

Linear Quadratic Game Theory Approach to Optimal Preview Control of Vehicle Lateral Motion

2011-01-0963

Published
04/12/2011Seyed Hossein Tamaddoni, Saied Taheri and Mehdi Ahmadian
Virginia Tech

Copyright © 2011 SAE International

doi:10.4271/2011-01-0963

ABSTRACT

Vehicle stability is maintained by proper interactions between the driver and vehicle stability control system. While driver describes the desired target path by commanding steering angle and acceleration/deceleration rates, vehicle stability controller tends to stabilize higher dynamics of the vehicle by correcting longitudinal, lateral, and roll accelerations. In this paper, a finite-horizon optimal solution to vehicle stability control is introduced in the presence of driver's dynamical decision making structure. The proposed concept is inspired by Nash strategy for exactly known systems with more than two players, in which driver, commanding steering wheel angle, and vehicle stability controller, applying compensated yaw moment through differential braking strategy, are defined as the dynamic players of the 2-player differential linear quadratic game. The optimal preview feedback gains are obtained, and the resulting controllers are evaluated by the nonlinear vehicle model of CarSim7 for the standard lane change maneuver.

INTRODUCTION

Over the last few years research into road safety has become increasingly important for solving problems related to the vehicle-road-driver system. Many driving aid systems, including Advanced Driver Assistance Systems (ADAS), Anti-lock Braking System (ABS) and Vehicle Stability Control System (VSC), are designed to avoid dangerous situations and improve vehicle stability.

Vehicle stability control systems, VSC, help drivers maintain vehicle stability and avoid spinning out during emergency braking and steering maneuvers. A comprehensive literature review conducted by Ferguson [1] reveals that VSC can effectively reduce single-vehicle crashes in cars and SUVs by

30-50%. Also, fatal rollover crashes are estimated to be about 70-90% lower with VSC regardless of vehicle type. Among all vehicle stability enhancement strategies, direct yaw control (DYC) is one of the most effective methods of active chassis control which can considerably enhance the vehicle stability and controllability [2]. For vehicle control, the yaw moment control is a way to control the lateral motion of a vehicle during severe maneuvers using active steering, e.g., front and/or four wheel steering, or active differential braking using ABS.

It is well-known that Game theory deals with the development of suitable concepts to describe and understand conflict situations with multiple decision makers. The advanced studies of this theory concern the non-cooperative linear quadratic, LQ, games, where the dynamics of the players are covered by a linear ordinary differential equation, and the cost functions are quadratic in state vectors as well as control actions [3]. Differential games were first introduced by Isaacs [4], within the framework of two-person zero-sum games. For systems governed by several controllers, i.e., decision makers or players, where each controller aims to minimize its own cost function, Nash strategy offers a nice framework to study control robustness. In fact, such a strategy has an inherent robustness property since no player can improve his/her payoff by deviating unilaterally from his/her Nash strategy once the equilibrium is attained.

Tamaddoni et al. [5,6] used Nash strategy to develop an optimal control strategy which takes into account the driver of the vehicle as an inherent part of the controller. In the presented method, the driver, commanding the steering wheel angle, and the vehicle stability controller, applying the corrective yaw moment, are defined as dynamic players in a 2-player differential linear quadratic game. They found out that the cooperative game theory method brings more optimal

performance by setting the driver's steering angle and the controller's corrective yaw moment compared to the independently optimized set of steering angle and corrective yaw moment obtained through linear quadratic regulation approach.

Following Tamaddoni et al. [5,6,7], the game theory framework is adapted and modified in this paper to include time-previewed driver model in collaboration with the VSC system. To do so, the equations of motion are put into discrete form and a preview system is defined. Using the Riccati equation for discrete difference game, the optimal set of the steering angle and the corrective yaw moment can be obtained.

BACKGROUND

For decades, driver modeling has been an interesting issue for both traffic and vehicle control research. Driver models are usually based on preview of the road ahead where the driver is represented as an optimal preview controller, constructing a path error functional by previewing the road over a known preview distance, and minimizing a weighted integral of squares of differences between the previewed path points and the corresponding estimated lateral position of the vehicle over the preview distance. Sharp et al. [8] introduced a new representation of optimal linear steering control where the standard lateral/yaw linear vehicle model was transformed into discrete-time formation that constructed a quadratic cost function consisting of terms describing path and attitude errors with respect to the road path. Based on this cost function, steering wheel angle control was minimized by linear quadratic regulator (LQR) control.

In 2001, Sharp et al. [8] proposed a simple linear vehicle model with absolute lateral position, lateral velocity, yaw angle, and yaw rate as the non-preview states in discrete form and the road sample inputs as the preview states.

$$x = (y, v_y, \psi, \dot{\psi}, y_{r0}, y_{r1}, y_{r2}, \dots, y_{rN})^T \quad (1)$$

with y_{r0} is the road reference position at one step before the current time, and y_{r1} is the current reference position, and $y_{r2}, \dots, y_{rN}$ are road reference positions at $(N - 1)$ steps ahead.

The main purpose of the controller is stable path following, the quality of which can be specified by the sum of the squares of the differences between the y-coordinate of the car's reference point and the corresponding value y_r for the lateral position of the road with respect to the fixed ground frame.

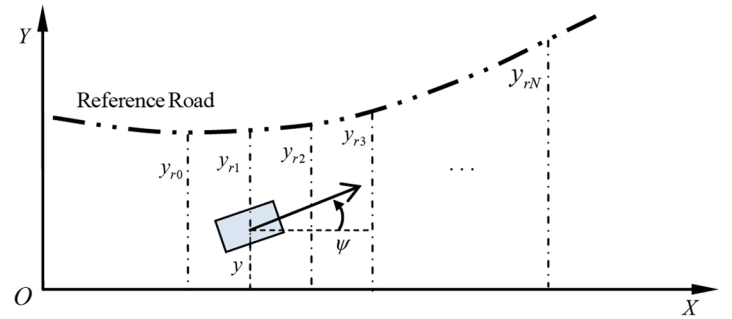


Figure 1. Schematic of Maneuver Task Geometry

Similarly, the human driver is assumed to have the tendency to minimize the corresponding sum of squares of attitude angle differences and minimize the higher-order dynamics, typical terms that can be found analytically. These priorities are reflected into a quadratic cost function [9] by setting:

$$Q = C^T q C, \quad (2)$$

where the matrix C is defined as,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/T_s & -1/T_s & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/v_x T_s & -1/v_x T_s & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & -1/v_x T_s^2 & 2/v_x T_s^2 & -1/v_x T_s^2 & 0 & \dots & 0 \end{bmatrix}_{4 \times (4+N)} \quad (3)$$

The relative importance attached to path errors, attitude errors and steer angle are set by choosing the diagonal values of q appropriately [9].

VEHICLE MODEL

A detailed standard nonlinear vehicle model of CarSim7 is used in numerical simulations to analyze the response of the controlled vehicle. The CarSim math model includes nonlinear tire models according to combined sideslip theory [10], nonlinear spring model, nonlinear front steering system, and incorporates the major kinematics and compliance effects in the suspension and steering systems including differential load transfer for each wheel. However, to design the controller, a widely used simplified linear single track vehicle model is considered which captures the essential vehicle steering dynamics. In this respect, the tire forces are assumed to be linear functions of tire slip angle.

EVALUATION MODEL

In order to study the handling and roll dynamic responses of the vehicle, the nonlinear CarSim model of a vehicle is used which includes longitudinal and lateral translational motions, and roll and yaw motions with rotational dynamics of each of the four wheels. It must be mentioned that the roll dynamics and the suspension compliance properties play an important role in providing a more realistic simulation environment for evaluating the control algorithm that will be developed in this paper.

Based on the vehicle coordinate system, parameters, and external forces depicted in [Figure 2](#), the nonlinear vehicle model is obtained by writing the translational and rotational equations in the vehicle fixed coordinate frame

$$m(\dot{v}_x - v_y \dot{\psi}) - m_\phi h(\dot{\phi} \dot{\psi} + 2\dot{\psi} \dot{\phi}) = (F_{xFR} + F_{xFL}) \cos \delta_F - (F_{yFR} + F_{yFL}) \sin \delta_F + (F_{xBR} + F_{xBL}), \quad (4)$$

$$I_z \ddot{\psi} + (I_{z,\phi} \theta_R - I_{xz,\phi}) \ddot{\phi} - m_\phi (h \dot{v}_x \dot{\phi} - v_y \phi \dot{\psi}) = I_F (F_{xFR} + F_{xFL}) \sin \delta_F + I_F (F_{yFR} + F_{yFL}) \cos \delta_F - I_B (F_{yBR} + F_{yBL}) + s_L (F_{xBL} + F_{xFL} \cos \delta_F - F_{yFL} \sin \delta_F) - s_R (F_{xBR} + F_{xFR} \cos \delta_F - F_{yFR} \sin \delta_F), \quad (5)$$

$$m(\dot{v}_y + v_x \dot{\psi}) + m_\phi h(\ddot{\phi} - \phi \dot{\psi}^2) = (F_{xFR} + F_{xFL}) \sin \delta_F + (F_{yFR} + F_{yFL}) \cos \delta_F + (F_{yBR} + F_{yBL}), \quad (6)$$

$$(I_{x,\phi} + m_\phi h^2) \ddot{\phi} + m_\phi h(\dot{v}_y + v_x \dot{\psi}) + C_\phi \dot{\phi} + (I_z \theta - I_{xz}) \ddot{\psi} - (m_\phi h^2 + I_{y,\phi} - I_{z,\phi}) \phi \dot{\psi}^2 + (K_\phi - m_\phi g h) \phi = 0. \quad (7)$$

The equation of motion that governs wheel dynamics is simply given by

$$I_w \dot{\omega} = -F_x R_w + T_{Driving} - T_{Braking} \quad (8)$$

Tire velocity for each wheel are approximated as

$$v_{wFR} = \begin{pmatrix} v_x - s_R \dot{\psi} \\ v_y + l_F \dot{\psi} \end{pmatrix}, \quad v_{wFL} = \begin{pmatrix} v_x + s_L \dot{\psi} \\ v_y + l_F \dot{\psi} \end{pmatrix}, \\ v_{wBR} = \begin{pmatrix} v_x - s_R \dot{\psi} \\ v_y - l_B \dot{\psi} \end{pmatrix}, \quad v_{wBL} = \begin{pmatrix} v_x + s_L \dot{\psi} \\ v_y - l_B \dot{\psi} \end{pmatrix} \quad (9)$$

Tire slip angle is defined as the angular difference between the treads in the contact patch and the direction the wheel is turned.

$$\sigma_{Fi} \dot{\alpha}_{Fi} + v_{wFi}^x \left(\alpha_{Fi} - \delta_F + \tan^{-1} \left(\frac{v_{wFi}^y}{v_{wFi}^x} \right) \right) = 0, \\ \sigma_{Bi} \dot{\alpha}_{Bi} + v_{wBi}^x \left(\alpha_{Bi} + \tan^{-1} \left(\frac{v_{wBi}^y}{v_{wBi}^x} \right) \right) = 0, \quad i \in \{R, L\} \quad (10)$$

Longitudinal slip of the tire is defined as the difference between the tire tangential speed and the speed of the axle relative to the road.

For each wheel represented by index $i \in \{FR, FL, BR, BL\}$, tire slip is calculated as

$$\kappa_i = \begin{cases} \left(\frac{R_w \omega_i - v_{wi}^x}{\max \{R_w \omega_i, v_{wi}^x\}} \right) \cdot \text{sign}(|v_{wi}^x| - |R_w \omega_i|), & v_{wi}^x \neq R_w \omega_i \\ 0 & v_{wi}^x = R_w \omega_i \end{cases} \quad (11)$$

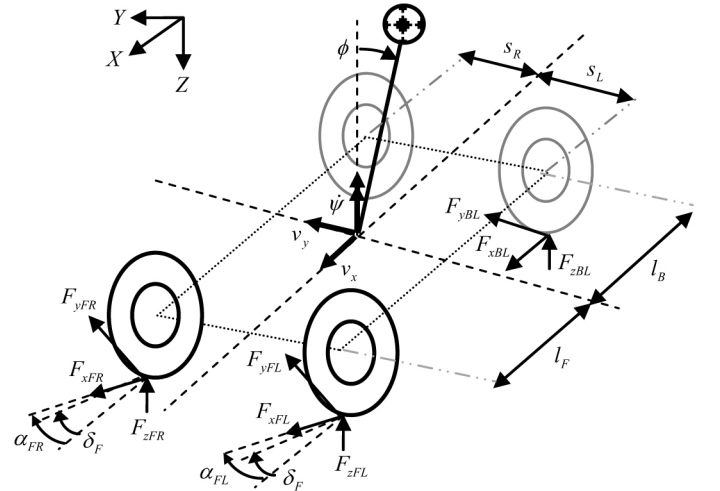


Figure 2. Vehicle evaluation model

Tire forces and moments are calculated based on a widely used semi-empirical tire model based on trigonometric functions known as Magic Formula [10]. Magic-Formula tire models are considered the state-of-the-art for modeling tire-road interaction in vehicle dynamics applications.

CONTROL MODEL

The commonly used single track bicycle model is considered in this paper [5,6,7,10]. This model captures the needed dynamic information for yaw as well as lateral degrees of freedom. In order to derive the equations, it is assumed that