

Three-pass Perforated Tube Muffler with End-resonator

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ABSTRACT

The one-dimensional analytical approach, three-dimensional finite element method (FEM) and boundary element method (BEM) are developed to predict and analyze the acoustic attenuation performance of three-pass perforated tube muffler with end-resonator. For an elliptical muffler, the predictions of transmission loss from the FEM and BEM agree well each other throughout the frequency range of interest, while the one-dimensional analytical solution shows a reasonable agreement with the numerical predictions at lower frequencies and deviates at higher frequencies. The FEM is then used to investigate the effects of geometrical parameters and internal structure on the acoustic attenuation performance of three-pass perforated tube muffler with end-resonator.

INTRODUCTION

The three-pass perforated tube mufflers are widely used in automotive exhaust systems to reduce the engine exhaust noise produced by engine operation. Dickey *et al* [1] developed the one-dimensional time-domain approach, Munjal [2, 3] and Selamet *et al* [4] presented the one-dimensional frequency-domain approaches to predict the acoustic attenuation performance of the typical three-pass perforated tube mufflers. For a prototype muffler, their predictions of transmission loss agree reasonably well with the experimental results within the plane wave region. The one-dimensional plane wave theories are valid at lower frequencies only, a multi dimensional approach is needed for the accurate prediction of acoustic attenuation performance of the mufflers at higher frequencies in view of the complex internal sound field. Ji and Selamet [5] developed the three-dimensional boundary element method to predict the acoustic attenuation performance of the typical three-pass perforated tube muffler, their BEM predictions agree well with

experiments for a prototype muffler in the frequency range of interest, and then the effects of geometrical parameters on the acoustic attenuation performance of typical three-pass perforated tube muffler are investigated numerically.

In order to improve the low frequency noise attenuation performance, the typical three-pass perforated tube muffler may combine an end-resonator to form the *three-pass perforated tube muffler with end-resonator*, which is also widely used in the modern vehicles to reduce the engine exhaust noise. However, no computational or experimental guidance has been provided to determine the acoustic attenuation performance of three-pass perforated tube muffler with end-resonator.

The objective of the present study is then (1) to develop the one-dimensional analytical approach, the three-dimensional FEM and BEM approaches to predict and analyze the acoustic attenuation performance of the three-pass perforated tube muffler with end-resonator, and (2) to investigate the effects of geometrical parameters and internal structure on the acoustic attenuation performance of three-pass perforated tube muffler with end-resonator.

ONE-DIMENSIONAL ANALYTICAL APPROACH

The three-pass perforated tube muffler with end-resonator is shown in Figure 1. Following the works of Selamet *et al* [4], the relationship of sound pressures and particle velocities at the inlet ($x = 0$) and outlet ($x = l_p$) of the common perforated section in the expansion chamber ④ may be expressed as

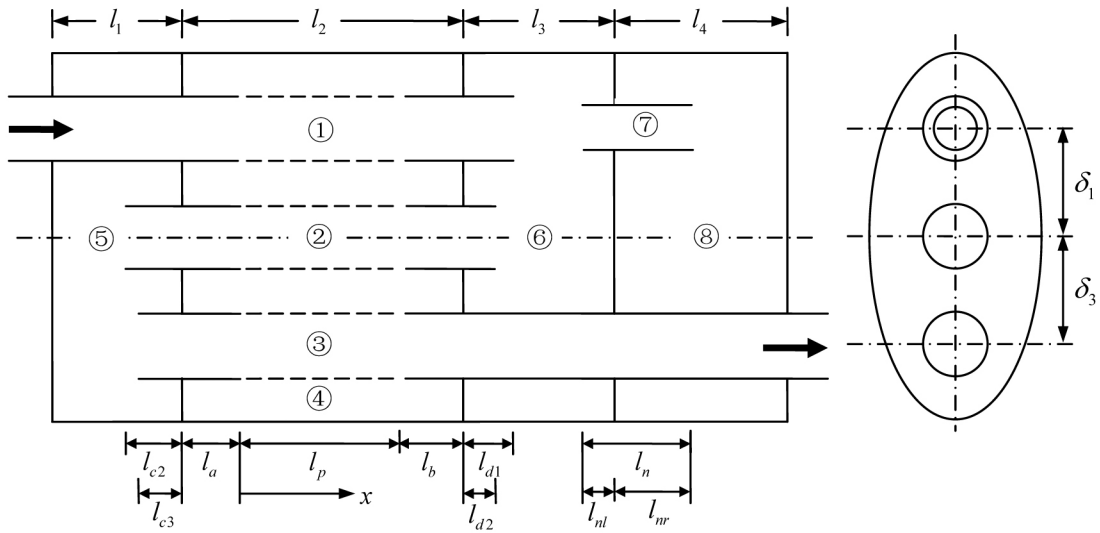


Fig.1. Three-pass perforated tube muffler with end-resonator.

$$\begin{Bmatrix} p_1(0) \\ \rho_0 c u_1(0) \\ p_2(0) \\ \rho_0 c u_2(0) \\ p_3(0) \\ \rho_0 c u_3(0) \end{Bmatrix} = \begin{bmatrix} [TR11] & [TR12] & [TR13] \\ [TR21] & [TR22] & [TR23] \\ [TR31] & [TR32] & [TR33] \end{bmatrix} \begin{Bmatrix} p_1(l_p) \\ \rho_0 c u_1(l_p) \\ p_2(l_p) \\ \rho_0 c u_2(l_p) \\ p_3(l_p) \\ \rho_0 c u_3(l_p) \end{Bmatrix} \quad (1)$$

where p is the sound pressure, u is the particle velocity, ρ_0 is the density of air, c is the speed of sound, and subscripts 1, 2, 3 represent the perforated tubes ①, ②, ③, respectively, $[TRij]$ are the 2×2 matrices ($i, j=1,2,3$).

For the transition chamber ⑥ and the right solid portions of tubes ① and ②, the following expression may be obtained by the transfer matrix method

$$\begin{Bmatrix} p_1(l_p) \\ \rho_0 c u_1(l_p) \end{Bmatrix} = [T_{ic}] \begin{Bmatrix} p_2(l_p) \\ -\rho_0 c u_2(l_p) \end{Bmatrix}, \quad (2)$$

where,

$$\begin{aligned} [T_{ic}] &= \begin{bmatrix} \cos kl'_{lr} & j \sin kl'_{lr} \\ j \sin kl'_{lr} & \cos kl'_{lr} \end{bmatrix} \cdots \\ &\begin{bmatrix} 1 & 0 \\ \frac{S-S_3}{S_1} A'_1 & \frac{S-S_1-S_3}{S_1} \end{bmatrix} \begin{bmatrix} \cos kl_{d1-2} & j \sin kl_{d1-2} \\ j \sin kl_{d1-2} & \cos kl_{d1-2} \end{bmatrix} \cdots \\ &\times \begin{bmatrix} 1 & 0 \\ j \frac{S-S_1-S_2-S_3}{S-S_1-S_3} \tan kl'_{d2} & \frac{S_2}{S-S_1-S_3} \end{bmatrix} \begin{bmatrix} \cos kl'_{2r} & j \sin kl'_{2r} \\ j \sin kl'_{2r} & \cos kl'_{2r} \end{bmatrix} \cdots \end{aligned} \quad (3)$$

is the transfer matrix for the case of $l_{d1} \geq l_{d2}$. Here, $j = \sqrt{-1}$ is the imaginary unit, $l'_{lr} = l_b + l_{d1} + \delta_{lr}$, $l_{d1-2} = l_{d1} - l_{d2}$, $l'_{d2} = l_{d2} + \delta_{2r}$, $l'_{2r} = l_b + l_{d2} + \delta_{2r}$, δ_{1r} , and δ_{2r} are the end corrections of extensions of tubes ① and ② into the transition chamber, respectively, S , S_1 , S_2 , S_3 are the cross-sectional areas of muffler chamber, tubes ①, ②, ③, respectively, A'_1 is the specific acoustic admittance on the cross-section of right end of tube ① in the transition chamber ⑥ and may be expressed as

$$A'_1 = \frac{j \tan kl_{1-7} + \left[j \frac{S-S_3-S_7}{S-S_3} \tan kl'_{nl} + \frac{S_7}{S-S_3} A_{er} \right]}{1 + j \tan kl_{1-7} \left[j \frac{S-S_3-S_7}{S-S_3} \tan kl'_{nl} + \frac{S_7}{S-S_3} A_{er} \right]}, \quad (4)$$

where $l_{1-7} = l_3 - (l_{d1} + \delta_{1r} + l_{nl} + \delta_{7l})$, $l'_{nl} = l_{nl} + \delta_{7l}$, δ_{7l} is the end correction of left side of tube ⑦, S_7 is the cross-sectional area of tube ⑦, and A_{er} represents the input specific acoustic admittance of end-resonator at the inlet of tube ⑦ and may be written as

$$A_{er} = \frac{j \tan kl'_n + j \left[\frac{S - S_3 - S_7}{S_7} \tan kl'_{nr} + \frac{S - S_3}{S_7} \tan kl'_{4r} \right]}{1 - \tan kl'_n \left[\frac{S - S_3 - S_7}{S_7} \tan kl'_{nr} + \frac{S - S_3}{S_7} \tan kl'_{4r} \right]}, \quad (5)$$

where $l'_n = l_n + \delta_{7l} + \delta_{7r}$, $l'_{nr} = l_{nr} + \delta_{7r}$, $l'_{4r} = l_4 - (l_{nr} + \delta_{7r})$, δ_{7r} is the end correction of right side of tube ⑦.

Similarly, for the case of $l_{d1} < l_{d2}$, we may have

$$\begin{aligned} [T_{tc}] &= \begin{bmatrix} \cos kl'_{1r} & j \sin kl'_{1r} \\ j \sin kl'_{1r} & \cos kl'_{1r} \end{bmatrix} \cdots \\ &\begin{bmatrix} 1 & 0 \\ j \frac{S - S_1 - S_2 - S_3}{S_1} \tan kl'_{d1} & \frac{S - S_2 - S_3}{S_1} \end{bmatrix} \cdots \\ &\times \begin{bmatrix} \cos kl_{d2-1} & j \sin kl_{d2-1} \\ j \sin kl_{d2-1} & \cos kl_{d2-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{S - S_3}{S - S_2 - S_3} A'_2 & \frac{S_2}{S - S_2 - S_3} \end{bmatrix} \cdots \\ &\begin{bmatrix} \cos kl'_{2r} & j \sin kl'_{2r} \\ j \sin kl'_{2r} & \cos kl'_{2r} \end{bmatrix}, \end{aligned} \quad (6)$$

where $l_{d2-1} = l_{d2} - l_d$, $l'_{d1} = l_{d1} + \delta_{1r}$, A'_2 is the specific acoustic admittance on the cross-section of right end of tube ② in the transition chamber ⑥ and may be expressed as

$$A'_2 = \frac{j \tan kl_{2-7} + \left[j \frac{S - S_3 - S_7}{S - S_3} \tan kl'_{nl} + \frac{S_7}{S - S_3} A_{er} \right]}{1 + j \tan kl_{2-7} \left[j \frac{S - S_3 - S_7}{S - S_3} \tan kl'_{nl} + \frac{S_7}{S - S_3} A_{er} \right]}, \quad (7)$$

where $l_{2-7} = l_3 - (l_{d2} + \delta_{2r} + l_{nl} + \delta_{7l})$.

For the left end-cavity ⑤ and the left solid portions of tubes ② and ③, the following expression may be obtained by the transfer matrix method

$$\begin{Bmatrix} p_2(0) \\ -\rho_0 c u_2(0) \end{Bmatrix} = [T_{ec}] \begin{Bmatrix} p_3(0) \\ \rho_0 c u_3(0) \end{Bmatrix}, \quad (8)$$

where,

$$\begin{aligned} [T_{ec}] &= \begin{bmatrix} \cos kl'_{2l} & j \sin kl'_{2l} \\ j \sin kl'_{2l} & \cos kl'_{2l} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j \frac{S - S_1}{S_2} \tan kl'_5 & \frac{S - S_1 - S_2}{S_2} \end{bmatrix} \cdots \\ &\begin{bmatrix} \cos kl_{c2-3} & j \sin kl_{c2-3} \\ j \sin kl_{c2-3} & \cos kl_{c2-3} \end{bmatrix} \cdots \\ &\times \begin{bmatrix} 1 & 0 \\ j \frac{S - S_1 - S_2 - S_3}{S - S_1 - S_2} \tan kl'_{c3} & \frac{S_3}{S - S_1 - S_2} \end{bmatrix} \begin{bmatrix} \cos kl'_{3l} & j \sin kl'_{3l} \\ j \sin kl'_{3l} & \cos kl'_{3l} \end{bmatrix}, \end{aligned} \quad (9)$$

for the case of $l_{c2} \geq l_{c3}$. Here, $l'_{2l} = l_a + l_{c2} + \delta_{2l}$, $l'_5 = l_1 - (l_{c2} + \delta_{2l})$, $l_{c2-3} = l_{c2} - l_{c3}$, $l'_{c3} = l_{c3} + \delta_{3l}$, $l'_{3l} = l_a + l_{c3} + \delta_{3l}$, δ_{2l} and δ_{3l} are the end corrections of left sides of tubes ② and ③, respectively. For the case of $l_{c2} < l_{c3}$,

$$\begin{aligned} [T_{ec}] &= \begin{bmatrix} \cos kl'_{2l} & j \sin kl'_{2l} \\ j \sin kl'_{2l} & \cos kl'_{2l} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j \frac{S - S_1 - S_2 - S_3}{S_2} \tan kl'_{c2} & \frac{S - S_1 - S_3}{S_2} \end{bmatrix} \cdots \\ &\times \begin{bmatrix} \cos kl_{c3-2} & j \sin kl_{c3-2} \\ j \sin kl_{c3-2} & \cos kl_{c3-2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j \frac{S - S_1}{S - S_1 - S_3} \tan kl''_5 & \frac{S_3}{S - S_1 - S_3} \end{bmatrix} \cdots \\ &\begin{bmatrix} \cos kl'_{3l} & j \sin kl'_{3l} \\ j \sin kl'_{3l} & \cos kl'_{3l} \end{bmatrix}, \end{aligned} \quad (10)$$

where $l'_{c2} = l_{c2} + \delta_{2l}$, $l_{c3-2} = l_{c3} - l_{c2}$, $l''_5 = l_1 - (l_{c3} + \delta_{3l})$.

The end corrections of tube extensions in the above expressions may be calculated by the following expression [6]

$$\begin{aligned} \delta/a &= 0.6165 - \frac{0.7046a}{a_v} + \\ &0.2051 \exp\left(-\frac{1.7227l_e}{a}\right) - \frac{0.3749a}{a_v} \exp\left(-\frac{1.3012l_e}{a}\right), \end{aligned} \quad (11)$$

where a is the radius of tube, a_v is the equivalent radius of chamber, and l_e is the tube extension into the chamber.

Eqs. (2) and (8) may be rewritten as

$$\begin{Bmatrix} p_1(l_p) \\ \rho_0 c u_1(l_p) \end{Bmatrix} = [T'_{tc}] \begin{Bmatrix} p_2(l_p) \\ \rho_0 c u_2(l_p) \end{Bmatrix}, \quad (12)$$

$$\begin{Bmatrix} p_2(0) \\ \rho_0 c u_2(0) \end{Bmatrix} = [T'_{ec}] \begin{Bmatrix} p_3(0) \\ \rho_0 c u_3(0) \end{Bmatrix}. \quad (13)$$

Finally, combining Eqs. (1), (12) and (13) yields the following relationship between the inlet and outlet of the muffler

$$\begin{Bmatrix} p_1(0) \\ \rho_0 c u_1(0) \end{Bmatrix} = [T] \begin{Bmatrix} p_3(l_p) \\ \rho_0 c u_3(l_p) \end{Bmatrix}, \quad (14)$$

where,

$$[T] = ([TR11][T'_{tc}] + [TR12])([T'_{ec}][TR31][T'_{ec}] + [TR32]) - [TR21][T'_{tc}] - [TR22]^{-1} \times ([TR23] - [T'_{ec}][TR33]) + [TR13]. \quad (15)$$

Thereby the transmission loss of the muffler can be calculated by

$$TL = 20 \log_{10} \left(\frac{1}{2} |T_{11} + T_{12} + T_{21} + T_{22}| \right) + 10 \log_{10} \left(\frac{S_1}{S_3} \right). \quad (16)$$

FINITE ELEMENT APPROACH

The three-dimensional sound field in the muffler is governed by Helmholtz equation [7]

$$\nabla^2 p + k^2 p = 0. \quad (17)$$

Eq. (17) may be solved numerically by the FEM. In order to apply the FEM to calculate the acoustic attenuation performance of perforated tube muffler, the muffler is divided into two acoustic domains by the perforated tubes: the internal domain Ω_1 and external domain Ω_2 , p_1 and p_2 are used to represent the sound pressures in the domains Ω_1 and Ω_2 , respectively.

For the acoustic domain Ω_1 , the Galerkin's method of weighted residuals is applied to Eq. (17) to give

$$[M_1]_e \{p_1\}_e - k^2 [S_1]_e \{p_1\}_e = \{F_1\}_e \quad (18)$$

where $[S_1]_e$ and $[M_1]_e$ are the acoustic stiffness and mass matrices of the element e , given by

$$[S_1]_e = \int_{\Omega_{1e}} \{N\} \{N\}^T dV, \quad (19)$$

$$[M_1]_e = \int_{\Omega_{1e}} \{\nabla N\} \{\nabla N\}^T dV, \quad (20)$$

and $\{F_1\}_e$ is the forcing vector given by

$$\{F_1\}_e = \int_{\Gamma_{1e}} \{N\} \frac{\partial p_1}{\partial n} dS. \quad (21)$$

where $\{N\}$ is the shape function vector, Γ_1 is the boundary of acoustic domain Ω_1 .

The boundary conditions of acoustic domain Ω_1 may be divided into four types:

(a). the rigid wall,

$$\frac{\partial p_1}{\partial n} = 0, \quad (22a)$$

(b). the velocity boundary,

$$\frac{\partial p_1}{\partial n} = -jk \rho_0 c u_{1n}, \quad (22b)$$

(c). the impedance boundary,

$$\frac{\partial p_1}{\partial n} = -jk / z_{1n} p_1, \quad (22c)$$

(d). the perforated tube wall,

$$\frac{\partial p_1}{\partial n} = -jk / \zeta_p (p_1 - p_2), \quad (22d)$$

where u_{1n} is the normal particle velocity on the boundary, z_{1n} is the normal specific acoustic impedance on the boundary, ζ_p is the specific acoustic impedance of perforation, p_1 and p_2 are the sound pressures on perforated tube wall of the acoustic domains Ω_1 and Ω_2 , respectively. For the above four