

Energy Finite Element Analysis Developments for Vibration Analysis of Composite Aircraft Structures

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ABSTRACT

The Energy Finite Element Analysis (EFEA) has been utilized successfully for modeling complex structural-acoustic systems with isotropic structural material properties. In this paper, a formulation for modeling structures made out of composite materials is presented. An approach based on spectral finite element analysis is utilized first for developing the equivalent material properties for the composite material. These equivalent properties are employed in the EFEA governing differential equations for representing the composite materials and deriving the element level matrices. The power transmission characteristics at connections between members made out of non-isotropic composite material are considered for deriving suitable power transmission coefficients at junctions of interconnected members. These coefficients are utilized for computing the joint matrix that is needed to assemble the global system of EFEA equations. The global system of EFEA equations is solved numerically and the vibration levels within the entire system can be computed. The new EFEA formulation for modeling composite laminate structures is validated through comparison to test data collected from a representative composite aircraft fuselage that is made out of a composite outer shell and composite frames and stiffeners. NASA Langley constructed the composite cylinder and conducted the test measurements utilized in this work.

INTRODUCTION

The EFEA is an emerging technology for analyzing complex structural-acoustic systems [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. The EFEA utilizes the spatially averaged vibrational or acoustic energy as a primary variable for developing governing differential equations for each wave type present within a system. The fundamental solution to the governing differential equations presents an exponential variation with space, which eliminates the requirement of using small finite elements at high frequencies. At discontinuities (i.e. connections between structural components, structural and acoustic interfaces, changes in material properties, changes in geometry, etc.) the primary variables in the EFEA become discontinuous. Joint matrices must be developed in order to introduce the power transfer characteristics at each discontinuity into the EFEA system of equations. For each finite element encountered at a discontinuity, the joint matrices represent relationships between the power flow at the element's boundaries and the corresponding EFEA primary variables. The joint matrices are developed from power transfer coefficients representative of the connections between the members encountered at the discontinuity. The derivation of power transfer coefficients is based on considering a connection between semi-infinite members, prescribing an impinging wave type originating from one of the members, and evaluating all transmitted and reflected wave types in all members. A library of element matrices and a library of joints can be combined for creating

the EFEA model for a complex system and computing its structural-acoustic response.

This paper aims to further extend the capability of the EFEA method for computing the vibro-acoustic response of aircraft type of structures made out of composite laminates and/or composite sandwich panels. A Spectral Finite Element Method (SFEM) [19, 20] is used for evaluating the group speed c_g^* and the damping η^* due to the effectiveness of SFEM for taking into account the layer-wise composition of composite panels along with the shear deformation effects.

The c_g^* and η^* values computed by the SFEM are used in the EFEA formulation for deriving the element level matrices of the composite materials. A method for deriving the power transmission coefficients between composite members is presented. The new method can take into account the shear deformation effects of composite plates. The power transfer coefficients are used for deriving the joint matrices in the EFEA formulation. Theoretical aspects of these two formulations are presented in this paper. A numerical EFEA implementation for composite materials is utilized for analyzing the NASA Composite Test-bed Cylinder (CTC) and comparing simulation results to test data for the vibration at several bays of the cylinder. The CTC has longitudinal stiffeners (representing stringers) and circumferential stiffeners (representing frames) and represents the dynamic characteristics of a composite aircraft fuselage [22]. Excitation is applied through a shaker at different locations on the CTC and measurements are collected using a laser vibrometer at twelve bays of the cylinder. The results from the EFEA simulations are compared to test data in the frequency range that covers the 1/3 octave bands of 800Hz-4,000Hz. The experimental set-up and the correlation study are discussed in this paper.

DERIVATION OF GROUP SPEED AND DAMPING FOR COMPOSITES

The c_g^* and η^* values computed by the SFEM are used in the EFEA formulation for composites for deriving the element level matrices. The basic steps of the SFEM approach are presented here. Figure depicts a plane wave propagating in the positive x direction with frequency ω and wavenumber k in a multilayer composite plate. The x direction comprises a reference for orienting the various layers. The through-thickness discretization in the z -axis along with the assumption of the harmonic wave motion in the x direction results in the following form of displacement field, $\mathbf{u}^T = [u, v, w]$ at any point, (x, y, z) within the plate,

$$\mathbf{u}(x, z, t) = \mathbf{N}(z)\hat{\mathbf{u}}e^{i(\omega t - kx)} \quad (1)$$

where $\mathbf{N}(z)$ is a matrix of shape functions and $\hat{\mathbf{u}}$ is a vector of the nodal degrees of freedom of the form:

$$\hat{\mathbf{u}} = [u_1, v_1, w_1 \quad u_2, v_2, w_2 \quad \cdots \quad u_{N_e+1}, v_{N_e+1}, w_{N_e+1}]^T \quad (2)$$

where N_e is the total number of linear finite elements through the thickness. It is noted that any of the displacement components in $\hat{\mathbf{u}}$ may be complex numbers.

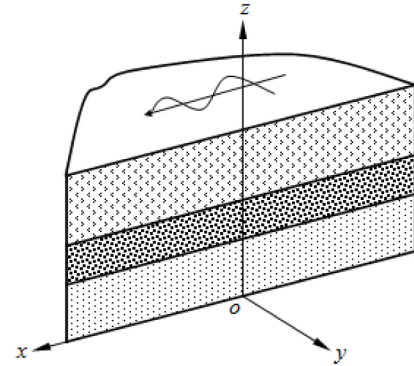


Figure 1. A plane wave propagation in a multilayer composite panel

The time-averaged total kinetic and potential energies, $\langle T \rangle$ and $\langle V \rangle$ are given by

$$\langle T \rangle = \frac{\omega^2}{4} \int_{\Omega} \rho \mathbf{u}^H \mathbf{u} d\Omega; \langle V \rangle = \frac{1}{4} \int_{\Omega} \boldsymbol{\sigma}^H \boldsymbol{\varepsilon} d\Omega \quad (3)$$

where ^{H} stands for the Hermitian transpose. Substitution of appropriate stress-strain and strain-displacement relations and the displacement field equation (1) into equation (3) results in:

$$\langle T \rangle = \hat{\mathbf{u}}^H \omega^2 \mathbf{M} \hat{\mathbf{u}}; \langle V \rangle = \hat{\mathbf{u}}^H \mathbf{K} \hat{\mathbf{u}} \quad (4)$$

where $\mathbf{K}$ and $\mathbf{M}$ are, respectively, the stiffness and mass matrices. The replacement of the spatial derivatives with respect to x and z by $-ik$ and $d\mathbf{N}/dz$ can yield the expressions for the stiffness and mass matrices. Using Hamilton's principle results in:

$$[\mathbf{K}(k) - \omega^2 \mathbf{M}] \hat{\mathbf{u}} = \mathbf{0} \quad (5)$$

Since $\mathbf{K} = \mathbf{K}(k)$, for a prescribed circular frequency ω , equation (5) can be solved for the eigenvalues ω^2 and eigenvectors $\hat{\mathbf{u}}$. The hysteretic damping model can be applied to each layer to yield the following form for the total time-averaged energy loss associated with an arbitrary wave type,

$$\langle \pi_{diss} \rangle = \sum_{l=1}^{N_e} \eta_l \omega \langle e \rangle_l \quad (6)$$

where η_l and $\langle e \rangle_l$ are, respectively, the structural loss factor and energy density of the l th layer of multi-layered composites. Since $\langle e \rangle = \langle T \rangle + \langle V \rangle = 2\langle V \rangle$, the damping loss factor associated with each propagating wave can be expressed as

$$\eta = \frac{\sum_{l=1}^{N_e} \eta_l \hat{\mathbf{u}}^H \mathbf{K}^{(l)} \hat{\mathbf{u}}}{\hat{\mathbf{u}}^H \mathbf{K} \hat{\mathbf{u}}} \quad (7)$$

Here, the distinction should be made that $\mathbf{K}^{(l)}$ is the stiffness matrix of the l th layer and $\mathbf{K}$ is the assembled global stiffness matrix of all layers. Equation (7) is derived for each wave heading angle θ_i , and thus the angle-average must be considered for accounting for the full range of wave propagation directions when deriving the angle averaged damping loss factor as follows:

$$\eta^* = \frac{\int_{\Theta} \eta(\theta_i) d\theta_i}{\int_{\Theta} d\theta_i} \quad (8)$$

The variation of the angle θ_i is obtained by rotating the relative orientation of the x-axis with respect to the composite material (Figure 1). The circular frequency for the wave number k can be written in terms of the Rayleigh quotient as

$$\omega^2 = \frac{\hat{\mathbf{u}}^H \mathbf{K}(k) \hat{\mathbf{u}}}{\hat{\mathbf{u}}^H \mathbf{M} \hat{\mathbf{u}}} \quad (9)$$

Figure 2 shows the wavenumber of an arbitrary wave propagating in an anisotropic medium as a function of wave heading, θ_i . The distribution and the direction of energy flow in anisotropic media like composites exhibit directional dependence, thus the direction of the energy flow, depicted as θ_e in Figure 2, is usually different from the wave heading, θ_i . Thus, an appropriate correction must be introduced in the calculation of the group speed.

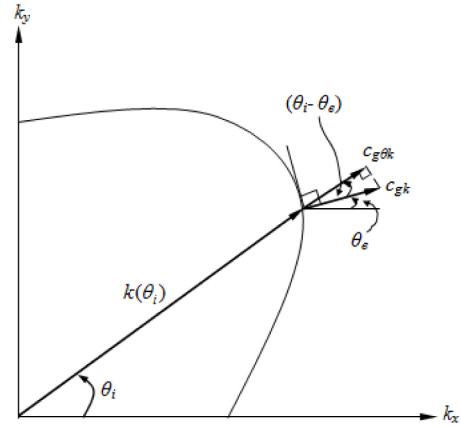


Figure 2. Wavenumber as a function of wave heading in the k (wave number) plane

The group speed in the direction of wave propagation is $c_g \theta = d\omega/dk$, and the differentiation of equation (9) with respect to the wavenumber, k , results in:

$$c_{g\theta} = \frac{\hat{\mathbf{u}}^H (\partial \mathbf{K}(k) / \partial k) \hat{\mathbf{u}}}{2\omega \hat{\mathbf{u}}^H \mathbf{M} \hat{\mathbf{u}}} \quad (10)$$

Referring to Figure 2, the following relationship holds:

$$c_g = \frac{c_{g\theta}}{\cos(\theta_i - \theta_e)} \quad (11)$$

where the heading of group speed, θ_e may be derived from the geometric interpretation of the wavenumber curve shown in Figure 2

$$\tan \theta_e = -\frac{\partial k_x / \partial \theta_i}{\partial k_y / \partial \theta_i} = -\frac{(\partial k(\theta_i) / \partial \theta_i) \cos \theta_i - k(\theta_i) \sin \theta_i}{(\partial k(\theta_i) / \partial \theta_i) \sin \theta_i + k(\theta_i) \cos \theta_i} \quad (12)$$

Therefore, the angle-averaged group speed can be evaluated for the full range of wave propagation directions as follows:

$$c_g^* = \frac{\int_{\Theta} c_g(\theta_i) d\theta_i}{\int_{\Theta} d\theta_i} \quad (13)$$

In the EFEA formulation for composites, c_g^* and η^* from Equations (13) and (8) respectively, are used when deriving the element level EFEA matrices.