

New Attempts on Vehicle Suspension Systems Modeling and Its Application on Dynamical Load Analysis

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ABSTRACT

Suspension system dynamics can be obtained by various methods and vehicle design has gained great advantages over the dynamics analysis. By employing the new Udwadia-Kalaba equation, we endeavor some attempts on its application to dynamic modeling of vehicle suspension systems. The modeling approach first segments the suspension system into several component subsystems with kinematic constraints at the segment points released. The equations of motion of the unconstrained subsystems are thus easily obtained. Then by applying the second order constraints, the suspension system dynamics is then obtained. The equations are of closed-form. Having the equations obtained, we then show its application on dynamical load analysis. The solutions for the dynamical loads at interested hard points are obtained. We use the double wishbone suspension to show the systematic approach is easy handling.

INTRODUCTION

Vehicle suspensions are classic multi-body systems on the complex part. Researches on dynamics modeling of vehicle suspension systems have started with multi-body dynamics since last century and still keep moving on. While Schiehlen summarized the developments from the quite beginning on multi-body dynamics [1], a detailed survey by Blundel in a series of papers reviews some of the main contributions

leading to the development of models and tools in vehicle dynamics [2,3,4,5]. The series papers specifically invested researches and developments of the commercial software ADAMS. A later book by Blundel and Harty gives a systematic introduction on multi-body dynamics applying on vehicle dynamics [6]. Such development offers the automobile engineering great benefits and becomes one of the main tools that are used in vehicle design. The benefits still drive researchers to work on vehicle dynamics and simulation tools up to date. The Udwadia-Kalaba equation was raised in the 90's of last century and gains much interest on its application to complex multi-body system modeling. A survey by Schutte and Udwadia shows a good process of research made by using the Udwadia-Kalaba equation on multi-body system modeling, as well as its advantages [7]. The equation is well formed and offers a systematic way of dealing with complex multi-body systems even there are multiple closed loops presented. It is such development that inspires us to attempt applying the Udwadia-Kalaba equation to modeling vehicle suspension systems. Once a system modeling is done, the model can be used for all kinds of performance analysis and system control design. Here in the current literature, by using the new approach, we just emphasize on the dynamical load analysis. Load analysis is important in processing vehicle design, e.g. in designing vehicle body as well as the chassis components. Dynamical load plays an important role in the sense of reliability analysis. After obtaining the dynamical loads, useful tools

like the FEM (Finite Element Method) can thus be used for analyzing vehicle structures.

The current literature endeavors new attempts on applying the Udwadia-Kalaba equation to model vehicle suspension systems. The suspension system is segmented into a series of subsystems which are kinematically connected. The equations of motion of each subsystem are very easy to obtain. The constraint forces, which are induced by the constraints, can be calculated without using any auxiliary variables. As a result, the equations of motion of the constrained subsystems can be obtained. In the sense of numerical simulations, the approach can show a numerical performance analysis over suspension systems, which may be taken parallel to the famous Lagrangian approach, but with the following advantages: 1) Instead of searching the Lagrange multiplier, the new equation employs the Moore-Penrose inverse of a matrix for solving the equations. State that the Moore-Penrose inverse can be calculated off-line and the approach can give a closed-form (analytical form) solution of the dynamic equation. 2) the equations of motion can be explicitly represented only by using physical coordinates, and no minimum generalized coordinates are required. This may make the modeling of each subsystem more accessible. 3) the modeling procedure is systematic and the dynamic model of each subsystem can be easily added, removed and reused.

We introduce the Udwadia-Kalaba equation in Section 2 and then the double wishbone suspension system is chosen to illustrate how the modeling approach works. The segmentation of the suspension system is done in Section 3, and the modeling of each component is followed in Section 4. Section 5 introduces the constraints that are released during the segmentation in a proper form. The system dynamics is then achieved in Section 6. Section 6 also shows how to perform dynamical load analysis by using the obtained equation. Numerical simulation results are then shown in Section 7.

THE UDWADIA-KALABA EQUATION

For an unconstrained mechanical multibody system, like the vehicle suspension system, the equation of motion is generally given by

$$M(q, t)\ddot{q} + C(q, \dot{q}, t)\dot{q} + g(q, t) = \tau(t) \quad (1)$$

Where t is a time variable, q is the n -vector coordinate, $\dot{q}$ is the n -vector velocity, $\ddot{q}$ is the n -vector acceleration. Furthermore, $M(q, t) > 0$ is the initial matrix, $C(q, \dot{q}, t)\dot{q}$ is the terms of centrifugal/Coriolis force, $g(q, t)$ is the gravity force vector and denotes the external force. The matrices and vectors M , C , g and τ are in proper dimensions. Note that the

coordinate can be selected based on the specifics of the problem and does not need to be the generalized coordinate [8].

We next suppose that the system is subjected to h holonomic constraints of the form

$$\gamma_i(q, t) = 0 \quad i = 1, 2, \dots, h \quad (2)$$

and $m - h$ non-holonomic constraints of the form

$$\gamma_i(q, \dot{q}, t) = 0 \quad i = h + 1, h + 2, \dots, m \quad (3)$$

The initial conditions $q_0 = q(t = t_0)$ and $\dot{q}_0 = \dot{q}(t = t_0)$ are assumed to satisfy these constraints so that $\gamma_i(q_0, t_0) = 0$ $i = 1, 2, \dots, h$ and $\gamma_i(q, \dot{q}, t) = 0$ $i = h + 1, h + 2, \dots, m$. Under the assumption of sufficient smoothness, take time derivative once on Eq. (3) and twice on Eq. (2), the constraint equation can then be, in the matrix form of

$$A(q, t)\ddot{q} = b(q, \dot{q}, t) \quad (4)$$

where the constraint matrix A , is a known m by n matrix and b is a known m -vector.

The presence of constraints in Eq. (4) imposes additional generalized force of constraint on the system so that the explicit equations of motion of the constrained system now take the form

$$M(q, t)\ddot{q} = Q(\dot{q}, q, t) + Q^c(\dot{q}, q, t) \quad (5)$$

Where $Q = \tau - C\dot{q} - g$ and the additional term Q^c on the right-hand side arises by virtue of the imposed constraints which are proscribed by Eq. (4). A recently work by Udwadia and Kalaba shows the constraint force is given in the closed-form as [9]

$$Q^c(\dot{q}, q, t) = M^{\frac{1}{2}}B^+(q, t) \times (b(q, \dot{q}, t) - A(q, t)M^{-1}(q, t)Q(\dot{q}, q, t)) \quad (6)$$

where $B(q, t) = A(q, t)M^{\frac{1}{2}}(q, t)$ and '+' denotes the Moore-Penrose inverse of a matrix. This in turn means the equation of motion of the constrained mechanical system is

$$M(q, t)\ddot{q} = Q(\dot{q}, q, t) + M^{\frac{1}{2}}B^+(q, t) \times (b(q, \dot{q}, t) - A(q, t)M^{-1}(q, t)Q(\dot{q}, q, t)) \quad (7)$$

This is the famous Udwadia-Kalaba equation which is a most recent development on analytical dynamics. Comparing to the classic Lagrangian approach of dynamics modeling which mainly based on solving the so called Lagrange multiplier, the difference is notable, e.g., the Udwadia-Kalaba equation gives the closed-form equations of motion while the Lagrangian approach only numerically searching the Lagrange multiplier. Note again that the B^+ in the above equations can be calculated off-line, thus the equations of motion of the constrained mechanical system can be explicitly given.

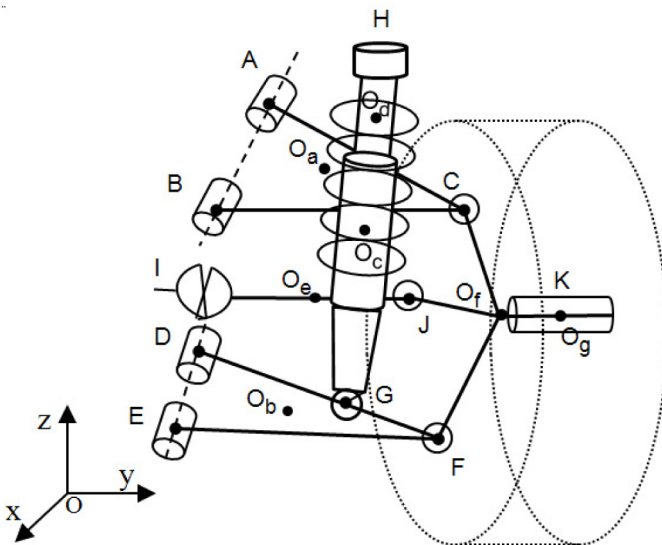


Figure 1. The double wishbone suspension system

DOUBLE WISHBONE SUSPENSION SYSTEM

Recent research shows an interest on using the Udwadia-Kalaba equation for dynamical modeling of complex mechanical systems, a mechanical system can be segmented into several unconstrained subsystems with kinematic constraints at the segment joints, the equations of motion of the unconstrained subsystems can be easily obtained, after applying the constraint in the form of Eq.(4), the equations of motion of the constrained mechanical system are then obtained. Thus the approach can be quite handy for dealing with mechanical system modeling even with the presence of closed-loop, this however, is usually difficult for other approaches or may be restricted by some inconvenience.

This inspires us to try the new approach on modeling vehicle suspension systems. To organize the current literature better, we now try to show the modeling procedure by using an example. We choose the double wishbone suspension system for illustration (shown in Figure 1).

The double wishbone suspension system is a classic multi-body mechanical system with the following components: the upper wishbone (component *a*), the lower wishbone (component *b*), the damper (lower component *c* and upper component *d*), the tie rod (component *e*), the steering knuckle (component *f*) and the wheel (component *g*). It is a quite complex one in a sense of presenting multiple closed kinematic loops in topology. Employing the Udwadia-Kalaba equation suggests us to segment the double wishbone suspension system into several subsystems. Here for convenience, we do the segmentation at all the possible joints and take each component as one subsystem, i.e. subsystems *a-f*. The segmentation is shown in Figure 2.

In later modeling use, without losing any generality, we give the following assumptions:

1. The preload at each bushing is set to be 0 when no additional displacement at each hard point occurs.
2. The translational stiffness of the bushings along all three axes and the rotational stiffness around all three axes are all linear.
3. The mass of the spring on the damper component only contributes to the center of gravity of the components *c* and *d*.

DOUBLE WISHBONE SUSPENSION SYSTEM

We model each component that is segmented in last section. Note that each component now is treated as free of constraint. The modeling process is then straightforward by using general dynamic modeling approaches, e.g., Newton-Euler approach, Lagrangian Approach etc. In the following modeling procedure we claim in ahead the following conventions for the notations:

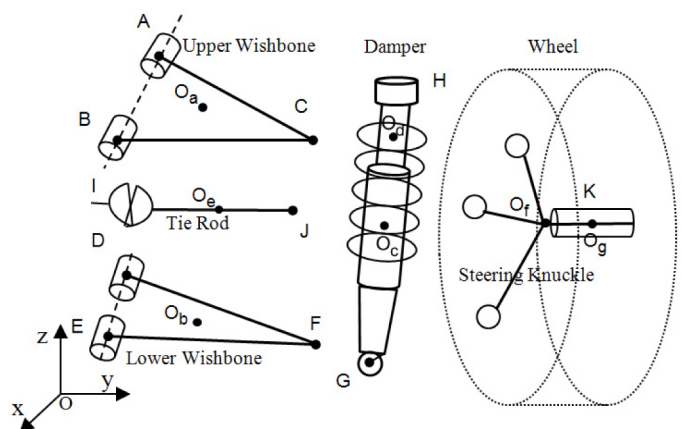


Figure 2. The segmentation of the double wishbone suspension system