

Model Approach for a Load and Frequency Dependent Stiffness in Friction Materials

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ABSTRACT

It is widely known that friction materials show a load and frequency dependent stiffness. The load dependency can be explained with a geometrical nonlinear material structure, like the existence of porosities that are compressed by increasing the load. The frequency dependency is principally due to the viscoelastic characteristics of the polymer matrix that binds the different components together. Today there are commonly three different measurement systems used to investigate the material stiffness in compression: Ultrasound, which measures in a MHz-range, piezoelectric actuator to measure in a kHz-range, and hydropulser used for low frequency and quasi-static conditions [1].

Comparing the results from different authors it can be observed, that the stiffness does not change dramatically in the frequency range of each system, but between systems. There is a great discrepancy in the observed results, and it is said, due to the material's frequency dependence. While ultrasound measurement systems usually provide the stiffest values, the hydropulser in the quasi-static range generates the softest values. The discrepancy between these measurements systems has been matter of discussion for quite some time in the industry.

In this paper, a model approach using rheological elements is investigated to explain the differences in material stiffness observed during previous works from different authors. The investigation considers the basic mechanical effects present during the loading of a composite material that leads to different response behaviors depending on the measurement system. It will be shown once again, that friction materials are not only viscoelastic, being the main reason for the observed discrepancy using different measurement methodologies.

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INTRODUCTION

Stiffness is a relation between the applied force and the displacement observed on a body. For metals, the stiffness is represented by the Young's modulus in the region of elastic deformation from the force-displacement relation. However, the response of friction materials is much more complicated than just to be defined with the Young's modulus, because there is not only one material deforming under a force but several different ones. Each of these materials has different mechanical properties, bounded in a viscoelastic resin matrix deforming under the same force. It can be e.g. a steel fiber or a silicon particle that are compacting together, with internal friction and micro-indenter effects due to the interaction between hard particles and soft matrix. This creates a nonlinear macro relation between force and displacement in almost every condition.

The complex material response can be seen as an interaction of elastic, plastic, viscoelastic and viscoplastic deformation, which has been observed in creep tests (quasi-

static), implemented in the past and reported in [2], [3] for a material characterization. However, at higher frequencies by dynamical testing, the effect of the viscosity is more relevant and the response diverged from the quasi-static observation.

It is clear that the material behavior will be strongly dependent on the load conditions. Today, simulations are principally needed for the prediction of the brake fluid displacement due to the material compression (quasi-static load condition) and for dynamic instability analysis using complex eigenvalue calculations (dynamic conditions), where the correct material parameters are fundamental for squeal prediction, as shown in [4].

An analysis of different viscoelastic models is done to reach a final explanation for the phenomena observed by different authors with different measurement systems. It appears that it is not only the viscoelastic effect but also the material's internal friction, which has an influence on the final stiffness observed by dynamical tests.

DYNAMIC BEHAVIOR OF VISCOELASTIC MATERIALS

SINUSOIDAL EXCITATION

The response of a friction material to a cyclic load, usually sinusoidal, is characterized by a phase shift between input (load) and output (displacement) representing the energy loss as shown in [Figure 1](#).

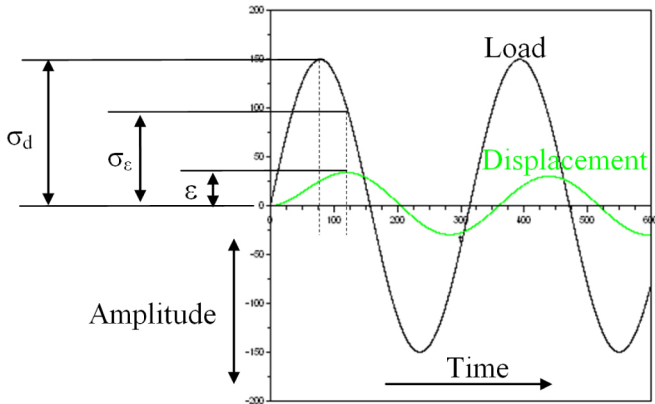


Figure 1. Measurement of a viscoelastic material for a dynamic load.

The *dynamic stiffness* obtained from the measurement is represented by [Eq\(1\)](#) and the *elastic stiffness* by [Eq\(2\)](#).

$$E_{dyn} = \sigma_d / \varepsilon \quad (1)$$

$$E_{el} = \sigma_e / \varepsilon \quad (2)$$

The elastic stiffness E_{el} corresponds to the response “in phase” with the load σ (storage modulus), while the dynamic stiffness E_{dyn} is produced by two values that occur in different times (absolute value of the complex stiffness). Therefore, the dynamic stiffness is a “result” of the material incapacity to follow the load while deforming, and it depends on the relation between elasticity and viscosity.

Parallel spring-dashpot representation

The dynamical behavior is usually idealized with a Kelvin-Voigt model (see [Figure 2](#)) to characterize the material using the observations from measurements. If the fitted parameters permit to reproduce the results with some accuracy, this model can be assumed as useful but only for that special measurement condition.

It does not answer the question about which mechanical effect is really behind the observed response, and it is maybe the reason for some confusion about the general material characterization.

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \quad (3)$$

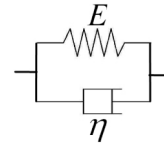


Figure 2. Kelvin-Voigt Model. Representation and differential equation.

An idealization of a material response with a Kelvin-Voigt model is done in [Figure 3](#) by changing three parameters: the frequency ω , the elastic stiffness E of the spring and the dashpot viscosity η observed in [Figure 2](#). Using a sinusoidal load as in [Figure 1](#) and putting it into [equation \(3\)](#) the following curves are obtained.

As observed in [Figure 2](#), the dashpot is responsible in the differential equation for the velocity or “frequency dependence”, and the spring for the low frequency response, which corresponds to the lowest stiffness this material can reach.

In [Figure 3a](#) only the frequency ω has been changed by fixing all other parameters. It can be observed that for higher frequencies the displacement is reduced, but the area enclosed by the ellipse increases. The elastic stiffness remains constant but the dynamic stiffness increases, according to [Eq. \(1\)](#) and [\(2\)](#). In contrast, [Figure 3b](#) shows a decrease of the displacement (from $E1$ to $E3$) and of the ellipse area, with no difference between dynamic and elastic stiffness (frequency is the same) due to the change in the spring's stiffness. [Figure 3c](#) shows no changes in the dynamic and elastic stiffness but an increasing ellipse area due to the different viscosities η used for the dashpot.

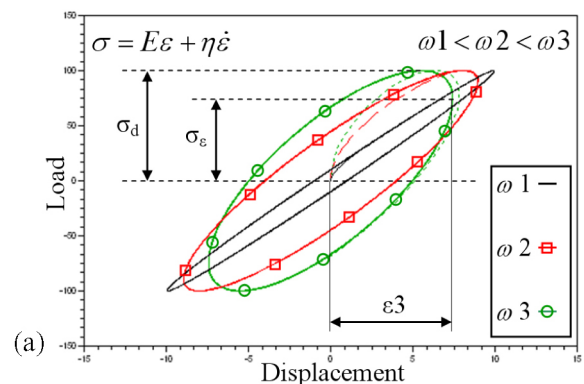


Figure 3. Simulation with a Kelvin-Voigt Model.
a) Frequency change. b) Spring stiffness change. c) Dashpot viscosity change.

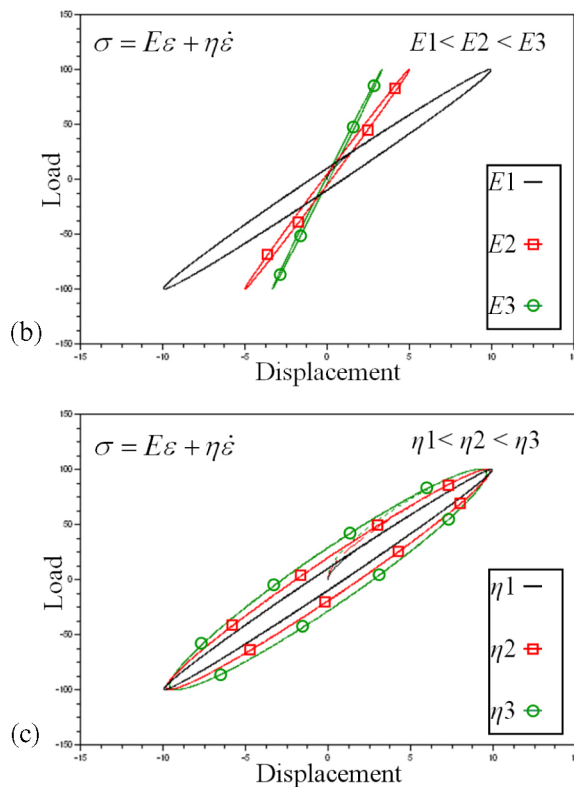


Figure 3 (Cont'd). Simulation with a Kelvin-Voigt Model. a) Frequency change. b) Spring stiffness change. c) Dashpot viscosity change.

The enclosed area of the corresponding ellipse is related to the phase shift between load and displacement in time. It is due to the material's internal friction that produces a loss of energy. For the Kelvin-Voigt model, the viscosity of the material represents the internal friction and an increasing frequency will increase the loss factor.

The measurements made by Nonaka [4] in Figure 4 show a very small increment of the dynamic stiffness in the range between 1 kHz and 4 kHz. Out of this range, we assume the measurement system is not able to give good results.

The small stiffness increment and the small phase angle through the frequency allow the conclusion that the material is deforming almost elastically with no difference between dynamic and elastic stiffness.

Comparing these results with the Kelvin-Voigt model in Figure 3, the observed response by Nonaka is more similar to the Figure 3b, where the stiffness is only dependent on the spring in the model (dynamic stiffness = elastic stiffness) rather than Figure 3a, where the increasing frequency also influences the dynamic stiffness.

The identification of the friction material's viscoelastic properties with a Kelvin-Voigt model is therefore incomplete and it can only serve to reproduce a certain material state.

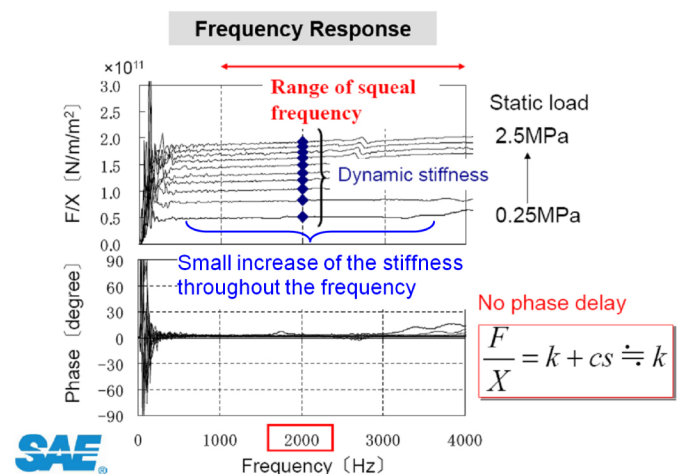


Figure 4. Dynamic stiffness depending on static preload. Measurements by Nonaka [4].

Standard solid model

A good improvement for the material characterization can be achieved with a standard solid model, which corresponds to a serial connection of a Kelvin-Voigt model and a spring as depicted in Figure 5.

$$\underbrace{\sigma + \frac{\eta_1}{E_0 + E_1} \dot{\sigma}}_{\sigma + p_1 \dot{\sigma}} = \frac{E_0 E_1}{E_0 + E_1} \varepsilon + \frac{E_1 \eta_1}{E_0 + E_1} \dot{\varepsilon} = q_1 \varepsilon + q_2 \dot{\varepsilon} \quad (4)$$

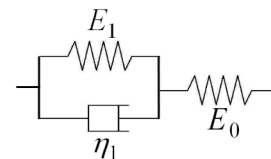


Figure 5. Standard Solid Model. Representation and differential equation.

In this case, the Kelvin-Voigt model, with the elastic stiffness E_1 and viscosity η_1 , will represent the frequency dependence, and the spring E_0 the elasticity when the frequency tends to infinite. An example of the Standard-Solid model behavior is presented in Figure 6 for three different frequencies.

It is observed that higher frequencies ω increase the material stiffness and decrease the loss factor (hysteresis). The difference between dynamic and elastic stiffness is also negligible, in contrast to the Kelvin-Voigt behavior observed in Figure 3a.