

Optimization of One-Dimensional Aluminum Foam Armor Model for Pressure Loading

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ABSTRACT

The primary objective of this investigation is the *optimum design of lightweight foam material systems for controlled energy absorption* under blast impact. The ultimate goal of these systems is to increase the safety and integrity of occupants and critical components in structural systems such as automotive vehicles, buildings, ships, and aircrafts. Although outstanding results have been achieved with the use of foams in blast protective systems, current design practices rely on trial and error as *there is an absence of a systematic design method*. While the governing equations are known for a variety of physical phenomena in appropriate length scales, there are no suitable methodologies to accomplish the aforementioned objectives. A promising approach to systematically design the material's microstructure is the use of structural optimization methods. This investigation presents an appropriate design methodology to optimally design foam material systems for blast mitigation. The objective function is expressed in terms of acceleration. Macroscopic effective material properties are used to drive the nonlinear analysis of the elasto-plastic material under time-dependent loading conditions. Gradient-based optimization methods are used to obtain the final density distribution of the foam material system. The application of this approach is shown through a two-dimensional optimization problem.

INTRODUCTION

The main objective of this investigation is the *development of novel lightweight heterogeneous material systems for controlled energy absorption* under extreme events. These events particularly include technological hazard such as blast (e.g., explosives threats) and impact (e.g., high velocity truck impact). Since the governing equations for the behavior of a

variety of materials are well-known for a range of physical time-dependent phenomena in proper length scales, the systematic design and generation of novel protective systems only requires the development or adoption of suitable design methods.

Due to advances in manufacturing processes, composite materials, metallic foams, and honeycomb structures have seen an increase in industrial applications. For their lightweight and energy-absorbing properties, these materials have been used in the design of crashworthy structures in the automotive and aerospace industries[1,2]. In particular, Aluminum foams have received a great deal of attention for their low weight-to-strength ratio, large scale deformation characteristics, and relatively low cost [3]. Applications to Aluminum foam in crash mitigation efforts have led to innovations in heterogeneous material systems such as Aluminum foam filled columns and frame rails for the purpose of dissipating crash energies in automotive vehicle structures [4]. Similar research initiatives have investigated the possibility of employing metallic heterogeneous materials for the purpose of attenuating the transmission of blast waves through a sacrificial cladding [5,6]. The basis for the use of sacrificial cladding is to design a material to be applied to the external face of the heterogeneous material system to absorb part of the kinetic energy of an incoming shock wave, similar to a crash event, by exploiting the large plastic regime of compressible foams [7]. However, a shock amplification effect has been experimentally observed with the use of foams and other cellular structures.

A number of studies have been done to characterize the ability of metallic foams to mitigate blast [8,9,10,11], as part of a wider effort to design armor out of cellular materials[5,12,13,14]. However, in none of these works is there a concerted effort to optimize the performance of the

material to the task of dissipating blast energy. The vast majority of studies involving foams or other cellular materials are based around characterizing the performance of those materials and developing stress models to describe the deformation.

The study by Main and Gazonas[12] is perhaps the closest in scope to the current study in that they study the effects of distributing the mass of a sandwiched foam armor between the face sheet (the sheet of solid material that receives the blast load) and the core material. However, in their study they use a rigid-perfectly-plastic-locking (R-P-P-L) assumption for the material model of the cellular material and the density is evenly distributed through the core region. When changing the fraction of mass in the core, they increase the length of the core rather than increasing the density of it. Furthermore, Main and Gazonas comment about the use of optimization to find the optimal ratio between core and cover plates but themselves do not perform any optimization.

In the area of blast mitigation, only a handful of applications of optimization exist in the literature [15,16]. These few only involve the shape and sizing optimization of homogenous monolithic armor plating and not the microstructure or material distribution of heterogeneous material. This current study proposes to use optimization techniques and the concept of material properties related to core density in order to control the dissipation of a pressure loading.

METHODOLOGY

PROBLEM FORMULATION

There is consistent agreement in the literature [3,17,18] that the properties of foam metals are related to the density of the foam, ρ_f , relative to the density of the base solid material, ρ_0 . McCullough uses a model that relates the properties of the foam directly to the properties of the base material:

$$\frac{\sigma_p}{\sigma_{y0}} = 0.3\phi^{3/2} \left(\frac{\rho_f}{\rho_0}\right)^{3/2} + (1 - \phi) \left(\frac{\rho_f}{\rho_0}\right) \quad (1)$$

$$\frac{E_f}{E_0} = 0.3\phi^2 \left(\frac{\rho_f}{\rho_0}\right)^2 + (1 - \phi) \left(\frac{\rho_f}{\rho_0}\right) \quad (2)$$

$$\varepsilon_d = 1 - 1.4 \left(\frac{\rho_f}{\rho_0}\right) \quad (3)$$

Where σ_p is the yield stress of the foam, ϕ is a distribution constant that indicates if the foam behaves like an open cell foam or a closed cell foam, E_f is the young's modulus of the foam, and ε_d is the compaction strain. Note here that the compaction strain goes negative at relative densities above

0.714, this indicates that the model is not valid for materials that are only slightly porous.

On the other hand, Hanssen [18] proposes a model, which is supported by the work of Gibson and Ashby [3] that describes the yield stress, young's modulus, and the entire hardening profile for compression:

$$\varepsilon_d = 1 - \frac{\rho_i}{\rho_0} \quad (4)$$

$$\sigma = \sigma_p + \gamma \frac{\varepsilon}{\varepsilon_d} + \alpha \ln \left(\frac{1}{1 - \left(\frac{\varepsilon}{\varepsilon_d}\right)^{1/\beta}} \right) \quad (5)$$

Where all the coefficients, $\{\sigma_p, \alpha, \frac{1}{\beta}, \gamma\}$, are calculated using a calibration function:

$$\{\sigma_p, \alpha, \frac{1}{\beta}, \gamma\} = C_0 + C_1 \left(\frac{\rho_f}{\rho_0}\right)^n \quad (6)$$

For hydrostatic pressure loading, Hanssen uses the y-direction calibration values because the stress-strain curves of the y-direction and the hydrostatic load cases are the most similar. Otherwise, the material is anisotropic and the calibration constants vary greatly between loading directions.

Because Hanssen's model more completely defines the material behavior, it will be used as the basis for this study. The system to be modeled is a plate of aluminum foam backed by a solid layer of aluminum (Figure 1). This system can be modeled as a chain of spring-mass elements where the mass of the element is lumped at the node and material model is represented by the spring (Figure 2). This representation is similar to one used Shim et al in their study on the crushing of cellular structures [13]. Although the many authors have modeled foam systems using a continuum momentum balance approach [9,12,19,20], this method is not employed here because it relies on the r-p-p-l simplification and neglects elastic effects.

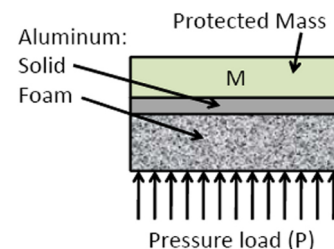


Figure 1. Depiction of scenario to be modeled

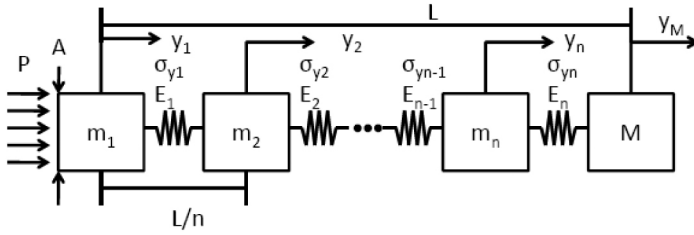


Figure 2. mass-spring equivalent of foam plate system

The boundaries employed are similar to those used by Fleck and Deshpande [19]. The model represents a 1 dimensional slice of armor and the reduction in impulse provided by the supports is neglected. The motivation behind this approximation is that the time period of the loading phase is much smaller than the period of the overall structure.

The equation of motion for the i^{th} element can be written as

$$m_i \ddot{y}_i = F_L - F_R = A(\sigma_L - \sigma_R) \quad (7)$$

where m_i is the mass of the i^{th} element and the subscripts L and R indicate the force or stress to the left or the right of the element. In this model, the stress to the right (σ_R) is the internal stress in the i^{th} element and resists the force from the left. The stress to the left can be written as

$$\sigma_L = \begin{cases} P & i = 1 \\ \sigma_{i-1} & i > 1 \end{cases} \quad (8)$$

where P is the pressure load. If we assume both an elastic and a plastic region, the stress in the element, σ_i , can be written as:

$$\sigma_i = \begin{cases} E_i \varepsilon_i & \text{Elastic } (\varepsilon \leq \varepsilon_y) \\ \sigma_{p,i} + \gamma_i \frac{\varepsilon_i - \varepsilon_{y,i}}{\varepsilon_{d,i} - \varepsilon_{y,i}} + \alpha_i \ln \left(\frac{1}{1 - \left(\frac{\varepsilon_i - \varepsilon_{y,i}}{\varepsilon_{d,i} - \varepsilon_{y,i}} \right)^{1/\beta_i}} \right) & \text{Plastic } (\varepsilon > \varepsilon_y) \end{cases} \quad (9)$$

where the yield strain ε_y has been subtracted in order to ensure continuity between the elastic region and the plastic region. The yield strain is calculated by the relation

$$\varepsilon_y = \frac{\sigma_p}{E_i} \quad (10)$$

The strain of the i^{th} element is given by

$$\varepsilon_i = \begin{cases} (y_i - y_{i+1}) \frac{n}{L} & i < n \\ (y_i - y_M) \frac{n}{L} & i = n \end{cases} \quad (11)$$

Now we will non-dimensionalize the system. Stresses will be non-dimensionalized by the yield stress of the base material, σ_{y0} . Length will be non-dimensionalized by the total system length, L , and mass by the density of the base material, ρ_0 . To non-dimensionalize time, we define a value T to be the duration over which load is applied. For simplicity, the loading profile is that seen in Figure 3.

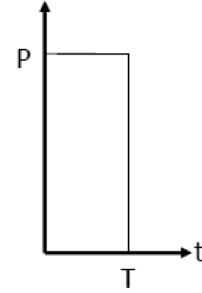


Figure 3. Loading profile applied to spring mass system

With the dimensional parameters defined, we can convert the system into a non-dimensional system using the following relations:

$$y_i = u_i L$$

$$t = \tau T$$

$$\dot{y}_i = \frac{\dot{u}_i L}{T}$$

$$\ddot{y}_i = \frac{\ddot{u}_i L}{T^2}$$

$$m_i = \frac{\rho_i}{\rho_0} \frac{L}{n} A \rho_0 = x_i \frac{L}{n} A \rho_0$$

Where the relative density of the element is defined as $x_i = \rho_i / \rho_0$ and the dot represents $\frac{d}{dt}$ when over y and $\frac{d}{d\tau}$ when over u . With these relations we can now write the equations of motion as

$$\frac{x_i A \rho_0 L}{n} \frac{L}{T^2} \ddot{u}_i = A \sigma_{y0} \left(\frac{\sigma_{i-1}}{\sigma_{y0}} - \frac{\sigma_i}{\sigma_{y0}} \right) \quad (12)$$

Where the right hand side was multiplied by $\frac{\sigma_{y0}}{\sigma_{y0}}$. Rearranging yields,

$$\ddot{u}_i = \frac{AT^2 \sigma_{y0} n}{x_i A \rho_0 L^2} \left(\frac{\sigma_{i-1}}{\sigma_{y0}} - \frac{\sigma_i}{\sigma_{y0}} \right) = \omega^2 \frac{n}{x_i} \left(\frac{\sigma_{i-1}}{\sigma_{y0}} - \frac{\sigma_i}{\sigma_{y0}} \right) \quad (13)$$