

A Model for Combined Tire Cornering and Braking Forces with Anisotropic Tread and Carcass Stiffness

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ABSTRACT

The objective of this paper is to enhance the accuracy of tire model combined tire cornering and braking forces with anisotropic tread and carcass stiffness. The difference of tire longitudinal slip stiffness and cornering stiffness will arouse that the direction of tire resultant shear stress in adhesion region is not the same as that in sliding region. Then the direction of total friction force in the whole tire-road contact patch will change under different combined cornering/braking situations.

Generally speaking, there is a basic premise: “the direction of resultant shear stress in sliding region will be the same as that in adhesion region” in the existing tire models, in which the anisotropy of tread and carcass stiffness is neglected. Therefore, these models don't work well when the tire tread and carcass stiffness has a strong anisotropy.

The direction of tire shear stress in the adhesion and the sliding region of the contact patch is discussed in this paper and a modification factor is proposed basing on the UniTire semi-physical model to modify the direction of resultant shear stress, which can describe the variation tendency of the total friction force direction in the tire-road contact patch with different combination of cornering and braking conditions. Finally, the UniTire semi-physical tire model contained modification factor is validated by the experimental data, which indicates the accuracy of combined tire forces has been enhanced significantly.

INTRODUCTION

The tire mechanics characteristics under combined cornering/braking situations have significant effects on the vehicle directional control during driving and braking [1]. Various tire models have been proposed to describe these mechanical properties [2,3,4], in which there is always an assumption: “the direction of resultant shear stress in sliding region and adhesion region is coincident”. It is true for the condition of equality of longitudinal slip and cornering stiffness, however, in most conditions, the longitudinal slip stiffness is different with the cornering stiffness. Thus, the premise will not be suitable. Especially, when the anisotropy of tread and carcass stiffness is strong, the direction of the shear force in adhesion region is totally different from that in sliding region, which will consequently result in the obvious deviation of both the longitudinal and the lateral force.

Therefore, in order to describe the combined tire cornering and braking forces, the relevant problem that how to describe the direction of total friction force precisely is a continued source of concern. In this paper, firstly, a simplified physical tire model is discussed to explain the basic and simplified relationship between tire shear forces and normalized slip ratio. Then, a modification factor expression is proposed to describe the variation tendency of the direction of total friction forces in different combined cornering/braking situations. And after knowing the magnitude and direction of total friction force, the longitudinal and lateral forces can be obtained eventually. In this paper, a set of test data with strong anisotropy of tire stiffness, in which the longitudinal stiffness is about three times larger than lateral stiffness, is used to verify the model results, which shows that the modification factor has good effect to enhance the accuracy of tire model at cornering/braking combination.

SIMPLIFIED PHYSICAL TIRE MODEL

TIRE SHEAR FORCE WITHOUT SLIDING

The physical model of a tire [2] is simplified as Fig.1, assuming that the carcass of the tire can merely be deformed along the directions of X and Y axes translationally, without any angular deflection.

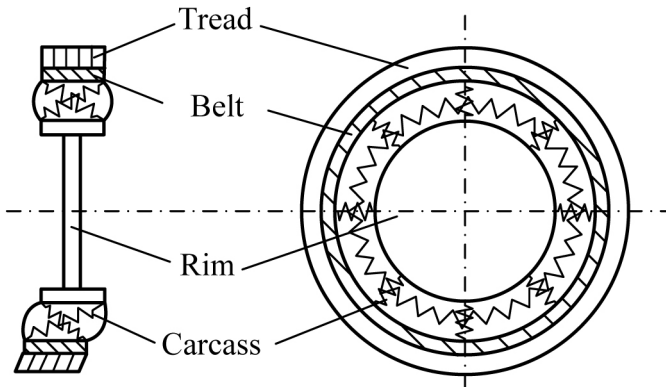


Figure 1. The physical model of a tire

The relationship between the tire force and velocity in the contact patch, under combined slip condition, is shown in Fig.2. F denotes the total friction force in the contact patch, the longitudinal and lateral components of which are represented by F_x , F_y respectively. The direction of wheel traveling speed, which is shown as V in the figure, gives a slip angle α with respect to the central wheel plane.

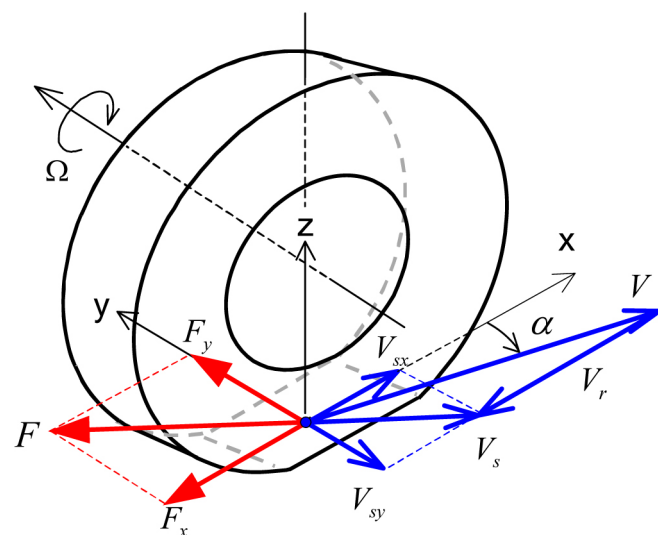


Figure 2. Tire forces and velocities

The rolling speed (updating speed of the contact patch) of the tire can be expressed as:

$$V_r = \Omega \cdot R_e \quad (1)$$

where Ω is the angular speed, R_e is the effective rolling radius.

The sliding speed $\vec{V}_s$ in the center of contact patch is a vector, could be expressed as:

$$\vec{V}_s = \vec{V} + \vec{V}_r \quad (2)$$

So, V_{sx} and V_{sy} , the longitudinal and lateral sliding speeds are obtained as:

$$\begin{cases} V_{sx} = V \cdot \cos \alpha - V_r \\ V_{sy} = V \cdot \sin \alpha \end{cases} \quad (3)$$

The longitudinal and lateral slip ratios can all be defined in an unified form as:

$$S_x = \frac{-V_{sx}}{\Omega R_e}, S_y = \frac{-V_{sy}}{\Omega R_e} \quad (4)$$

The longitudinal slip ratio usually used in tire force test is defined as:

$$\kappa = \frac{-V_{sx}}{V \cos \alpha} = \frac{\Omega R_e - V \cos \alpha}{V \cos \alpha} \quad (5)$$

The relationship between S_x and κ is:

$$S_x = \frac{\kappa}{1 + \kappa} \quad (6)$$

In order to obtain the shear force in the contact patch, the deformations of the tread and carcass along X and Y axes must be known firstly. The longitudinal and lateral deformations of the tread and carcass, under combined slip condition, are shown in Fig.3. In this figure, $X_t O_t Y_t$ is a coordinate system for describing the deformations of both carcass and tread, and XOY is a relative coordinate system for describing the tread deformation with respect to the carcass. The origin 'O' is set at the leading point of contact patch for convenience which coincided with point 'O_t' before it is deformed. 'ABC' is the central line of the contact patch under combined slip condition. V is the wheel traveling speed, the direction of which gives a slip angle, α , with respect to the wheel plane. In general case, the whole length

of contact patch, $2a$, is divided into two parts, the adhesion region ‘AB’, and the sliding region ‘BC’, by the initial sliding point ‘B’. x_c and y_c are the deformations of carcass along axes X_t and Y_t caused by the longitudinal and lateral forces, which are expressed as:

$$x_c = \frac{F_x}{k_{cx}}, \quad y_c = \frac{F_y}{k_{cy}} \quad (7)$$

where k_{cx} and k_{cy} are the longitudinal and lateral stiffness of the carcass.

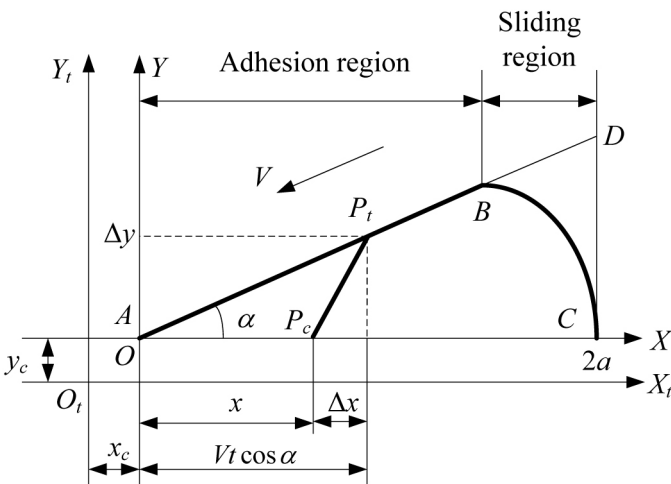


Figure 3. Deformations of tread and carcass

Now, F_x and F_y are assumed to be quite small, so that there is no sliding region in the contact patch (the point 'B' coincides with the point 'D' in Fig.3). A point in contact patch, which begins to contact at the point 'A', is now reaching the position 'P_t', after rolling for a period of time t . Meanwhile, the corresponding point on carcass moves from the point 'O_t' to 'P_C', the coordinates of which is x_t and can be calculated by

$$\begin{cases} x_t = x_c + x \\ x = V_r \cdot t = \Omega \cdot R_e \cdot t \end{cases} \quad (8)$$

To account for the force in the contact patch, we need to calculate the deformations of the tread along X and Y axes:

$$\begin{cases} \Delta x = Vt \cos \alpha - x = V_{sx} \cdot t = S_x \cdot x \\ \Delta y = Vt \sin \alpha = V_{sy} \cdot t = S_y \cdot x \end{cases} \quad (9)$$

If the stiffness of tread material in X and Y directions are k_{tx} and k_{ty} , the shear stresses of point P_t in both directions can be expressed as:

$$q_x = k_{tx} \cdot \Delta x, \quad q_y = k_{ty} \cdot \Delta y \quad (10)$$

Then the magnitude of resultant shear stress becomes:

$$q = \sqrt{q_x^2 + q_y^2} = x \sqrt{(k_{tx} S_x)^2 + (k_{ty} S_y)^2} \quad (11)$$

The shear force in the direction X and Y can be determined as follows:

$$\begin{cases} F_x = \int_0^{2a} q_x dx = K_x S_x \\ F_y = \int_0^{2a} q_y dx = K_y S_y \end{cases} \quad (12)$$

where $K_x = 2a^2k_{tx}$, $K_y = 2a^2k_{ty}$ represent the longitudinal slip and cornering stiffness.

SHEAR FORCE IN GENERAL CASE WITH SLIDING REGION

Considering that the sliding region might exist, the shear force in this area should be expressed as $q(x) = \mu q_z(x)$, based on which the coordinate of initial sliding point, $x = x^*$, or the relative coordinate $u^* = x^*/a$, can be solved [5]. In this expression, $q_z(x)$ denotes the vertical load distribution along the contact patch length, which can be expressed in a unified form as:

$$q_z = \frac{F_z}{2a} \cdot \eta(u) \quad (13)$$

where F_z is tire vertical load; $u=x/a$ is relative coordinate and $\eta(u)$ is the normalized pressure distribution function.

Then, the coordinate of initial sliding point will satisfy the following equation:

$$q = \sqrt{q_x^2 + q_y^2} = x \sqrt{(k_{ix} S_x)^2 + (k_{iy} S_y)^2} = \frac{\mu F_z}{2a} \eta(u) \quad (14)$$

The normalized longitudinal slip ratio ϕ_x , lateral slip ratio ϕ_y , and combined slip ratio ϕ are defined as:

$$\begin{cases} \phi_x = \frac{2a^2 k_{tx} S_x}{\mu_x F_z} = \frac{K_x S_x}{\mu_x F_z} \\ \phi_y = \frac{2a^2 k_{ty} S_y}{\mu_y F_z} = \frac{K_y S_y}{\mu_y F_z} \\ \phi = \sqrt{\phi_x^2 + \phi_y^2} \end{cases} \quad (15)$$

Substituting the given expressions into equation (14), it yields

$$\phi = \eta(u^*)/u^* \quad (0 \leq u^* \leq 2) \quad (16)$$

If the ϕ_x and ϕ_y are given, the relative coordinate of the initial sliding point, u^* , could be solved. Then total tire shear force can be expressed as:

$$\begin{aligned} F &= a \int_0^{u^*} q du + \int_{u^*}^2 \frac{\mu F_z}{2} \cdot \eta(u) du \\ &= \int_0^{u^*} \frac{\mu F_z}{2} \cdot \phi \cdot u du + \int_{u^*}^2 \frac{\mu F_z}{2} \cdot \eta(u) du \\ &= \frac{\mu F_z}{4} \cdot \phi \cdot u^{*2} + \frac{\mu F_z}{2} (2 - m_0) \end{aligned} \quad (17)$$

or in a normalized form as follow:

$$\bar{F} = \frac{F}{\mu \cdot F_z} = \frac{1}{4} \cdot \phi \cdot u^{*2} + 1 - \frac{m_0}{2} \quad (18)$$

where m_0 can be defined as $m_0 = \int_0^{u^*} \eta(u) du$

It is notable that m_0 is a function of coordinate of initial sliding point (u^*), which can be solved as a function of normalized combined slip ratio (ϕ). So the normalized tire force ($\bar{F}$) can be expressed as a function of the single variable ϕ .

Considering the assumption that the direction of the resultant shear stress in the sliding region will be the same as that in the adhesion region, the normalized longitudinal and lateral force can be obtained as follow:

$$\begin{cases} \bar{F}_x = F_x / (\mu_x F_z) = \bar{F} \cdot \phi_x / \phi \\ \bar{F}_y = F_y / (\mu_y F_z) = \bar{F} \cdot \phi_y / \phi \end{cases} \quad (19)$$

MAGNITUDE OF THE TOTAL FRICTION FORCE

To obtain the longitudinal and lateral forces under combined slip situations, the basic question is: what are the magnitude and direction of total friction force?

In the last section, the magnitude of friction force can be calculated by Eq.(17), however, it is not accurate enough to be used in practice due to neglecting of bending deformation of the carcass and other simplifications. Therefore, a more effective way for engineering application is to present a semi-physical model, based on the theoretical results, with some macro-parameters, which can be identified from available tests.

It is seen from previous section that the magnitude of $\bar{F}$ is a function of ϕ , that is $\bar{F} = f(\phi)$. And from the simplified physical model, the boundary conditions of $\bar{F}$ can be derived as[6,7,8]:

$$\begin{cases} \lim_{\phi \rightarrow 0} \bar{F} = 0 \\ \lim_{\phi \rightarrow 0} \frac{d\bar{F}}{d\phi} = 1 \\ \lim_{\phi \rightarrow 0} \frac{d^2 \bar{F}}{d\phi^2} = \frac{2}{D} \\ \lim_{\phi \rightarrow 0} \frac{d^3 \bar{F}}{d\phi^3} = \frac{2}{D^2} \left(3 - \frac{2D'}{D} \right) \\ \lim_{\phi \rightarrow \infty} \bar{F} = 1 \\ \lim_{\phi \rightarrow \infty} \frac{d\bar{F}}{d\phi} = 0 \end{cases} \quad (20)$$

Where

$$D = \frac{d\eta(u)}{du} \Big|_{u=2}, \quad D' = \frac{d^2 \eta(u)}{du^2} \Big|_{u=2} \quad (21)$$

Satisfactory with the above boundary conditions, the UniTire semi-physical tire shear force model is written as[6,7,8]:

$$\begin{cases} \bar{F} = 1 - \exp \left[-\phi - E\phi^2 - \left(E^2 + \frac{1}{12} \right) \phi^3 \right] \\ F = \bar{F} \cdot \mu F_z \end{cases} \quad (22)$$