but characterized by a lower saturation level of the lateral force (lower friction coefficient).

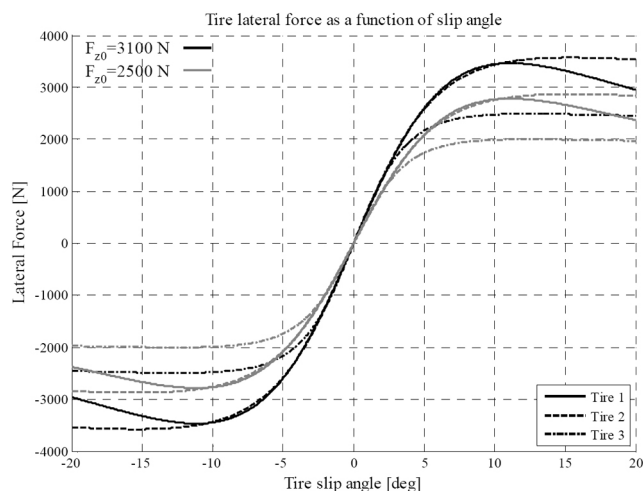


Figure 2. Non-linear tire lateral force characteristics vs. slip angle for two vertical loads

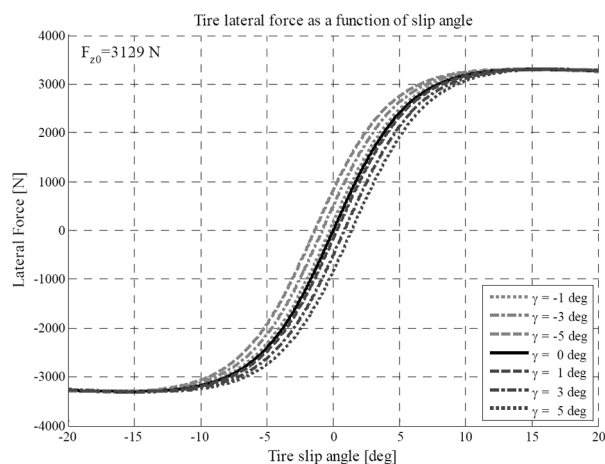


Figure 3. Non-linear tire lateral force characteristics vs. slip angle for different camber angle values (Tire 2)

The characteristics of the three tires, if linearized about the origin of the cornering force vs. slip angle graph, would give origin to the same parameterization within the linear tire and vehicle model (Linear Vehicle Model, LVM in the Figures), and the same results from the viewpoint of the vehicle dynamics response. Figures 4 and 5 show the vehicle yaw rate and vehicle body sideslip angle response (normalized with the amplitude of the steering input, as for the rest of this article) for the linear and non-linear vehicle models, the later equipped with the three types of tires (the same type of tire for the front and the rear axle, lateral force characteristics shown in Figure 2). All the rigid vehicle model simulations are carried out with vehicle forward velocity kept constant at 100 km/h ($U=100$ km/h).

The following observations can be pointed out:

- For small values of steering angle, all the models give origin to similar responses due to the fact that tire cornering

stiffness is the same and almost linear (for small values of tire slip angle and slip ratio) for all of them;

- For higher values of steering angle, the non-linear vehicle model is found to be more underdamped and more understeering in comparison with the linear model. The significant yaw rate oscillations imply lower controllability of the vehicle and would provoke interventions of the Vehicle Dynamics Control, absent in these simulations. This happens for any tire configuration and is due to the reduction of the rate of the cornering force vs. slip angle graph for high values of slip angle;

- For the non-linear model, the yaw rate steady-state gain decreases as a function of steering angle, consistently with the characteristics of a non-linear tire, whereas it remains constant in the case of the linear model;

- For the tires with slight fall in lateral force (Tire 1), vehicle response, for very high values of steering input, exhibits a lower damping in comparison with the other two tires. Tire 1 also provokes a significant increase of the frequency of vehicle yaw rate oscillations, during extreme step steer maneuvers;

- The peak-to-peak amplitude of the yaw rate oscillations is higher for Tire 2 and Tire 3;

- Vehicle body sideslip angle response (Figure 5) is in accordance to the linear response for small values of steering angle input and exhibits the same characteristics as yaw rate response.

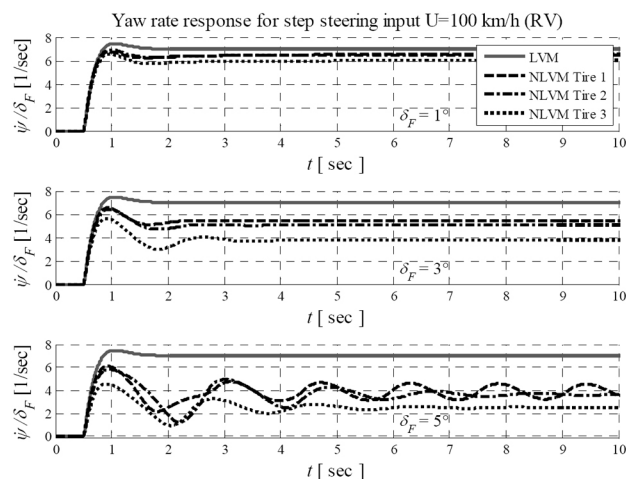


Figure 4. Yaw rate comparison for different tire models

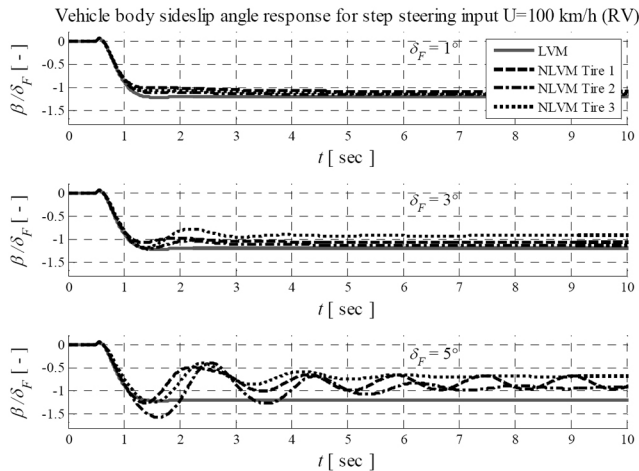


Figure 5. Body sideslip angle comparison for different tire models

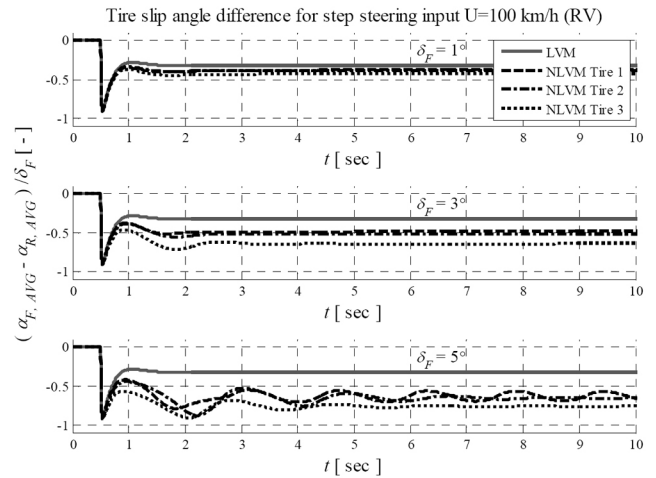


Figure 6. Tire slip angle difference for the linear model and the non-linear model equipped with the three different tires

The difference of the front and rear tire slip angles can be adopted as one of the main parameters to measure the degree of vehicle understeer/oversteer in dynamic conditions (Figure 6). For the non-linear model, the average between the angles of the two tires of the same axle is considered as equivalent slip angle for that axle. It may be observed that:

- For the linear vehicle model, the negative steady-state value of tire slip angle (slip angles are negative when cornering on the left) difference (divided by the amplitude of the steering input) shows an understeering behavior, the same for any value of steering amplitude, given the linearity of the model;
- High negative peaks for transient response prove that the vehicle becomes highly understeering when the steering angle input is initially applied, and then the level of understeer reaches its minimum when vehicle response is at the maximum yaw rate. This trend is valid for all the vehicle models.

A comparison of the settling time, percent overshoot and damped natural frequency of vehicle yaw rate response for different tires is presented in Figure 7, which confirms the comments of the previous Figures.

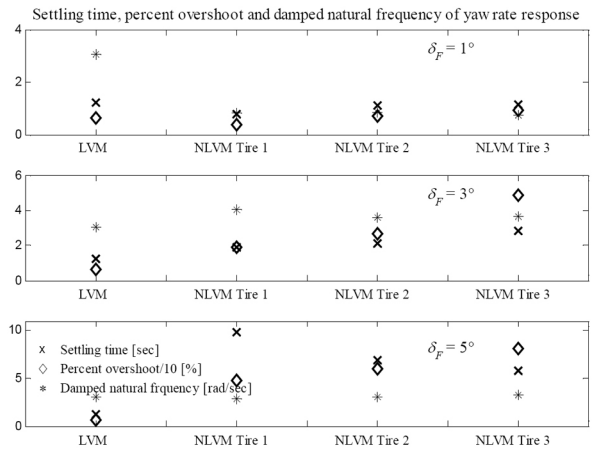


Figure 7. Settling time, percent overshoot and damped natural frequency for the yaw rate response of the linear model non-linear vehicle models

TOE AND CAMBER ANGLES

One of the main suspension parameters to be tuned in order to adjust vehicle handling is toe angle. Different static values of front toe-in have been adopted within the simulations presented in Figure 8. On an axle, toe-in provokes an increase of the absolute value of slip angle on the tire external to the bend, and the same decrease of the absolute value of slip angle on the tire internal to the bend. The net effect of toe-in would be zero if the cornering stiffness of the two tires was the same, but it is not, due to the load transfer which occurs when the vehicle is subjected to a lateral acceleration, and which provokes a variation of tire cornering stiffness. As a consequence, for the same average value of the slip angle of the tires of the axle, toe-in provokes an increase of cornering force on the axle, proportional to the difference between the cornering stiffness of the two tires and to the amount of toe-