

Hybrid Automata Modeling of SI Gasoline Engines towards State Estimation for Fault Diagnosis

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ABSTRACT

Mean Value Engine Models, commonly used for model based fault diagnosis of SI engines, fail to capture the within-cycle dynamics of engines, often resulting in reduced fault sensitivity. This paper presents a new Hybrid Automata based modeling approach for characterizing the within-cycle dynamics of the thermo-fluidic processes in a Spark Ignition Gasoline Engine, targeted for use in model based fault diagnosis. Further, using a hybrid version of the Extended Kalman Filter (EKF), the states from the nonlinear hybrid automata based dynamic model are estimated and their results validated w.r.t standard industrial simulation software, AMESim. It is observed that due to the switching of within cycle engine dynamics, causing mode change, there is a corresponding change in model's structure which in turn can cause change in system's observability. The changes in observability (with modes) and the characterization of the evolving instantaneous engine dynamics into distinct system major modes and sub modes is expected to provide better sensitivity and faster detection and isolation of faults, when estimating the state variables in real time.

CITATION: Sengupta, S., Mukhopadhyay, S., Deb, A., Pattada, K. et al., "Hybrid Automata Modeling of SI Gasoline Engines towards State Estimation for Fault Diagnosis," *SAE Int. J. Engines* 5(3):2012, doi:10.4271/2011-01-2434.

1. INTRODUCTION AND MOTIVATION

Availability, performance and compliance to emission regulations (like Euro 5 and OBD-II requirements for automotives) [1, 2, 3] demand effective and fast detection and isolation systems for engine faults. Since there exist well characterized models for automobile engines, a model based approach is promising for fault diagnosis. Models of engines for treating faults can be largely classified as physics based and data driven based. The physics based approach used in this paper models the instantaneous dynamics of the thermo-fluidic processes prevailing during engine operation.

Research works on diagnosis of engines reported in the literature predominantly use the Mean Value Engine Model (MVEM) [4, 5, and 6]. MVEMs which only consider time evolution of engine dynamics effectively averaged over several engine cycles, describe the comparatively slower dynamics of the averaged physical variables such as mass

flow rates, intake manifold pressure, temperature, etc. In particular they replace periodic heat and mass transfer processes in the cylinder that arise due to combustion and motion of the crank shaft and valves by the average heat and mass quantities over several engine cycles. Further, the MVEM based approaches usually deal with faults from a single engine sub-system, like, intake [7], exhaust [8], etc. Another class of approaches [9, 10, 11] considers the crank angle based cylinder-to-cylinder within-cycle instantaneous variations of the engine variables for treating faults. They are often used for diagnosing misfires, compression faults, combustion quality etc. in the engines. In some cases, physics and data driven based methods have been combined for achieving improved performance at comparatively lower computational and modeling costs [12].

There exists another class of dynamical system modeling approaches called Hybrid Automata based Modelling used to characterize Hybrid Systems [13] that capture discrete and continuous behaviors of dynamical systems which exhibit

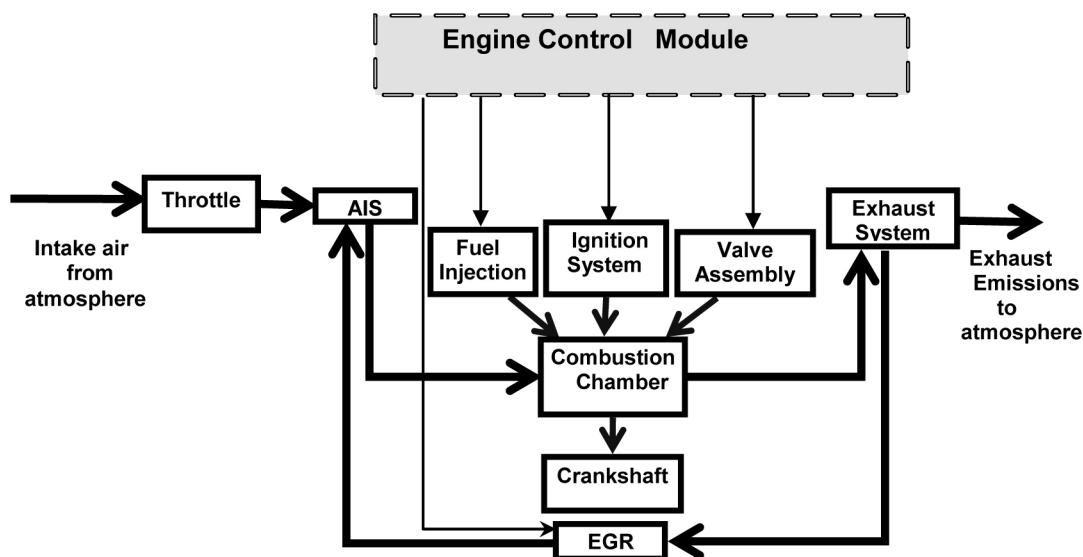


Figure 1. Block Diagram of a Naturally Aspirated SI Engine

fast mode transitions followed by relatively slower and durable modes of continuous variable dynamics. In SI engine operations, there are a set of dynamic modes which can be modeled as discrete meta-states and transitions with guard conditions between modes of a Hybrid Automata. While being in a particular dynamic mode or discrete state, a set of continuous state variables evolve over time according to a continuous dynamic model, represented by a set of differential equations which is the characteristic of that dynamic mode. Hybrid Automata models for SI engines [14, 15, 16, 17, 18, 19] mostly deal with aspects of controller design rather than fault diagnosis. A switched linear hybrid system model of the gasoline engine for misfire detection that does not consider details of within-cycle cylinder dynamics has been recently reported in [20].

In this paper, for the purpose of estimation and fault diagnosis, a new approach of Hybrid Automata based modeling of SI gasoline engine has been proposed. Issues associated with the switching modes in the model during estimation have been analyzed in detail and some of their effects discussed in the context of fault diagnosis. This paper is organized as follows. Section 2 describes a typical SI gasoline engine and compares the two commonly used physics based modeling approaches. Section 3 describes the instantaneous model of an SI engine, identifying the state variables and the structural changes in the model due to the switching dynamics of the engine, throughout the entire engine cycle operation. Section 4 explains how the switching dynamics of the instantaneous model of engine can be perceived as hybrid automata. Section 5 illustrates the estimation of such a hybrid model using the Extended Kalman Filter and discusses implementation details including observability issues and relevance of such representation to fault diagnosis. Section 6 shows simulation results. Finally, section 7 concludes the paper.

2. PHYSICS BASED MODELING APPROACHES

A naturally aspirated four stroke SI gasoline engine with gasoline direct injection (GDI) consists of components like the air intake system (AIS), the combustion chamber (consisting of cylinder, piston) and the exhaust system with the exhaust gas recirculation (EGR) physically connecting the intake and exhaust systems. The block diagram of such a system is shown in Fig. 1. The bold lines depict the mass or energy flow for the physical engine variables. There are several electronic subsystems which control the engine processes like the fuel injector, ignition and valve actuation systems. They, in turn, receive their actuation signals (represented by thin lines in Fig.1) from the Electronic Control Module (ECM) of the engine.

The physics based modeling approaches commonly used in fault diagnosis are either those considering instantaneous within cycle behavior or averaged behavior of engine dynamics.

2.1. ENGINE AS A SWITCHED DYNAMICAL SYSTEM

The dynamics of an SI engine operation is nearly periodic in terms of the rotational angle of the crank-shaft over two rotations (0° – 720°) for a 4-stroke engine, with each stroke approximately of 180° duration, (Fig. 2). The engine operation cycle can broadly be divided into four major modes, namely the Intake mode, followed by the Compression mode, the Expansion mode and finally ending in the Exhaust mode [21, 22]. In each of these modes the time evolution of the engine variables are governed by a distinct set of nonlinear dynamical equation related to heat and mass transfer. The engine dynamical modes undergo transitions in response to events such as opening and closing of intake and

exhaust valves, fuel injection, spark ignition, etc. over the engine cycle.

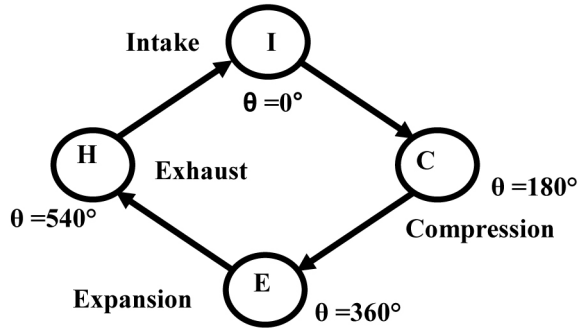


Figure 2. A simple representation of the mode evolution of engine over a cycle

The dynamic model that captures nonlinear and switched dynamics of the instantaneous time evolution of the process variables within crank angle cycles [23, 24] has been termed here as the Within Cycle Crank-angle-based Model (WCCM). It is evident that events and their effects occurring within the cycle will be captured by such a model.

2.2. MODELLING OF ENGINE DYNAMICS BY MVEM

The MVEM, under the assumption that the cylinder operates as a pump, describes the time evolution of averaged enthalpy and mass transfers only by conventional continuous dynamical model. This simplified modeling which can only capture the low frequency components of the evolution of the process variables is valid during steady state operation. Also, the large time constant of the engine dynamics (due to volume of the manifolds, pipes, etc.) and the sensor dynamics suppress high frequency components of engine dynamics in the measured data coming from the actual engine, making MVEM ideal for representing the low frequency response of the engine. Since the MVEM approach considers average values of the physical variables across specific time intervals, it is simpler than the Within Cycle Crank Angle Based model (which considers high frequency in-cylinder dynamics) in having fewer states and requiring fewer inputs for its propagation. This enables one to obtain the torque and speed response of the vehicle without involving high frequency complex engine dynamics. Thus, this model will have lower computational requirements as compared to the crank angle based model. Hence, it is conventionally used in automobile engines for control purposes.

However, compared to instantaneous within cycle model, the averaging inherent in the MVEM results in suppression of important high frequency dynamics which can often carry the fault signatures thereby reducing fault sensitivity, as shown in [25].

3. WCCM FOR A 4-CYLINDER 4-STROKE SI ENGINE

The WCCM of an engine in this work comprises:

- i. The set of pressure, temperature and mass flow equations governing the heat and mass transfer dynamics in the various modes of an engine cycle;
- ii. The set of events that causes transitions from one dynamical mode to another.

From the governing equations it maybe noted that the dynamic modes of the engine transitions and the associated set of continuous dynamical equations change not only due to control actions (e.g. valve motion, ignition) but also due to change in operating regimes of state variables (e.g. sub-sonic or sonic, positive flow or negative flow).

The engine dynamics has been captured in terms of a 'state vector', 'x' defined as,

$$x = \begin{bmatrix} P_{im} & P_{cyl}^{(1-4)} & T_{im} & T_{cyl}^{(1-4)} & P_{exm} & T_{exm} \end{bmatrix}^T \quad (1)$$

Pressure and temperature of engine enclosures are considered as states since the entire process dynamics of the engine can be captured by them (states).

The physical meanings of state and input variables are described in the nomenclature.

The control inputs are:

1. Crank angle (θ)
2. Throttle angle (α)
3. Angular engine speed (ω)
4. Intake and exhaust valve lifts ($L_{v(in)}$) and ($L_{v(exh)}$), respectively for each individual cylinder.
5. Fuel Injection Pulse Width (FUELPW) for each individual cylinder.
6. EGR valve lift ($L_{v(EGR)}$).

Additionally, the environmental inputs are ambient pressure (P_a) and ambient temperature (T_a). This corresponds to a total of 18 inputs for a 4 cylinder engine. Spark angle advance and combustion duration are assumed to be known. It is to be noted that, as torque is not modeled here (to reduce computational complexity), the requirement has been met by taking extra control inputs of crank angle ' θ ' and speed ' ω ' which provide the necessary information to the model.

The state equations for a gasoline engine with natural aspiration of air and direct injection of fuel (GDI) are given by the following subsections.

3.1. FLOWS

Flows have been modeled as single-dimensional, compressible and turbulent flow through orifices [26]. The

following cases of subsonic and sonic flow may arise and their mass flow rate is given by: $\dot{m}_{flow}$ is given by:

a. For subsonic flow: when $\frac{P_D}{P_U} > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$$\dot{m}_{flow} = C_d A \frac{P_U}{\sqrt{RT_U}} \left(\frac{P_D}{P_U}\right)^{1/\gamma} \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{P_D}{P_U}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (2.1)$$

b. For sonic flow: when $\frac{P_D}{P_U} \leq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$$\dot{m}_{flow} = C_d A \frac{P_U}{\sqrt{RT_U}} \sqrt{\gamma} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (2.2)$$

Note that,

1. $P_D > P_U$ corresponds to backflow which require swapping of P_U and P_D in (2.1, 2.2). The flow direction is however reversed requiring a negative sign of the flow rate.

2. When $P_D/P_U > 0.9$ or $P_U/P_D > 0.9$, turbulent flow does not hold well since laminar flow sets in, as supported by [27-28].

3. γ should correspond to the upstream medium.

4. Pipe and wave dynamics have been neglected.

5. Transport delays in gas flows are considered negligible.

The flow equations (2.1, 2.2) have been used to model the various flow processes of intake, exhaust, EGR, throttle and muffler flow.

3.2. GAS MASS FRACTIONS IN INTAKE MANIFOLD, EXHAUST MANIFOLD AND CYLINDER

Instantaneous gas mass fractions in cylinder, intake or exhaust manifold can be expressed as:

$$GMF^{(j)} = \frac{m_j}{\sum_{j=1}^3 m_j} \quad (3)$$

where, m_j is the net instantaneous mass of the j^{th} component. The component ' j ' equals 1 for air, 2 for burnt gas and 3 for fuel.

Note:

1. For each ' j ', m_j can be obtained by integrating the final rate of change of the individual component mass for each subsystem, which evolves due to inflows, outflows or combustion (in case of cylinders).

2. C_p , C_v , γ and R of air, burnt gases and fuel are assumed independent of temperature; since their temperature coefficients are small. Also, effective C_p , C_v , γ and R for gas mixtures in the volumes are computed by weighting of those of the individual component by its gas mass fraction as in (4).

$$C_p = C_{p(air)} \times GMF^{(1)} + C_{p(burn)} \times GMF^{(2)} + C_{p(f)} \times GMF^{(3)} \quad (4)$$

3.3. THE EVOLUTION OF EACH STATE VARIABLE

This subsection describes the dynamic equations for the state variables of the engine, valid throughout the cycle. The structure of equation of each state variable changes due to events such as flow reversal, valve operation, ignition, etc. It is evident from the expressions that such events cause C1 discontinuity in the state variables. The reversals in flow can be brought about by events such as piston motions, pressure change and opening of valves. Also, ideal gas (in all cases) and negligible heat loss in the EGR system is assumed.

3.3.1. Intake Manifold Pressure, P_{im}

By extending the approach based on enthalpy balance [29], the dynamic equation for intake manifold pressure can be derived (details in Appendix A) by incorporating effects of backflow, intake flows to multiple cylinders and EGR flow.

$$\begin{aligned} \dot{P}_{im} = \frac{\gamma_{im} R_{im} T_a}{V_{im}} & \left[\dot{m}_{th} \times Co_{PmThrot} + \dot{m}_{egr} \times Co_{PmEGR} \right. \\ & \left. - \sum_{i=1}^4 \dot{m}_{gasToCYL}^{(i)} \times Co_{PmIntToCYL}^{(i)} - \frac{\dot{Q}_{IM}}{C_{pim} * T_a} \right] \\ & - P_{im} \frac{R_{im}}{C_{v(im)}} \frac{d}{dt} \left[\frac{C_{v(im)}}{R_{im}} \right] \end{aligned} \quad (5)$$

Note:

1. The coefficients in (5), under various conditions are given by:

$$\begin{aligned} Co_{PmThrot} &= (T_{im}/T_{air}) & \dot{m}_{th} < 0 \\ &= (C_{p(air)}/C_{p(im)}) & \dot{m}_{th} > 0 \end{aligned} \quad (5.1)$$



$$Co_{PmIntToCYL}^{(i)} = \left(C_{p(cyl)}^{(i)} / C_{p(air)} \right) * \left(T_{cyl}^{(i)} / T_{air} \right) \quad \dot{m}_{gasToCYL}^{(i)} < 0$$

$$= \left(T_{im} / T_{air} \right) \quad \dot{m}_{gasToCYL}^{(i)} > 0 \quad (5.2)$$

$$Co_{PmEGR} = \left(T_{im} / T_{air} \right) \quad \dot{m}_{egr} < 0$$

$$= \left(C_{p(exm)} / C_{p(im)} \right) * \left(T_{exm} / T_{air} \right) \quad \dot{m}_{egr} > 0 \quad (5.3)$$

3.3.2. Intake Manifold Temperature, T_{im}

Following the same approach, as for P_{im} , and using the

relation, $P_{im} = \frac{m_{im}}{V_{im}} R_{im} T_{im}$ it can be derived (details in Appendix B) that:

$$\dot{T}_{im} = \frac{R_{im} T_{im}}{V_{im} P_{im}} \left[\dot{m}_{th} \times Co_{TmThrot} + \dot{m}_{egr} \times Co_{TmEGR} - \left(\sum_{i=1}^4 \dot{m}_{gasToCYL}^{(i)} \times Co_{TmIntToCYL}^{(i)} \right) - \frac{\dot{Q}_{IM}}{C_{v(im)}} \right] - T_{im} \frac{\dot{C}_{v(im)}}{C_{v(im)}} \quad (6)$$

Note:

1. Similarly, the coefficients in (6) are given by

$$Co_{TmThrot} = C_{p(im)} * T_{im} \quad \dot{m}_{th} < 0$$

$$= C_{p(air)} * T_{air} \quad \dot{m}_{th} > 0 \quad (6.1)$$

$$Co_{TmEGR} = (\gamma_{im} - 1) * T_{im} \quad \dot{m}_{egr} < 0$$

$$= (C_{p(exm)} / C_{v(im)}) * T_{exm} - T_{im} \quad \dot{m}_{egr} > 0 \quad (6.2)$$

$$Co_{TmIntToCYL}^{(i)} = \left(C_{p(cyl)}^{(i)} / C_{v(im)} \right) * T_{cyl}^{(i)} - T_{im} \quad \dot{m}_{gasToCYL}^{(i)} < 0$$

$$= (\gamma_{im} - 1) * T_{im} \quad \dot{m}_{gasToCYL}^{(i)} > 0 \quad (6.3)$$

3.3.3. Temperature of i^{th} Cylinder, $T_{cyl}^{(i)}$

Assuming all cylinders to be identical, by generalizing the expression for $\dot{T}_{cyl}^{(i)}$ [30], rate of change of temperature for the i^{th} cylinder is given by:

$$\dot{T}_{cyl}^{(i)} = \frac{1}{m_{cyl}^{(i)} C_{v(cyl)}^{(i)}} \left[\dot{Q}_{comb}^{(i)} - \dot{Q}_{heatLoss}^{(i)} + \dot{H}_{in}^{(i)} + \dot{H}_f^{(i)} - \dot{H}_{exh}^{(i)} \right] \quad (7)$$

where, using Weibe's law [21],

$$\dot{Q}_{comb}^{(i)} = \eta_c^{(i)} m_f^{(i)} Q_{LHV} \omega \frac{dx_b^{(i)}}{d\theta}; \text{ during combustion}$$

$$= 0 \quad ; \text{ otherwise} \quad (7.1)$$

Note that latent heat of vaporization of gasoline has been accounted for in (7.1).

$$\dot{Q}_{heatLoss}^{(i)} = \text{Heat loss from } i^{th} \text{ cylinder,} \quad (7.2)$$

using Annand's convective and radiative heat loss model [31].

$$\dot{H}_{in}^{(i)} = \dot{m}_{gasToCYL}^{(i)} \times C_{p(im)} \times T_{im}; \quad \text{if } \dot{m}_{gasToCYL}^{(i)} > 0$$

$$= \dot{m}_{gasToCYL}^{(i)} \times C_{p(cyl)}^{(i)} \times T_{cyl}^{(i)}; \quad \text{if } \dot{m}_{gasToCYL}^{(i)} < 0 \quad (7.3)$$

$$\dot{H}_{exh}^{(i)} = \dot{m}_{exh}^{(i)} \times C_{p(cyl)}^{(i)} \times T_{cyl}^{(i)}; \quad \dot{m}_{exh}^{(i)} > 0$$

$$= \dot{m}_{exh}^{(i)} \times C_{p(exm)} \times T_{exm}; \quad \dot{m}_{exh}^{(i)} < 0 \quad (7.4)$$

$$\dot{H}_f^{(i)} = \dot{m}_{f-inj}^{(i)} \times C_{p(f)} \times T_f; \quad (7.5)$$

The original reference [30] contains several typographical errors in the expression for $\dot{T}_{cyl}$ and is reproduced with corrections as obtained through correspondence from their authors in Appendix C for ease of reference.

3.3.4. Pressure of i^{th} Cylinder, $P_{cyl}^{(i)}$

A general expression for $\dot{P}_{cyl}^{(i)}$ can be obtained using the relation, $P_{cyl}^{(i)} = \rho_{cyl}^{(i)} R_{cyl}^{(i)} T_{cyl}^{(i)}$.

$$\dot{P}_{cyl}^{(i)} = \rho_{cyl}^{(i)} R_{cyl}^{(i)} \dot{T}_{cyl}^{(i)} + T_{cyl}^{(i)} \dot{\rho}_{cyl}^{(i)} + P_{cyl}^{(i)} \frac{\dot{R}_{cyl}^{(i)}}{R_{cyl}^{(i)}} \quad (8)$$

where,

$$\dot{p}_{cyl}^{(i)} = \frac{\dot{m}_{cyl}^{(i)} V_{cyl}^{(i)} - m_{cyl}^{(i)} \dot{V}_{cyl}^{(i)}}{V_{cyl}^2} \quad (8.1)$$

Expression for $V_{cyl}^{(i)}$ is given in [21].

Note that events that cause change in equations for $\dot{T}_{cyl}^{(i)}$ also cause corresponding changes in $\dot{p}_{cyl}^{(i)}$.

3.3.5. Exhaust Manifold Pressure, P_{exm}

The evolution of P_{exm} (details in [Appendix A](#)) can be derived following the same approach as used for P_{im} :

$$\begin{aligned} \dot{P}_{ex} = \frac{R_{exm}}{C_{v(exm)} V_{exm}} & \left[\left(\sum_{i=1}^4 \dot{m}_{exh}^{(i)} \times Co_{ExmExhFrCyl}^{(i)} \right) - Co_{ExmEGR} \times \dot{m}_{egr} \right. \\ & \left. - Co_{ExmMUF} \times \dot{m}_{muf} - \dot{Q}_{EXM} \right] \\ & - P_{ex} \frac{R_{exm}}{C_{v(exm)}} \frac{d}{dt} \left[\frac{C_{v(exm)}}{R_{exm}} \right] \end{aligned} \quad (9)$$

3.3.6. Exhaust Manifold Temperature, T_{exm}

Its evolution can be derived following the same approach as for T_{im} ([Appendix B](#)):

$$\begin{aligned} \dot{T}_{exm} = \frac{R_{exm} T_{exm}}{P_{exm} V_{exm} C_{v(exm)}} & \left[\left(\sum_{i=1}^4 \dot{m}_{exh}^{(i)} \times Co_{ExmExhFrCyl}^{(i)} \right) - \right. \\ & \left. \dot{m}_{egr} \times Co_{ExmEGR} - \dot{m}_{muf} \times Co_{ExmMUF} - \dot{m}_{exm} C_{v(exm)} T_{ex} - \dot{Q}_{EXM} \right] \\ & - \frac{T_{ex} \dot{C}_{v(exm)}}{C_{v(exm)}} \end{aligned} \quad (10)$$

The coefficients for both the exhaust manifold pressure and temperature evolution equations are identical and are given as follows:

$$\begin{aligned} Co_{ExmExhFrCyl}^{(i)} &= C_{p(exm)} T_{exm} & \dot{m}_{exh}^{(i)} < 0 \\ Co_{ExmExhFrCyl}^{(i)} &= C_{p(cyl)} T_{cyl} & \dot{m}_{exh}^{(i)} > 0 \end{aligned} \quad (10.1)$$

$$\begin{aligned} Co_{ExmEGR} &= C_{p(im)} T_{im} & \dot{m}_{egr} < 0 \\ Co_{ExmEGR} &= C_{p(exm)} T_{exm} & \dot{m}_{egr} > 0 \end{aligned} \quad (10.2)$$

$$\begin{aligned} Co_{ExmMUF} &= C_{p(air)} T_{air} & \dot{m}_{muf} < 0 \\ Co_{ExmMUF} &= C_{p(exm)} T_{exm} & \dot{m}_{muf} > 0 \end{aligned} \quad (10.3)$$

The expression for heat loss rates ($\dot{Q}_{IM}$, $\dot{Q}_{EXM}$, $\dot{Q}_{heatLoss}$) are assumed to remain constant throughout the cycle. The accuracy of the above model has been validated with respect to a realistic car engine model configuration, simulated in AMESim [32] having the same parameters and under identical operating conditions. The details of the results of validation are given in section 6.

4. HYBRID AUTOMATA BASED SI ENGINE MODEL

In this section the overall dynamics of the engine that evolves cyclically through the thermal and gas mass flow processes described in section 3 has been modeled as Hybrid Automata to characterize its Hybrid System nature. In a Hybrid System framework, a system evolves through a set of continuous dynamic modes through instantaneous switching. The Hybrid System based framework paradigm is appropriate here since phenomena that cause mode switching, such as valve opening and closing, cylinder piston reversals etc. are discrete events that take negligible time with respect to an engine cycle.

Thus the engine dynamics has been modeled as Hybrid Automata with crank-angle-based continuous dynamic modes changing cyclically in response to discrete engine events causing corresponding changes in the dynamic equations of state. Since the engine dynamics evolve through numerous conditions, for clarity of presentation it is necessary to classify the engine's dynamic modes into distinct major modes and sub-modes within each major mode using a set of algebraic conditions involving engine variables, commonly referred to as the characterizing invariant condition for a mode, in Hybrid System literature. These are described in detail in the following sections, for a single cylinder engine, for simplicity.

4.1. COMPONENT MODES TO CHARACTERIZE SYSTEM MODES' INVARIANT CONDITIONS FOR SINGLE CYLINDER ENGINE

The dynamics of an engine cycle can be represented by the evolution and switching of system modes, each created by combining the statuses of the appropriate conditions of all the component modes (CM) as described in Section 3 and characterized in [Table 1](#). Within each system mode, engine states evolve according to a distinct set of dynamic equations.

The discrete instantaneous changes in continuous dynamic modes in a Hybrid System can be brought about by causes such as externally controlled changes (e.g. inputs) or autonomous changes (defined by guard conditions) due to causes within the system, typically involving the state variables [33]. External causes can be valve input events and example of autonomous internally generated event can be a change from sub-sonic to sonic flow regime, brought about by change in relative pressure. Out of the resulting four

Table 1. CM conditions of a normally operating single cylinder engine

CM No.	Modes of engine sub-system/components	Possible statuses of CM (status tag)	CM status tag
1	Throttle flow regimes (Refer section 3.1) (Intake manifold is affected)	Negative flow and Sonic (0)	1-0
		Negative flow and Sub-Sonic (1)	1-1
		Positive flow and Sonic (2)	1-2
		Positive flow and Sub-Sonic (3)	1-3
2	Cylinder Intake valve condition (Intake manifold and cylinder are affected)	Closed (0)	2-0
		Open (1)	2-1
3	Cylinder Exhaust valve condition (Exhaust manifold and cylinder are affected)	Closed (0)	3-0
		Open (1)	3-1
4	Combustion condition (Cylinder is affected)	Not occurring (0)	4-0
		Occurring (1)	4-1
5	Fuel injection condition (Cylinder is affected)	Not occurring (0)	5-0
		Occurring (1)	5-1
6	Condition for any gas flow exchange in cylinder	No flow, compression (0)	6-0
		No flow, Expansion (1)	6-1
		Flow (2)	6-2
7	Cylinder Intake flow regimes* (Refer section 3.1) (Intake manifold and cylinder are affected)	Negative flow direction (0)	7-0
		Positive flow direction (1)	7-1
		No flow (4)	7-4
8	Cylinder Exhaust Flow regimes* (Refer section 3.1) (Exhaust manifold and cylinder are affected)	Negative flow direction (0)	8-0
		Positive flow direction (1)	8-1
		No flow (4)	8-4
9	EGR valve open/closed (Exhaust manifold and Intake Manifold are affected)	Closed (0)	9-0
		Open (1)	9-1
10	EGR flow (Refer section 3.1) (Exhaust manifold and Intake Manifold are affected)	Negative flow and Sonic (0)	10-0
		Negative flow and Sub-Sonic (1)	10-1
		Positive flow and Sonic (2)	10-2
		Positive flow and Sub-Sonic (3)	10-3
		No flow (4)	10-4
11	Muffler flow (Refer section 3.1) (Exhaust manifold is affected)	Negative flow and Sonic (0)	11-0
		Negative flow and Sub-Sonic (1)	11-1
		Positive flow and Sonic (2)	11-2
		Positive flow and Sub-Sonic (3)	11-3
12	Crank Angle Range	$\text{Min}(\theta_{IVO}) < \theta \leq \text{Max}(\theta_{EVC})$	12-1
		$\text{Min}(\theta_{IVO}) < \theta \leq \text{Max}(\theta_{IVC})$	12-2
		$\text{Max}(\theta_{IVC}) < \theta \leq 360^\circ$	12-3
		$\text{Min}(\theta_{SFIND}) < \theta \leq \text{Max}(\theta_{SFIND}) + \text{FUELPW}$	12-4
		$360^\circ - \theta_{SPARKADV} < \theta \leq \theta_{EOC}$	12-5
		$\text{Max}(\theta_{EOC}) < \theta \leq \text{Max}(\theta_{EVO})$	12-6
		$\text{Min}(\theta_{EVO}) < \theta \leq \text{Max}(\theta_{EVC})$	12-7

*Subsonic and sonic flow regimes for these flows are not distinguished as the open duration for valves is small.

possible types of occurrences of discrete transition phenomena in hybrid systems described by [33], only autonomous switching and controlled switching is considered for the dynamic mode transitions in the present work since no discontinuous changes occur in instantaneous physical state variables like temperature and pressure. Rather, only

transitions that cause instantaneous changes in the state derivative function(s) (due to change in equation structure, causing C1 discontinuity) are considered here.

Table 1 lists the various possible condition statuses corresponding to each engine CM, such that the combined statuses of the 12 CMs can uniquely characterize the