

Determining Isolator Damping for Minimized Response from Impulse: A Theoretical Approach

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ABSTRACT

Several recent product developments for vibration and motion control have needed passive viscous damping, in addition to traditional elastomer-based hysteretic damping, to be successful in their respective applications. In addition to attenuating steady-state vibration, an important function of these recent product developments is to control motion from impulsive or mechanical shock input. Examples are the cab mounts of off-highway vehicles that need damping in the vertical direction to control cab motion from ground input through the vehicle and some torsionally flexible couplings that need damping to control torque spikes from shift shocks or other transient events. In this work, the theoretical damped impulse response quantities of displacement, velocity, acceleration, force, jerk, yank, and jounce are investigated. This work shows that, for certain response quantities, there is a specific magnitude of damping that minimizes response from impulsive or mechanical shock input. Further, this work shows that the specific magnitude of damping to minimize response depends upon the quantity considered. This work also shows that the magnitudes of damping needed to minimize response from impulsive or mechanical shock input are higher than what is readily attainable with the hysteretic damping of elastomeric products, substantiating the need to add passive viscous damping in recent elastomeric product developments.

INTRODUCTION

Elastomer-based products which have damping of the hysteretic form have long been used in many industrial applications to attenuate impulsive or mechanical shock input. Examples are the cab mounts of off-highway vehicles that need damping in the vertical direction to attenuate response from ground input through the vehicle and some

torsionally flexible couplings that need damping to attenuate response from shift shocks or other transient events. However, several recent product developments that have traditionally been elastomer-based have needed additional passive damping of the viscous form to be successful in their respective applications. In these developments, designers often don't know what magnitude of damping is best for their application, thereby resorting to costly and time-consuming trial and error solutions.

The engineering goal of impulse attenuation varies. In some applications the engineering goal may be to minimize displacement due to space limitations while in other applications the engineering goal may be to minimize acceleration due to fragility limitations. Yet another engineering goal may be to minimize jerk for the sake of human comfort. In this work, the theoretical impulse response quantities of displacement, velocity, acceleration, force, jerk, yank, and jounce are investigated. This work investigates the theoretical effect of damping magnitude for various engineering goals and shows that, for certain engineering goals, there is a specific value of damping magnitude that minimizes peak response to impulse. Those values of damping are presented herein and can be used as a starting point in product development so that trial and error solutions can be reduced.

The analysis and conclusions of this work are made in the context of a pure impulse to a single degree-of-freedom translational system. The results and trends from the single degree-of-freedom analysis can be extended to rotational systems and even to more complicated multi-degree-of-freedom translational and rotational systems. For example, the suspended mass of the analysis could represent an off-highway vehicle cab or it could represent the rotor of a grid-connected generator.

BACKGROUND

The effect of viscous damping in hydraulic mounts for engine suspensions is well studied, especially for the automotive industry [1,2,3]. Hydraulic mounts are often elastomer-based or have an elastomer spring component but have viscous damping of an adaptable or active nature [4, 5]. Many sources exist for investigating and determining the proper tuning of an adaptive or active hydraulic mount [6, 7]. Most of the sources focus on attenuation of response to steady-state vibration.

In addition to attenuating response to steady state vibration, the purpose of elastomer-based products in many applications is to attenuate transient response to impulse. The effect of damping in attenuating transient response is not as well studied, at least for the vehicle industries. The effect of damping, hysteretic and viscous, on transient response is more studied in the seismic motion control industry [8, 9] and in the marine industry [10]. This work presents a generic investigation of damped transient response to impulse. The results can be used for any range of applications in various industries.

THE SYSTEM MODEL

A single degree-of-freedom rigid body is suspended by a spring and damper as shown in Fig. 1. This work will assume that the mass quantity m cannot be changed and that the spring rate k has been chosen to properly attenuate steady-state vibration and cannot be changed. As such, the natural frequency ω_n is established. Suppose that the system is also subjected to impulse loading. The damping coefficient c is the only quantity that can be varied to attenuate the system response to impulse. This work will assume that the impulse cannot be changed and that it can be treated as a pure impulse I .

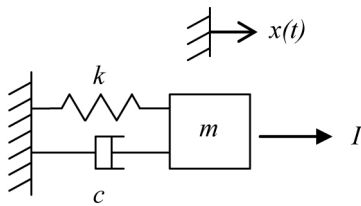


Figure 1. 1-DOF system subjected to impulse

Textbooks [11, 12] readily show that the response of a viscously damped single degree-of-freedom system to a pure impulse is a sine wave of decaying amplitude and that the displacement is expressed as Eq. (1).

$$x(t) = \frac{I}{m\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t \quad (1)$$

Textbooks also readily show the basic relationships of Equations (2), (3), (4).

$$\omega_n = \sqrt{\frac{k}{m}} \quad (2)$$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_n} \quad (3)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (4)$$

With Equations (2), (3), (4) substituted into Eq. (1), the expression for displacement is given by Eq. (5). Note that since m and k are constant, the damping ratio ζ is proportional to the damping coefficient c . For reference, the subsequent analysis of response is done with $m = 5.7$ kg, $k = 90,000$ N/m, and $I = 1,140$ N-s.

$$x(t) = \frac{I}{m\omega_n} e^{-\zeta\omega_n t} \left(\frac{1}{\sqrt{1 - \zeta^2}} \sin \omega_d t \right) \quad (5)$$

Figure 2 is a family of curves showing displacement response for selected values of damping ratio ζ . Note that the deformation of the spring and damper will be equal to the displacement of the mass. The curves also show how the peak displacement continually decreases as the damping is increased. So, if the application or engineering goal is to minimize the peak displacement of the mass or to minimize the peak deformation of the spring and damper, the magnitude of damping should be as high as possible. Ideally, the damping ratio should approach unity for the goal of minimizing peak displacement or peak deformation.

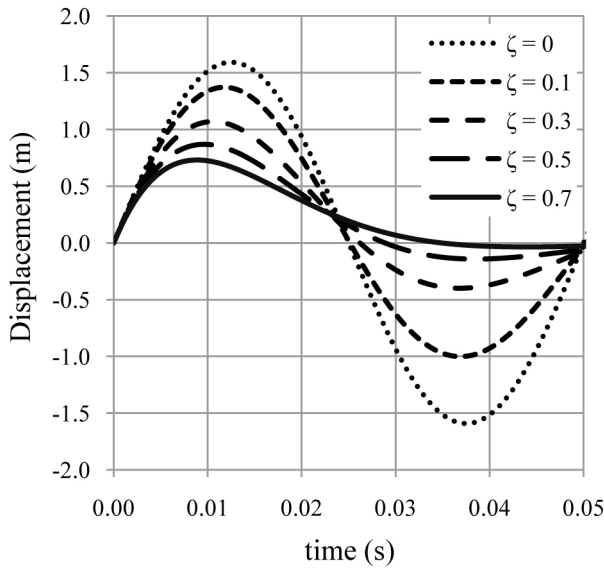


Figure 2. Displacement vs. time for selected values of damping ratio

The time derivative of displacement $x(t)$, [Eq. \(5\)](#), can be set equal to zero to get time t for displacement minimums and maximums. Doing so yields [Eq. \(6\)](#).

$$t = \frac{1}{\omega_d} \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \quad (0 \leq \zeta \leq 1) \quad (6)$$

Substituting the condition of [Eq. \(6\)](#) into $x(t)$, [Eq. \(5\)](#), produces [Eq. \(7\)](#) for peak displacement.

$$x_{peak} = \frac{I}{m\omega_n} \exp\left(-\frac{\zeta}{\sqrt{1-\zeta^2}} \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}\right) \quad (0 \leq \zeta \leq 1) \quad (7)$$

THE PROBLEM

Other application or engineering goals equally common to minimizing peak displacement might be to:

- 1). minimize the peak *velocity* of the mass. Such a goal might exist in rotating equipment where the speed fluctuations after a torsional impulse may adversely affect the performance of the equipment. A practical example might be grid or fault impulses to the rotor of a generator.
- 2). minimize the peak *acceleration* of the mass. Such a goal might arise from some fragility requirements of components or structures within the suspended mass. Practical examples may occur in the avionics or shipping container industries.
- 3). minimize the peak *force* transmitted through the spring and damper combination. Such a goal might be due to attachment strength limitations at either end of the spring and damper. Practical examples may be all those previously listed.
- 4). minimize the peak *jerk* of the mass. Jerk is the time rate of change of acceleration and in some applications it is thought to be correlated to human comfort [\[13\]](#). Such a goal might be applicable in vehicle cab and operator platform suspensions.

Consequently, the following quantities are investigated from a pure impulse to a single degree-of-freedom system.

- 1). $\dot{x}(t)$
- 2). $\ddot{x}(t)$
- 3). $F(t) = kx(t) + c\dot{x}(t) = -m\ddot{x}(t)$
- 4). $\ddot{\ddot{x}}(t)$

It is noted the quantity *yank* is the product of mass m and jerk, the quantity *jounce* is the time derivative of jerk, and the quantity *tug* is the product of mass m and jounce [\[14\]](#).

RESPONSE FROM IMPULSE

VELOCITY

The time derivative of displacement $x(t)$, [Eq. \(5\)](#), produces [Eq. \(8\)](#) for *velocity* of the mass.

$$\dot{x}(t) = \frac{I}{m} e^{-\zeta\omega_n t} \left(\cos\omega_d t - \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d t \right) \quad (8)$$

[Figure 3](#) is a family of curves showing *velocity* response for the same selected values of damping ratio ζ . The curves show that the peak *velocity* always occurs at $t = 0$ and that the peak

$\frac{I}{m}$ *velocity* is $\frac{I}{m}$ regardless of the damping ratio ζ ; it cannot be minimized with damping.