

Dynamics of Coupled Nonlinear Hypoid Gear Mesh and Time-varying Bearing Stiffness Systems

2011-01-1548

Published
05/17/2011Junyi Yang and Teik Lim
Univ of Cincinnati

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doi:10.4271/2011-01-1548

ABSTRACT

A new capability to analyze the dynamic interaction of nonlinear hypoid gear mesh characteristics and time-varying bearing stiffness is proposed. Both backlash nonlinearity and time-varying mesh parameters, such as mesh stiffness, mesh point and line-of-action, are included in the nonlinear hypoid gear mesh model. The time-varying bearing stiffness behavior due to the changing orbital position of rolling elements is also modeled. A practical application is studied to reveal the dynamic characteristics of the complex interactions. Dynamic simulation results show that dynamic mesh force is relatively insensitive to the temporal variation in the bearing stiffness. On the other hand, the dynamic bearing loads are affected significantly by the time-varying bearing stiffness, especially in the case of heavy drive torque load without the occurrence of jump response phenomenon. However, for the case of light load condition assuming time-invariant mesh parameters, the dynamic bearing loads are insensitive to time-varying bearing stiffness. The analysis also reveals that the presence of appropriate backlash nonlinearity can stabilize the parametric resonance caused by the time-varying bearing stiffness for the case of light load condition assuming the mesh parameters are time-invariant.

INTRODUCTION

The dynamic mesh force of engaging gear teeth, excited by transmission error, is believed to be one of the major sources of vibration and noise coming from the geared transmission systems. Vibration of gear pair can be transmitted through shaft and bearing system into the gearbox housing and subsequently to the rest of the vehicle structures. The structure-borne noise is then radiated from the vibrating housing and vehicle body structures. To be able to control this response, it is imperative that we understand the

underlying physics controlling the generation of the gear excitation. In geared rotor systems, gears and rolling element bearings are precision elements and are some of the most complex components inside the gearbox. Both of these machine elements are inherently nonlinear. However, their effects on the generation of gear excitation are still not well defined. Therefore, gaining an accurate representation and understanding of the dynamic interactions between the gears and bearing systems are essential in having the capability to predict the severity of gear vibration and noise produced.

The general stiffness matrix approach of representing the dynamic characteristics of rolling element bearing was formulated analytically by Lim and Singh [1]. Their theory was further implemented in general mechanical rotor and linear spur geared rotor systems [2, 3, 4]. For high frequency applications, the statistical energy analysis method was combined with the stiffness matrix representation to study the vibration energy transmitted through rolling element bearing [4]. Later on, the formulation was refined by considering the effect of load distribution along the roller length [5]. Based on those work [1, 2, 3, 4, 5], Liew and Lim [6] developed the time-varying stiffness formulation considering the raceway rotation, and implemented the new theory into a linear parallel geared rotor system. An alternative approach was used by Lahmar and Velez [7] to study the interaction between rolling element bearing and linearly behaving helical gear dynamics. In their study, time-varying property of rolling element bearing was formulated by recalculating the bearing stiffness matrix at each time step of the numerical integration process. Interaction of bearing radial clearance and spur gear pair backlash clearance was studied by using multiple degrees of freedom nonlinear dynamic model with multiple clearances [8, 9, 10]. However, no prior publications that directly deal with the issue of dynamic interaction between non-linear hypoid gear mesh and time-varying

bearing model was found in the open literature. The attempt to fill this gap is one of the primary objectives of the present investigation.

An analytical framework for analyzing both linear and nonlinear pure vibration characteristics of hypoid and bevel geared rotor system was developed by Cheng and Lim [11, 12, 13, 14, 15]. The single-point mesh model was formulated to represent the coupling between engaging gear pair. Due to the curvilinear feature of gear tooth and kinematics, the coupling between engaging gear pair is time-varying. Backlash was also modeled to represent the clearance of the working gear pair. Later, Wang and Lim [16] further developed a multi-point mesh model. Instead of using a single mesh stiffness coupling for the whole gear pair, a multi-point mesh representation is aimed at constructing an individual mesh stiffness coupling for each tooth pair that is in contact. As a result, the dynamic mesh force at each individual tooth pair can be obtained. More recently, a coupled multi-body dynamics and vibration model is developed by Tao[17]. In that formulation, the gear perturbation responses are analyzed simultaneously with the rigid body rotation of the gear pair. This requires the mesh model be defined as a function of gear angular position rather than strictly time. Thus, the dynamic load of teeth in contact for practical load and operating cases can be simulated. However, the major drawback of both the multi-point mesh theory and coupled multi-body dynamics and vibration model is the high computational cost due to the significantly more complex mathematical expressions. In fact, the designs of these complex models make them more suitable for a wide variety of problems including fatigue, duty cycle and transient issues in addition to simply noise and vibration response. In this paper, since the main focus is on the vibration and noise response, the single-point mesh representation and pure vibration assumption are sufficient.

In this investigation, the dynamic interaction between nonlinear hypoid gear mesh and time-varying bearing stiffness is studied. Both time-varying mesh parameters and backlash nonlinearity of hypoid gear are also considered. The dynamic characteristics of the rolling element bearing are represented by a set of time-varying stiffness matrices. Numerical result seems to show that dynamic mesh force is insensitive to time-varying bearing stiffness effect. On the other hand, the dynamic bearing load is found to be affected significantly by the temporal variation in its stiffness, as expected. The jump response frequency and amplitude of both the dynamic mesh force and dynamic bearing load are affected only slightly by the time-varying bearing stiffness property. On the other hand, both the dynamic mesh force and bearing loads are found to be sensitive to backlash.

NONLINEAR GEARED ROTOR SYSTEM MODEL

BASIC ASSUMPTIONS

A generic hypoid geared rotor system consisting of a coupling gear pair, shafts, bearings, engine and load is modeled with a nonlinear, 14 degrees of freedom (DOF) lumped parameter representation as shown in Figure 1.

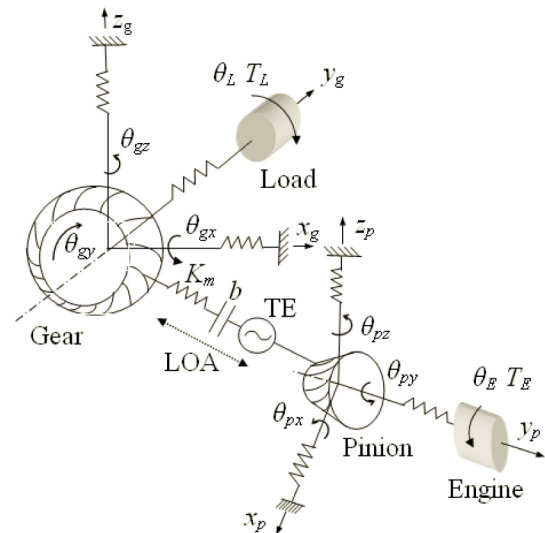


Figure 1. Schematic of a 14-DOF nonlinear dynamic model of hypoid geared rotor system.

Pinion and gear are considered as rigid bodies, and the engine and load are each represented by a rotational coordinate. The flexibility of both pinion and gear shaft bearing assemblies is condensed into a set of stiffness matrices to support the pinion and gear bodies. The coupling between the engaging gear pair is represented using single-point mesh model. The proposed mesh model consists of mesh point, line-of-action, mesh stiffness, backlash and transmission error. The model used in this paper is similar to the one proposed by Tao[17] except for the following feature. Due to the time-varying property of rolling element bearing, in this analysis, the support stiffness matrices are time-varying. This differs from the constant coefficient model previously employed by Tao[17].

MESH MODEL

The gear mesh model can be obtained by several ways, including pitch cone design theory, unloaded tooth contact analysis and loaded tooth contact analysis [15, 16, 17]. The first two approaches use idealized tooth surface geometry only. The third one uses both detailed tooth surface geometry and the effect of material elasticity. In this study, the mesh model is condensed from the results of a loaded tooth contact analysis performed using a formulation that combines a semi-

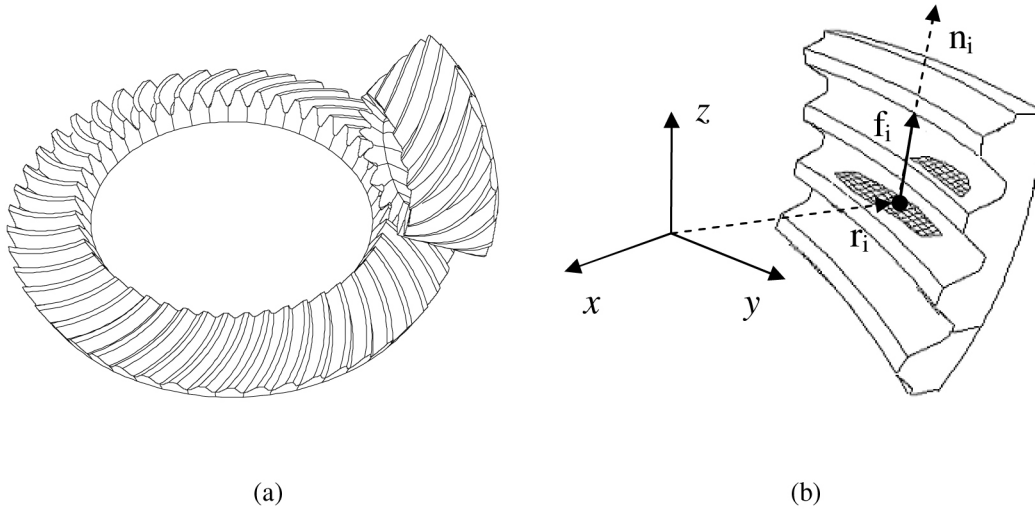


Figure 2. Loaded tooth contact analysis model: (a) Gear pair geometry, and (b) contact cells on engaging tooth surface.

analytical theory with a 3-dimensional finite element (FE) approach [18].

The geometry of hypoid gear pair is shown in Figure 2(a), and Figure 2(b) shows the result of contact analysis qualitatively. For contact cell i in the mesh coordinate system S_m , its position vector is represented by $r_i(r_{ix}, r_{iy}, r_{iz})$, contact force is represented by f_i , and $n_i(n_{ix}, n_{iy}, n_{iz})$ stands for its normal vector.

A vector summation process is performed to obtain the total mesh force F given by

$$F_j = \sum_{i=1}^N n_{ij} f_i, \quad F = \sqrt{\sum_{j=x}^z F_j^2}, \quad (j = x, y, z), \quad (1)$$

while the line-of-action $N(n_x, n_y, n_z)$ can be obtained from

$$n_j = F_j / F, \quad (j = x, y, z). \quad (2)$$

The mesh point $R(x_m, y_m, z_m)$ is computed by applying a method that is commonly used to compute the mass center of gravity of a group of rigid bodies. It can be obtained by solving the following algebraic equations

$$x_m = (M_z + F_x y_m) / F_y, \quad (3a)$$

$$y_m = (M_x + F_x z_m) / F_z, \quad (3b)$$

$$z_m = (M_y + F_z x_m) / F_x, \quad (3c)$$

where $M_j (j = x, y, z)$ is the moment about the axis j caused by the combined loads at all the contact cells. The expression for the moment can be explicitly given by

$$\{M_x, M_y, M_z\} = \sum_{i=1}^N f_i \cdot (r_i \times n_i). \quad (4)$$

The angular transmission error is a measure of the deviation of the relative rigid body rotational motions of the gear pair from the ideal position as dictated by the gear ratio. The loaded and unloaded translational transmission errors e and e_0 are the projections of respective angular transmission error along the line of action of the net mesh force. Finally, the mesh stiffness is define by

$$k_m = F / (e - e_0). \quad (5)$$

The mesh model will be utilized in the dynamic model to represent the mesh coupling between pinion and gear. Note that the mesh formulation established above is in the mesh model coordinate system denoted by S_m . Therefore, it should be transformed into pinion or gear coordinate systems when assimilated into the dynamic model.

SHAFT-BEARING ASSEMBLY MODEL

Support stiffness matrices for the pinion $[k_p]$ and gear $[k_g]$ can be obtained computationally, analytically or experimentally. In this study, the FE approach utilizing beam elements is applied to extract the flexibility of shat-bearing assemblies. This proposed approach is applied to model the pinion/gear overhung position as shown in Figure 3. Figure

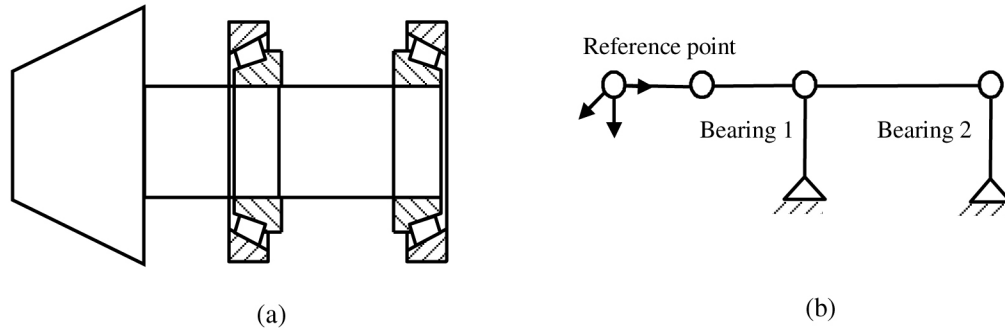


Figure 3. Shaft bearing assembly: (a) physical structure, (b) beam finite element representation.

3(a) is the physical structure and Figure 3(b) is the corresponding FE representation.

In the shaft-bearing assembly, each rolling element bearing is represented by a set of time-varying stiffness matrices. Pinion/gear shaft is modeled as beam elements, and each node has 6-DOF. The relation between the applied load and deflection at the reference point will give the support stiffness matrix. In this investigation, six load cases are designed. In each load case, one column of a 6×6 identity matrix is taken as the excitation force vector $\{F_i\}$, $i = 1, 2, \dots, 6$ exerted on the reference point. For each load case, the finite element equations are solved for the displacement vector of reference point represented by $\{d_i\}$, $i = 1, 2, \dots, 6$. The support stiffness matrix can be calculated using

$$[k_l] = [\{d_1\}, \{d_2\}, \{d_3\}, \{d_4\}, \{d_5\}, \{d_6\}]^{-1},$$

$$l = p, g. \quad (6)$$

This modeling process for the mean support stiffness matrices combined with the geared rotor system dynamic model was previously validated by experimental results [17]. However, in this paper, the resultant support stiffness matrices are time-varying due to the time-varying property of rolling element bearings.

TIME-VARYING BEARING STIFFNESS

The kinematics and coordinate system of a rolling element bearing is shown as Figure 4. The same nomenclature employed by Lim and Singh [1, 2, 3, 4, 5], and Liew and Lim [6] are adopted here. Raceways and rolling elements are assumed to be rigid except at the localized contact area. The Hertzian contact theory is used to calculate the normal contact forces.

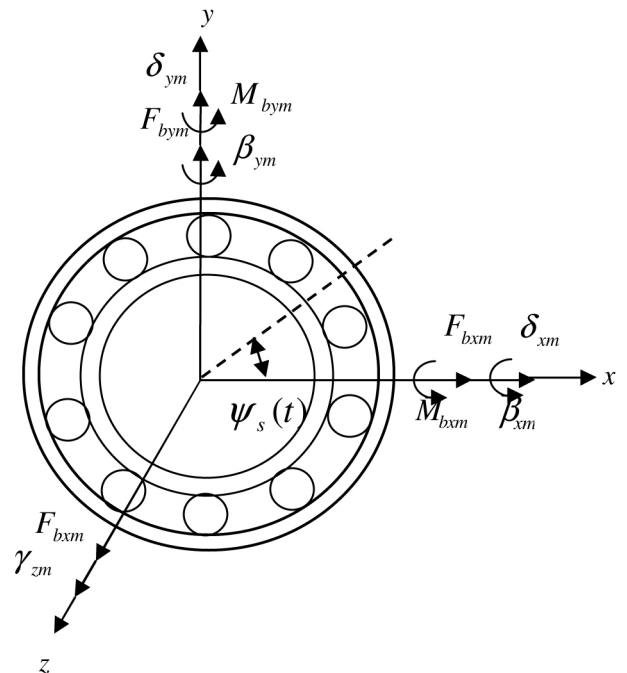


Figure 4. Rolling element bearing kinematics and the corresponding coordinate systems.

The bearing system is assumed to be able to rotate freely about the z -axis, and $\{F_{bxm}, F_{byy}, F_{bzm}, M_{bxm}, M_{byy}, M_{bzm}\}$ is the net load vector on the rolling element bearing. The vector $\{\delta_{xm}, \delta_{ym}, \delta_{zm}, \beta_{xm}, \beta_{ym}\}$ is used to represent the displacements of the bearing system. The analytical relation between the load and displacement vector was previously formulated as a function of geometrical parameters by Lim and Singh [1, 2, 3, 4, 5]. The elements of the bearing stiffness matrix $[k_b]$ are defined as the partial derivatives of load with respect to displacement. For the diagonal element for the x translational coordinate, this partial derivative is given by

$$k_{xx} = \frac{\partial F_{bxm}}{\partial \delta_{xm}}, \quad (7)$$

and for the coupling element between x and y translational coordinates, it is given by

