

Anthropomorphic Traction Control: Quarter Car Model

2011-01-2178

Published
09/13/2011William Kirchner and Steve Southward
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doi:10.4271/2011-01-2178

ABSTRACT

Human expert drivers have the unique ability to combine correlated sensory inputs with repetitive learning to build complex perceptive models of the vehicle dynamics as well as certain key aspects of the tire-ground interface. This ability offers significant advantages for navigating a vehicle through the spatial and temporal uncertainties in a given environment. Conventional traction control algorithms utilize measurements of wheel slip to help insure that the wheels do not enter into an excessive slip condition such as burnout. This approach sacrifices peak performance to ensure that the slip limits are generic enough such that burnout is avoided on a variety of surfaces: dry pavement, wet pavement, snow, gravel, etc. In this paper, a novel approach to traction control is developed using an anthropomorphic control synthesis strategy. The proposed anthropomorphic traction control algorithm operates on the same correlated input signals that a human expert driver would in order to maximize traction; however, this is a very different strategy than any conventional traction control algorithms based on regulating wheel slip. A gradient ascent approach is at the heart of the proposed anthropomorphic control algorithm, and a real-time implementation is described using linear operator techniques, even though the tire-ground interface is highly non-linear. Performance of the proposed anthropomorphic traction control algorithm is demonstrated using a simple longitudinal traction case study.

INTRODUCTION

The human ability to learn and adapt to spatially and temporally changing environments is a significant advantage when compared to a vast number of automatic control systems. This ability to construct and exploit intuitive, albeit often crude, models of the environment, differentiates

humans from most automatic control systems. This work is an attempt to leverage the best of both worlds, i.e. the human ability to correlate vehicle longitudinal acceleration to throttle input [1] and the repeatability and bandwidth of an automatic control system. Most traction control and anti-lock brake systems control the slip to an acceptable range [2, 3], often sacrificing performance to do so. A popular area of research has been estimation techniques to determine the tractive force at the tire-ground interface [4, 5, 6], resulting in a more robust friction model that allows the control engineer to push the limits of the more traditional traction control algorithms. Traditional algorithms such as PID, full state feedback, and even sliding mode control have been very successful at handling low level tasks where the physics of the dynamic system are known and stationary.

In fact the adaptive signal processing community has a variety of techniques to address non-stationary signals ranging from the Filtered X LMS algorithm [7] and model reference control to FIR filters used in noise cancellation [8]. The underlying motivation for this work is that there are advantages to framing the traction control problem in a manner that is closer to how a human expert drives a vehicle. In this particular study, this motivation results in a method that is similar to many adaptive signal processing solutions. The key difference is that instead of minimizing a performance surface we are maximizing a performance surface, resulting in a gradient ascent algorithm to control engine torque. An important feature of the proposed algorithm is that it is continually hunting for the peak of the performance surface. The controller developed in this work uses apriori knowledge of the surface properties and the same correlated input and output signals that a human uses [1] to perform traction control. A simulation using a 1 DOF model is presented, where the objective is to start with zero initial velocity and achieve peak longitudinal vehicle acceleration

for the duration of the simulation. The simulation starts on dry pavement, but goes through a discrete transition to wet pavement after one second. The controller shows the ability to track the slip ratio at which the peak tractive force is achieved on both surfaces provided the necessary apriori knowledge of the tire-ground interface is available.

EQUATION DEVELOPMENT

Any realistic traction control study must at least address the non-linear dynamics present at the tire-ground interface. To this end, a novel dynamic system formulation has been developed to leverage the use of linear state-space system dynamics while still including the non-linear terms. For this general formulation, the continuous time state space system is defined as [Equations 1](#) and [2](#), where N is the number of states, P is the number of outputs, M is the number of inputs and Q is the number of non-linear feedback terms. This allows us to define $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{y} \in \mathbb{R}^P$, $\mathbf{u} \in \mathbb{R}^M$ and $\mathbf{u}' \in \mathbb{R}^Q$.

$$\dot{\mathbf{x}} = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u} + \mathbf{F}_c \mathbf{u}' \quad (1)$$

$$\mathbf{y} = \mathbf{C}_c \mathbf{x} + \mathbf{D}_c \mathbf{u} + \mathbf{G}_c \mathbf{u}' \quad (2)$$

It is possible to define the non-linear feedback terms $\mathbf{u}' \in \mathbb{R}^Q$ and $\mathbf{y}' \in \mathbb{R}^{N+M}$, shown in [Figure 1](#), according to [Equations 3](#) and [4](#).

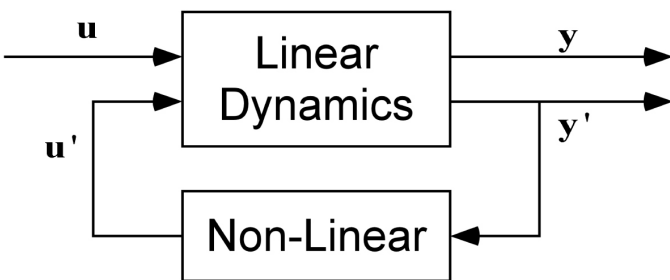


Figure 1. Block diagram of linear system with non-linear feedback.

$$\mathbf{y}' = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_M \end{bmatrix} \mathbf{u} = \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \end{bmatrix} \quad (3)$$

$$\mathbf{u}' = \begin{bmatrix} f_1(\mathbf{y}') \\ \vdots \\ f_Q(\mathbf{y}') \end{bmatrix} = \mathbf{f}(\mathbf{y}') \quad (4)$$

Furthermore, it is possible to discretize the linear system of [Equations 1](#) and [2](#). The method of discretization of this

system is not the subject of this paper, with the choice of an appropriate discretization method, the discrete time state space system of [Equation 5](#) can be obtained.

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A} \mathbf{x}_k + \mathbf{B} \mathbf{u}_k + \mathbf{F} \mathbf{u}'_k \\ \mathbf{y}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{D} \mathbf{u}_k + \mathbf{G} \mathbf{u}'_k \end{aligned} \quad (5)$$

Often times it is desirable to maximize or minimize the output of a system governed by [Equation 5](#). For this particular problem, we desire to maximize the longitudinal acceleration output. An adaptive algorithm, such as [Equation 6](#) [8], where the gradient is calculated as a discrete time process that is allowed to vary with time is one way to accomplish this. To develop the necessary mathematics to do so, we start with the objective function $J_k(\mathbf{u}_k)$ defined as [Equation 7](#).

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \mu \nabla_k \quad (6)$$

$$J_k(\mathbf{u}_k) = \frac{1}{2} \mathbf{y}_k^T \mathbf{y}_k \quad (7)$$

The gradient of the objective function, ∇_k , is then defined as [Equation 8](#).

$$\nabla_k = \frac{\partial \mathbf{y}_k^T}{\partial \mathbf{u}_k} \mathbf{y}_k \quad (8)$$

After formulating the gradient as the discrete time process of [Equation 8](#), it is necessary to expand and represent $\frac{\partial \mathbf{y}_k}{\partial \mathbf{u}_k}$ as a discrete time filter. To start, it is possible to define the state equation of [Equation 5](#) as [Equation 9](#), where z is the unit advance operator.

$$\mathbf{x}_k = [z\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{B} \mathbf{u}_k + \mathbf{F} \mathbf{u}'_k] \quad (9)$$

Substituting the result from [Equation 9](#) into the output equation results in [Equation 10](#), which should look familiar as it is exactly what would be expected from linear systems theory with the addition of the non-linear feedback term(s),

$$\begin{aligned} \mathbf{y}_k &= \mathbf{C} [z\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{B} \mathbf{u}_k + \mathbf{F} \mathbf{u}'_k] + \mathbf{D} \mathbf{u}_k + \mathbf{G} \mathbf{u}'_k \\ &= [\mathbf{C} [z\mathbf{I} - \mathbf{A}]^{-1} \mathbf{B} + \mathbf{D}] \mathbf{u}_k + [\mathbf{C} [z\mathbf{I} - \mathbf{A}]^{-1} \mathbf{F} + \mathbf{G}] \mathbf{u}'_k \end{aligned} \quad (10)$$

At this point it becomes convenient to define the substitution $\mathbf{H} = [z\mathbf{I} - \mathbf{A}]^{-1}$. When combined with the system of [Equation](#)

5 it is possible to define an equivalent system in the form of [Equation 11](#).

$$\begin{bmatrix} \mathbf{x}(z) \\ \mathbf{y}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{H}(z)\mathbf{B} & \mathbf{H}(z)\mathbf{F} \\ \mathbf{CH}(z)\mathbf{B} + \mathbf{D} & \mathbf{CH}(z)\mathbf{F} + \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{u}(z) \\ \mathbf{u}'(\mathbf{y}'(z)) \end{bmatrix} \quad (11)$$

Again, it is important to note that there is no loss in generality from linear systems theory, for the case when $\mathbf{u}'(\mathbf{y}'(z)) = 0$ or $\mathbf{F} = \mathbf{G} = 0$, the system reduces to a traditional linear time invariant system. From here it is possible to differentiate the output equation to get an expression for $\frac{\partial \mathbf{y}(z)}{\partial \mathbf{u}(z)}$.

$$\frac{\partial \mathbf{y}(z)}{\partial \mathbf{u}(z)} = [\mathbf{CH}(z)\mathbf{B} + \mathbf{D}] \frac{\partial \mathbf{u}(z)}{\partial \mathbf{u}(z)} + [\mathbf{CH}(z)\mathbf{F} + \mathbf{G}] \frac{\partial \mathbf{f}(\mathbf{y}'(z))}{\partial \mathbf{u}(z)} \quad (12)$$

By definition, the non-linear feedback term $\mathbf{f}(z)$ is a function of both the states $\mathbf{x}(z)$ and the inputs $\mathbf{u}(z)$. Expanding the partial derivative of this term results in [Equation 13](#).

$$\frac{\partial \mathbf{f}(\mathbf{y}'(z))}{\partial \mathbf{u}(z)} = \frac{\partial \mathbf{f}(z)}{\partial \mathbf{u}(z)} + \frac{\partial \mathbf{f}(z)}{\partial \mathbf{x}(z)} \frac{\partial \mathbf{x}(z)}{\partial \mathbf{u}(z)} \quad (13)$$

Substituting [Equation 13](#) into the output equation, [Equation 12](#), results in an expanded expression for $\frac{\partial \mathbf{y}(z)}{\partial \mathbf{u}(z)}$ in [Equation 14](#).

$$\frac{\partial \mathbf{y}(z)}{\partial \mathbf{u}(z)} = [\mathbf{CH}(z)\mathbf{B} + \mathbf{D}] + [\mathbf{CH}(z)\mathbf{F} + \mathbf{G}] \left[\frac{\partial \mathbf{f}(z)}{\partial \mathbf{u}(z)} + \frac{\partial \mathbf{f}(z)}{\partial \mathbf{x}(z)} \frac{\partial \mathbf{x}(z)}{\partial \mathbf{u}(z)} \right] \quad (14)$$

The partial derivative of the state equation, which shows up in [Equation 14](#), can be expressed as [Equation 15](#).

$$= \mathbf{H}(z)\mathbf{B} + \mathbf{H}(z)\mathbf{F} \left[\frac{\partial \mathbf{f}(z)}{\partial \mathbf{u}(z)} + \frac{\partial \mathbf{f}(z)}{\partial \mathbf{x}(z)} \frac{\partial \mathbf{x}(z)}{\partial \mathbf{u}(z)} \right] \quad (15)$$

[Equation 15](#) is recursive and it is necessary to break the loop using a little algebra, which leads to [Equation 16](#).

$$\frac{\partial \mathbf{x}(z)}{\partial \mathbf{u}(z)} = \left[\mathbf{I} - \mathbf{H}(z)\mathbf{F} \frac{\partial \mathbf{f}(z)}{\partial \mathbf{x}(z)} \right]^{-1} \mathbf{H}(z) \left[\mathbf{B} + \mathbf{F} \frac{\partial \mathbf{f}(z)}{\partial \mathbf{u}(z)} \right] \quad (16)$$

Substituting [Equation 16](#) back into [Equation 14](#), results in [Equation 17](#), where the notational dependence on the unit advance operator z has been dropped for ease in notation.

$$\frac{\partial \mathbf{y}}{\partial \mathbf{u}} = [\mathbf{CHB} + \mathbf{D}] + [\mathbf{CHF} + \mathbf{G}] \left[\frac{\partial \mathbf{f}}{\partial \mathbf{u}} + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \left[\mathbf{I} - \mathbf{HF} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right]^{-1} \mathbf{H} \left[\mathbf{B} + \mathbf{F} \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right] \right] \quad (17)$$

[Equation 17](#) appears to require the computation of multiple inverses; however, this is not the case. Numerically it is more robust and efficient to express [Equation 17](#) as a set of three connected discrete time state space sub-systems. Before proceeding, two additional definitions are made in [Equations 18](#) and [19](#).

$$\mathbf{\Omega} = \mathbf{F} \frac{\partial \mathbf{f}}{\partial \mathbf{x}_k} \quad (18)$$

$$\mathbf{\Psi} = \mathbf{B} + \mathbf{F} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} \quad (19)$$

Substituting the definitions for $\mathbf{\Omega}$ and $\mathbf{\Psi}$ into [Equation 17](#) results in [Equation 20](#).

$$\frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} + \left[\mathbf{I} - \mathbf{HF} \frac{\partial \mathbf{f}}{\partial \mathbf{x}_k} \right]^{-1} \mathbf{H} \left[\mathbf{B} + \mathbf{F} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} \right] = \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} + [\mathbf{I} - \mathbf{H}\mathbf{\Omega}]^{-1} \mathbf{H}\mathbf{\Psi} \quad (20)$$

In order to realize the discrete time state space sub-systems, input and output vectors are defined as $\mathbf{q}(z)$ and $\mathbf{r}(z)$, respectively. If these vectors are then considered as sub-system inputs and outputs to the discrete time filter of [Equation 20](#), it is possible to arrive at [Equation 21](#).

$$\begin{aligned} \mathbf{r}^{(1)}(z) &= \left[\frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} + [\mathbf{I} - \mathbf{H}\mathbf{\Omega}]^{-1} \mathbf{H}\mathbf{\Psi} \right] \mathbf{q}^{(1)}(z) \\ [\mathbf{I} - \mathbf{H}\mathbf{\Omega}] \mathbf{r}^{(1)}(z) &= \left[[\mathbf{I} - \mathbf{H}\mathbf{\Omega}] \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} + \mathbf{H}\mathbf{\Psi} \right] \mathbf{q}^{(1)}(z) \end{aligned} \quad (21)$$

We can now substitute the previous definition of $\mathbf{H} = [z\mathbf{I} - \mathbf{A}]^{-1}$ back into [Equation 21](#) and with a little algebra rearrange [Equations 22](#) and [23](#), to arrive at [Equation 24](#):

$$\begin{aligned} \mathbf{r}^{(1)}(z) - [z\mathbf{I} - \mathbf{A}]^{-1} \mathbf{\Omega} \mathbf{r}^{(1)}(z) &= [z\mathbf{I} - \mathbf{A}]^{-1} \mathbf{\Psi} \mathbf{q}^{(1)}(z) + \\ &\quad \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} \mathbf{q}^{(1)}(z) - [z\mathbf{I} - \mathbf{A}]^{-1} \mathbf{\Omega} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k} \mathbf{q}^{(1)}(z) \end{aligned} \quad (22)$$