

Propulsion-Airframe Integration Using Statistical Surrogates from Computer Simulations

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ABSTRACT

A unique perspective of system integration is presented in terms of statistical design and analysis. Advanced statistical concepts are employed to quantify the variance of the statistical models as well as to specify model truncation error. Three models are developed for this study: 1) a supersonic wing section; 2) a supersonic turbojet system and; 3) an integrated supersonic wing section and supersonic turbojet. The three models are analyzed and separately and surrogate models are developed for each model independently using Design of Experiments and advanced statistical analyses. The individual surrogate models are statistically validated compared to their respective models. The individual wing and turbojet surrogate models are then used to estimate the performance of the combined wing and turbojet system surrogate model performance. Conversely, the combined wing and turbojet system surrogate is used to estimate the individual performance of the wing section and turbojet surrogate models' results. It must be noted that the system interactions in the simpler models presented in this study were minimal but if disparate systems are considered for integration the same concepts will apply. If strong interactions are likely to be present, than the integrated model must be developed accordingly.

Statistical surrogates are developed from turbojet and wing section models and used to demonstrate integration of major system components using statistical superposition principles. Fourth-order optimal response surfaces were developed for a supersonic turbojet model as well as a supersonic wing section model. Independent parameters of Mach number, compressor pressure ratio, and maximum turbine inlet temperature were used for the Design of Experiments inputs

for the turbojet model. For the wing section model, independent parameters of Mach number, wedge half-angle, and angle of attack were used for the Design of Experiments inputs. The common parameter of Mach number was used along with fixed altitude to provide the system integration environment. The statistical surrogate models were developed for the wing and turbojet system individually as well as for the overall combined system of wing plus turbojet. The combined system of turbojet and wing section used the independent input parameters of Mach number, compressor pressure ratio, maximum turbine inlet temperature, wedge half-angle, and angle of attack. All models maintained the same truncation error and level of variance, providing minimal propagation of uncertainty throughout the process.

The lift-to-drag ratio of the wing is used to demonstrate the statistical integration process. First, the surrogate model for lift-to-drag ratio for the wing is statistically developed and corroborated for uncertainty in model prediction. Second, the surrogate model for lift-to-drag ratio of the combined wing plus turbojet is statistically developed and corroborated for uncertainty in model prediction. Lastly, the lift-to-drag predictions of both surrogates are statistically compared to determine if the integrated model maintains the same level of precision as the individual models. All models are interchangeable in terms of statistical prediction intervals to within prescribed uncertainty and precision levels.

INTRODUCTION

Statistical concepts have been applied in the aerospace engineering process for many years. Rheinfurth and Howell present an overview and summary of statistical engineering applications¹. Inclusion of statistical precision and

uncertainty, however, is only recently gaining the attention of the modeling and simulation community. Additionally, physics-based *designs* have only received more emphasis in the recent years^{2,3,4,5,6,7,8,9}. A major issue with the implementation of *all* of these concepts simultaneously is that advanced inferential statistics are just that-advanced, and require requisite education and experience to actually implement on complex aerospace systems, which are in turn themselves complex. Couple the requirement for advanced knowledge with the capability to perform advanced physics-based modeling and simulation in computational fluid dynamics (CFD), and very few individuals possess the expertise to oversee and/or implement such a multi-disciplinary analysis and design program. As a result, we often perform such advanced studies in teams comprised of Subject Matter Experts (SMEs) that are individually knowledgeable but not necessarily *systems-oriented*. The overall product of the process is often a non-integrated and often sub-optimal system-of- systems. The methodology outlined in this investigation applies a top-down systems approach to integration implemented using statistically-based as well as physics-based analyses that are modular and extensible. For example, a wing module from one system may be used in another system operating under similar (but not necessarily the same) conditions in a different system. Any level of model physics may be used to develop the statistical surrogate model, making the process scalable, realistic, and adaptable to design an optimization processes.

A methodology for combined statistical and physics-based analysis and design is used to efficiently study advanced concepts where experimental data is not available. Herein, a physics-based approach is referred to as one that incorporates the conservation equations along with 2nd Law feasibility constraints as well as operational requirements for performance. While many newer models and methods are being deployed that used this physics-based approach^{10,11,12,13,14,15,16,17}, much more needs to be done in order to put the *process* in the hands of working engineers. The present focus is to extend these concepts in such a fashion that the engineer may benefit from the development of these advanced techniques while still maintaining relevance at the project or design level. The ultimate benefit is to leverage the SME (advanced statistical and work flow knowledge) in order to increase engineering productivity and raise the individual engineer's awareness level to the *systems* level by being able to perform multiple interdependent system interactions efficiently in a modular fashion.

The process methodology will be presented in a workflow manner such that the development of the modular system-level components is clear. The process is itself modular and various sets of computational and analytical tools may be used at any point in the process, depending upon the desired

level of accuracy and/or precision for the end product. This modular concept supports the typical "building block" approach employed in systems engineering (e.g. subcomponent, component, subsystem, system, and "system-of- systems"). With a system identified in such a manner, a system-level assessment may be performed from the individual integrated parts. Note that the process is "open box" in nature, enabling the engineer to choose the requisite level of physics and/or data source for each component and/or subsystem. The goal is to be able to identify the system statistically, use the surrogate model in lieu of the time-intensive computational tools, "assemble" the system, interface the components, prescribe the inputs and select the desired outputs, and "run" the assembled model. The value added in this methodology is that the engineer can focus on the *interpretation* of the results, perform impact assessments and trades easily, and help form the basis for learning the important parameters to guide preliminary design and optimization.

CONCEPT

Incorporating a statistics-based approach into advanced analyses and designs offers the opportunity to determine uncertainty quantification and its concomitant propagation for system integration. In addition to satisfying the governing equations for a system, the physical feasibility criteria are automatically satisfied, ensuring that the process considered may actually occur. [Figures 1a](#) and [1b](#), respectively, present a conceptual and architectural illustration of the system concept detailed in this investigation. [Figure 1a](#) conceptually represents a generic "black-box" system which may have multiple inputs, outputs, constraints, as well as performance goals for the system. [Figure 1b](#) represents an actual system, that of a wing section.

For the system represented in [Figure 1b](#), we may consider, for example, the input analysis or design parameters to be the flight Mach number (M_∞), angle of attack (AOA or α), and wing thickness angle (ϵ). Similarly, constraints may be maximum angle of attack for no separation (α_{max}), and minimum and maximum wing thickness angle (ϵ_{min} , ϵ_{max}), minimum and maximum M . Outputs might be lift coefficient, drag coefficient, lift-to-drag coefficient, etc. Additionally, a performance goal (which may also be a combination of output responses), might be maximum lift-to-drag with minimum losses (exergy destruction). Similarly for the system represented in [Figure 1c](#), we may consider the input analysis or design parameters to be the flight Mach number (M_∞), compressor pressure ratio (CPR) and turbine inlet temperature (T_{Turbin}). Constraints may be maximum compressor pressure ratio as limited by spool speed or similar (CPR_{Max}), maximum turbine inlet temperature ($T_{Turbin,Max}$), as well as minimum and maximum Mach number (M_{min} , M_{max}). Outputs might be thrust, fuel flow rate, jet exit

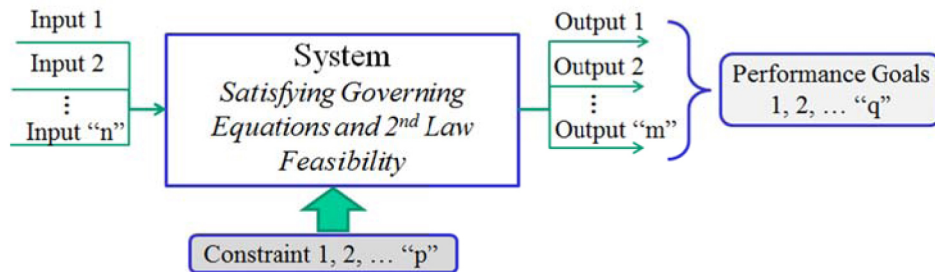


Figure 1a. Conceptual flow of a generic system.

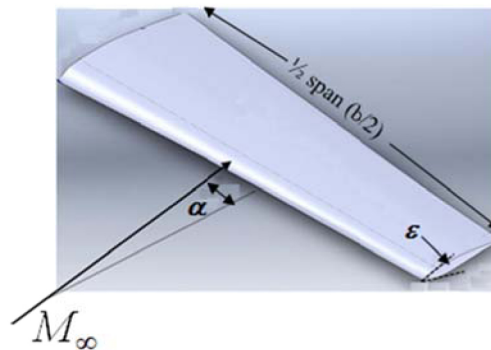


Figure 1b. Illustration of a wing section.

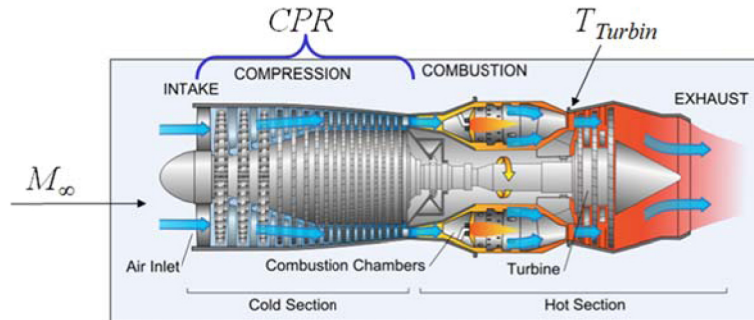


Figure 1c. Illustration of a turbojet, http://commons.wikimedia.org/wiki/File:Jet_engine.svg

Figure 1. Conceptual illustration of integration concept: 1a) generic inputs and outputs of a “black box” system with possible performance goals; 1b) sketch of wing with input variables M_∞ , α , and ϵ and 1c) sketch of turbojet with input variables M_∞ , CPR , and T_{Turbin} .

velocity, etc. Additionally, a performance goal might be thrust specific fuel consumption (TSFC).

Varying levels of complexity may be employed in the computational models to simulate the response of the system for a given set of inputs. A typical low- complexity model might be a one-dimensional, steady flow, perfect gas air model. On the other hand, a much more complex physical model might focus on an unsteady real gas model in three dimensions. Additionally, one may employ actual experimental data for the operation of the system¹⁸. Regardless of the *source* of the data, the conceptual model of the system illustrated in Figure 1a remains valid. We seek to employ statistical design of experiments and advanced inferences to determine the surrogate system identification model(s) that emulate the system and its interaction with the

surroundings. In general terms, system identification attempts to determine interactions that characterize the system's response for given bounds¹⁹.

There are three major areas of system identification that are typically used to characterize a system: the “white box” model, the “grey box” model, and the “black box” model (which is the focus of this paper). The white box model is the most fundamental and highest complexity, of the system identification subset and assumes one knows sufficient mathematical physics to fully describe the behavior of the system. Therefore, the white box model is essential its own system identification and is usually the result of detailed experimental and/or validated computational models. A simple example of a white box system model is a fully-specified mass-spring-damper system with known properties,

characteristics, and system responses (could be modeled or determined from experimental data). The grey box model is a lower complexity model of the system than the white box model. In the grey box model, one seeks reasonable approximations and physically consistent trends, rather than “truth” models. The grey box models are often empirical in nature and use extensive reduced models and correlations to characterize a system's behavior. It is presumed one knows the general mathematical physics to partially describe the behavior of the system. A simple example of a grey box system model is a heat transfer model with known properties and empirical coefficients, such as Newton's law of cooling²⁰. Most engineering models are classified as grey box. On the other hand, black box models are often quite general in nature and apply over a wide class of inputs and responses. The goal is to choose a basis function that is scaled and/or normalized such that the model provides an approximation for many varied classes of problem. An example of such a low complexity black box model may be the probabilistic density function such as a sigmoid function (Gaussian “bell” distribution).

Note: The generalized approach suggested herein may employ any level of system identification for each module for each of the “box models” in any combination, although it is typically best to be consistent in application of level of complexity for the system-of-system integration process. This paper focuses on the black box model in order to emphasize the process rather than the model itself.

Extending the concept from a simple process or subsystem to a more complicated system-of-systems is reasonably straightforward using statistically-realized system identification concepts. The consistency of system integration is aided by employing physics-based models and exergy accounting as a consistent measure of *integrated* performance⁷. Additionally, the physics-based approach also helps illuminate areas where loss mechanisms reduce system-level performance.

The concept of combining single subsystems into an integrated system is illustrated in [Figure 2](#), with [Figure 2a](#) representing the wing subsystem, [Figure 2b](#) representing a turbojet subsystem, and [Figure 2c](#) representing the combined wing-plus-turbojet system.

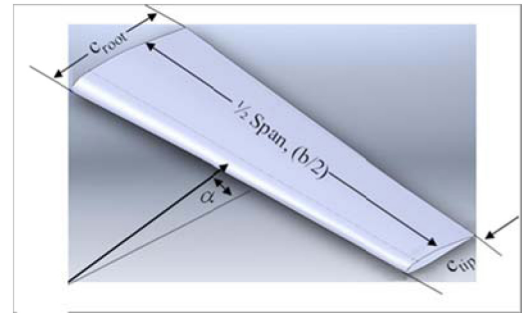


Figure 2a. Illustration of wing section system.

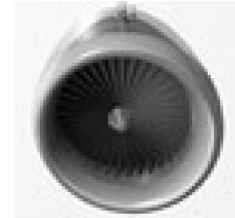


Figure 2b. Illustration of turbojet system.



Figure 2c. Illustration of combined wing section and turbojet system.

Figure 2. System-of-systems concept. The individual system of the wing (a) and turbojet (b) are combined into an integrated system (c).

The two subsystems illustrated in [Figure 2a](#) and [2b](#) were analyzed individually and then combined into a system-of-systems illustrated in [Figure 2c](#) in order to demonstrate the surrogate methodology as it supports the system identification process.

A supersonic turbojet was modeled using simplified thermodynamic cycle analyses employing perfect gas air computations and utilizing typical component efficiency parameters²¹. The station notation and illustration of the turbojet model are illustrated in [Figure 3](#). Note the inclusion of the reference dead state for exergy-based computations. Also, the normal shock wave standing off the inlet in supersonic flow is assumed to act as a “device” with requisite loss characteristics acting as part of the engine system. Each other device is clearly labeled in the figure in preparation for the modular development of the surrogate models for each subcomponent of the turbojet system.

The wing was modeled as a simple rectangular wing with diamond airfoil shape, as illustrated in [Figure 4](#). For the

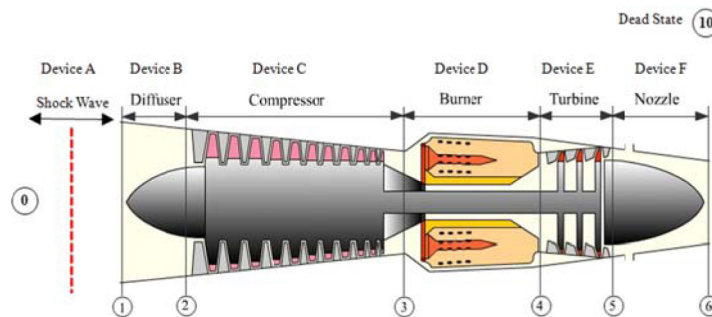


Figure 3. Illustration of turbojet model with station notation and device enumeration.

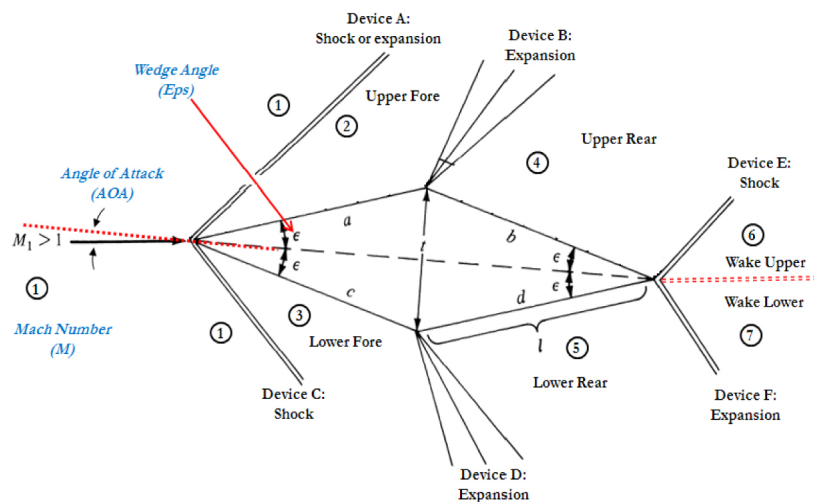


Figure 4. Illustration of wing section model with station notation and device enumeration.

rectangular wing section, the taper ratio (ratio of tip-to-root chord) is equal to unity. In order to be consistent with the turbojet gas dynamic model, the perfect gas model was again employed along with shock-expansion theory for the analysis of the external flow field around the wing section²². The shocks and expansions were again treated as “devices” in preparation for the statistical surrogate models required in the system identification process.

Often, each individual subsystem is analyzed/optimized separately and then combined into a total system. This “off-the-shelf” design concept often negatively impacts integrated system precision and/or uncertainty quantification (UQ). This degradation of precision may be avoided via the statistically-based system identification process by first specifying the precision of the turbojet and the wing as *separate* systems, then specifying the combined system precision under the same conditions. The propagation of uncertainty into the combined model may then be controlled to an acceptable level of precision²³:

INTEGRATION METHODOLOGY

The overall methodology required to combine individual systems into an integrated system will be outlined in this section. The overall goal is to characterize each subsystem statistically as a surrogate for the actual analysis which, itself, obeys the governing physical laws of the system subject to the appropriate constraints and boundary conditions. Incorporating a physics-based approach into advanced analyses and designs offers several advantages compared to other methodologies^{7,8,11,12,13,14,15,16,17}. Any model and level of complexity representation of the physical system may be employed in this process, making it extensible, general, and robust. The result of the overall methodology is a relatively simple set of equations that represent the individual subsystems and/or components in a modular fashion. As long as sufficiently broad and relevant range of operating conditions are utilized in the development of the models and modules-the combination of the modules may be performed after-the-fact (“*a posteriori*”). This enables the working engineer to utilize the modules in any desired computational environment with ease and portability (e.g. spreadsheet, calculator, computer subprogram, etc.). One can effectively “drag & drop” a surrogate module from one system to

another as long as the operating environments are similar to that in which the models were developed. Depending upon the complexity of the computational or experimental data employed in the development of the surrogates, there is minimal loss of accuracy and/or precision of the original calculation compared to the surrogate calculations.

A work flow representation of the system identification process employed in this study is illustrated in Figure 5. The process follows a sequentially ordered methodology as follows:

1). Development of the framework for determination of the surrogate model

a. Calibration data is developed using Design of Experiments (DoX²⁴) that supports uncertainty quantification (UQ) via truncation error specification and variance identification in the surrogate model,

b. A class of surrogate models is proposed along with a set of unknown parameters for those models.

2). The calibration process produces results for the class of proposed models along with statistical metrics of goodness for how well each model fits the calibration data as well as how well the model is likely to properly predict future data.

3). Use statistical processes (e.g. ANalysis Of VAriance²⁵, statistical significance along with hypothesis testing, *a posteriori* transformation, etc.) to select the most parsimonious model and parameter set that best minimizes the variance of the models as well as best estimates the current and future data.

4). Develop a set of pseudo-random input data for corroboration of the parsimonious surrogate model.

5). Perform predictions of the surrogate model using the corroboration data and compare the results to those of the original model using the same corroboration data.

Note: The keys in the system identification process is proper identification of operating ranges, correctly pre-identified analysis points, data sufficiency and quality, correct inferential statistical interpretation of results, and proper surrogate model development. A Subject Matter Expert (SME) sufficiently versed in Designed Experiments and advanced statistical inference should be employed during this portion of the process. Thereafter, using the models is relatively straightforward.

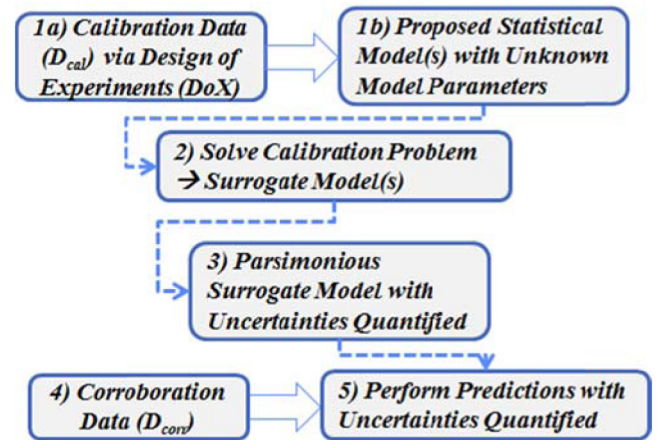


Figure 5. Illustration of work flows for individual system and combined system methodologies. Statistical truncation error, uncertainty quantification, and variance reduction are the over-arching concepts that enable the process to be utilized for integration of disparate models and/or data sources.

As might be expected from a quick perusal of the above-outlined surrogate methodology, there is a noted overhead in the process. As in all engineering efforts, there exists a tradeoff between alternative methodologies. The purpose and intent of the development of the surrogate models and system identification process is to enable *repeated* utilization of the modules in preliminary design. In such cases, the efficiencies gained upon utilization of the surrogate models more than makes up for the initial overhead for the development time. Qualitatively, a properly developed surrogate model may yield accurate results in microseconds compared to the hours/weeks that may be required for experimental and/or high-complexity computations. Additionally, the portability and usability of the results in modular fashion by non-Subject Matter Experts more than justifies the development effort of the surrogate modules. Lastly, the system identification process readily provides a basis from which to study the influence of uncertainty and error propagation from a systems level interpretation.

The process illustrated in Figure 5 is utilized to develop the surrogate models for the coefficient of lift-to-drag ratio (C_L/C_D) of the wing system as well as for the combined wing-and-turbojet system. The goal is to determine if the combined surrogate model predicts the magnitude and uncertainty of C_L/C_D as well as the wing system surrogate model. Additionally, it will be determined how well each surrogate model predicts future data on a point-by-point basis with uncertainty quantification.

THEORY

In order to focus on the statistical process, relatively simple engineering-level models (“grey box”) for the wing section and turbojet were selected for demonstration purposes. Each model of the individual system has its own requisite theory and was developed in detail in an earlier publication²⁶. Therefore, only an overview of the theory is presented herein. The statistical process, however, is presented in sufficient detail to illustrate the details involved. The systems considered in this investigation consist of a supersonic turbojet and supersonic diamond-shaped rectangular wing. While grey box models are suitable for preliminary trades, models with more complex physics and/or experimental data may be easily substituted in the modular surrogate model development supporting the system identification process. Further details of the development of the theory is presented in an earlier publication²⁶ and therefore only a summary is provided herein.

A. TURBOJET EQUATIONS

An engineering-level model is used to develop the surrogate model supporting the system identification process. The following general assumptions are used for the turbojet model:

- 1). Steady flow,
- 2). Local macroscopic thermodynamic equilibrium,
- 3). One-dimensional, uniform flow at each station,
- 4). Air is treated as a thermally and calorically perfect gas with constant properties,
- 5). Fuel addition influences mass and energy addition, but *not* mixture properties,
 - a. I.e. the combustion products are still treated as high temperature air,
- 6). Losses in each component (“device” as noted in [Figure 3](#)), are incorporated via standard engineering relationships
- 7). Kinetic energy of the flow is neglected except at stations 0, 1, and 6 ([Figure 3](#)) relative to other energy forms,
- 8). Turbulent, well-mixed flow,
- 9). Potential energy changes of the flow are neglected relative to other energy forms,
- 10). Body forces and viscous effects are negligible.

With reference to [Figure 3](#), the conservation equations are augmented with the appropriate equations of state and entropy/exergy equations in order to determine the flow path and quantities of interest.

Turbojet Performance Parameters

With the flow path evaluation performed, the overall quantities of interest for the turbojet may be characterized. Assuming the exit flow equilibrates to atmospheric pressure ($P_6 = P_{10}$), the thrust, propulsive power, and specific thrust are, respectively:

$$Thrust_{Net} = \dot{m}_6 V_6 - \dot{m}_1 V_0 = (\dot{m}_{air} + \dot{m}_{fuel}) V_6 - \dot{m}_{air} V_0 \quad (1)$$

$$Propulsive\ Power = (Thrust_{Net}) V_0 = (\dot{m}_6 V_6 - \dot{m}_1 V_0) V_0 \quad (2)$$

$$Specific\ Thrust_{Net} \equiv \frac{Thrust_{Net}}{\dot{m}_{air}} = (1 + \frac{\dot{m}_{fuel}}{\dot{m}_{air}}) V_6 - V_0 \quad (3)$$

B. WING SECTION EQUATIONS

The external supersonic air flow over the wing section illustrated in [Figure 4](#) is modeled using two-dimensional shock-expansion theory²². Depending upon the wedge angle (ϵ) and angle of attack (α) for the wing section, each station flow path (or device) may be a shock wave or an expansion fan. Therefore, careful logic must be implemented to check for flow path turning before applying the appropriate equations for a shock wave or an expansion fan.

Wing Section Model

With reference to [Figure 4](#), we make the following assumptions typical for supersonic shock-expansion theory:

- 1). Steady flow,
- 2). Local macroscopic thermodynamic equilibrium,
- 3). Two-dimensional, uniform flow at each station,
- 4). Air is treated as a thermally and calorically perfect gas with constant properties,
- 5). Losses in each component (“device” as noted in [Figure 4](#)), are due to shock waves only, if present,
- 6). Expansions are treated as reversible Prandtl-Meyer fans,
- 7). Flow is assumed to be adiabatic with no work interactions,
- 8). Potential energy changes of the flow are neglected relative to other energy forms,
- 9). Body forces and viscous effects are negligible,

The relevant conservation equations are augmented with the appropriate equations of state and entropy/exergy equations in order to determine the flow path. Each “device” for the

wing section is either an oblique shock wave or an expansion fan, depending upon the relative flow turning.

Note: There are two possible flow paths for the external flow over the wing section: over the upper wing or below the lower wing. For smaller angles of attack and thinner airfoils, the shock and expansion structures will be similar.

Wing Section Performance Parameters

With the flow path evaluation performed, the overall performance of the wing section may be characterized. Assuming the flow properties are uniform downstream of each shock wave and expansion wave, the lift coefficient and drag coefficient are calculated based upon the pressure fields and projected geometries. For given chord length (c) and diamond wedge half angle (ϵ) the wing section lift (L) and lift coefficient (C_L) are calculated, respectively, as²³:

$$L = P_3 \left(\frac{c/2}{\cos(\epsilon)} \right) \cos(\alpha + \epsilon) + P_5 \left(\frac{c/2}{\cos(\epsilon)} \right) \cos(\alpha - \epsilon) - P_2 \left(\frac{c/2}{\cos(\epsilon)} \right) \cos(\alpha - \epsilon) - P_4 \left(\frac{c/2}{\cos(\epsilon)} \right) \cos(\alpha + \epsilon) \quad (4)$$

$$C_L = \frac{L}{\frac{1}{2} \gamma P_1 M_1^2 c} \quad (5)$$

Similarly, the wing section drag and drag coefficient are calculated, respectively, as²³:

$$D = P_3 \left(\frac{c/2}{\cos(\epsilon)} \right) \sin(\alpha + \epsilon) + P_5 \left(\frac{c/2}{\cos(\epsilon)} \right) \sin(\alpha - \epsilon) - P_2 \left(\frac{c/2}{\cos(\epsilon)} \right) \sin(\alpha - \epsilon) - P_4 \left(\frac{c/2}{\cos(\epsilon)} \right) \sin(\alpha + \epsilon) \quad (6)$$

$$C_D = \frac{D}{\frac{1}{2} \gamma P_1 M_1^2 c} \quad (7)$$

The wing section lift-to-drag ratio is formulated as the ratio of lift coefficient to drag coefficient from [Equations 5 and 7](#), respectively as

$$\text{Lift-to-drag coefficient} = \frac{C_L}{C_D} \quad (8)$$

With the models for both individual systems developed, the analysis of each system may be performed. This independent analysis then serves as a basis for comparison of the integrated system. The comparison of the individual system's performance will be compared to the integrated performance in the next section.

C. STATISTICAL SURROGATE EQUATIONS

During the course of analyses and optimization procedures, it is helpful to isolate the influences of operating conditions and independent variables that influence the output response of a system^{27,28,29,30}. Operating conditions typically are fixed for a given study. There are two major types of variables in design: parameters and design variables, which serve different roles in the surrogate model development process. Parameters often define the bounds within which the study is performed or are chosen as fixed at a particular value for the entire study. For example, flight altitude is known to be a key variable which influences turbojet and wing performance. It may be varied parametrically within a range to be studied but held fixed for each analysis point. Hence, flight altitude in this case may be considered as a fixed parameter. On the other hand, a design variable is allowed to vary as a "free" variable during the course of the analysis and/or optimization. For the turbojet, such typical design parameters are chosen that tend to dominate the performance of the engine and therefore characterize the system on an overall basis. These are the variables of interest in the surrogate model development. An example of a typical design variable for the turbojet is the compressor pressure ratio (CPR). Both the turbojet and wing section illustrated in [Figure 2](#) have unique characteristics that define their operation and are discussed further in terms of fixed parameters and variable design variables (also known as factors in statistics).

Each of the two systems selected for this study have different parameters and design variables of interest. Careful pre-screening (or prototyping) of the parameters of interest and the design variables must be performed in order preclude errant statistical design and analysis points from the surrogate model. This prototype testing ensures maximal utility and extensibility of the developed modules for later use. During this prototyping process, one must also consider the integrability of the individual systems into a system-of-systems. The operating conditions, fixed parameters, and design variables for each individual system and combined system-of-systems are now detailed.

Operating Conditions

The operating conditions for the development of the surrogate models for system identification were selected to represent supersonic flight and are presented in [Table 1](#). Both the turbojet and wing section are assumed to operate under the

Table 1. Operating conditions for turbojet and wing section systems.

Operating Condition	Value	Comment
Flight Altitude (km)	6	Both turbojet and wing
Working Fluid	Air	Both turbojet and wing
Thermodynamic Model	Perfect Gas	Both turbojet and wing
Air Specific Heat Ratio (γ)	1.4	Both turbojet and wing
Air Specific Gas Constant (R_{air} , kJ/kg/K)	0.287	Both turbojet and wing
Air Specific Heat ($C_{p,air}$, kJ/kg/K)	1.00349	Both turbojet and wing
Reference Temperature for enthalpy, entropy (T_{ref} , K)	298.15	Both turbojet and wing
Free Stream Property Values	1976 US Standard Atmosphere *	Both turbojet and wing
Fuel Heating Value (kJ/kg) ⁺	42,700	Turbojet only

*[Reference³¹], [Reference³²]

Table 2. Fixed parameters for turbojet system.

Turbojet Fixed Parameters	Value	Comment
Inlet Diameter (m)	0.5	Arbitrary reference size
Dead State Reference Entropy (kJ/kg/K)**	6.9202	Significant digits used for validation
Diffuser total pressure loss ratio	0.98	$r_{Diff} = P_{Tot,2}/P_{Tot,1}$
Compressor Isentropic Efficiency	0.9	$\eta_{Comp} = (T_{3,s} - T_2)/(T_{3,act} - T_2)$
Combustor Pressure Loss Factor	~0.05	$P_{Loss,Comb} = 1 - P_4/P_3$
Turbine Isentropic Efficiency	0.95	$\eta_{Turb} = (T_4 - T_{5,act})/(T_4 - T_{5,s})$
Nozzle Isentropic Efficiency	0.975	$\eta_{Noz} = (T_5 - T_{6,act})/(T_5 - T_{6,s})$
Exit pressure / Atmospheric pressure	1	$P_6 = P_{10}$

**[Used for validation of turbojet cycle performance³⁴]

Table 3. Fixed parameters for wing section system.

Wing Section Fixed Parameters	Value	Comment
Dead State Reference Entropy (kJ/kg/K)**	6.9202	Significant digits used for validation
Chord length (c, m)	1	Arbitrary reference value
Wing span (b, m)	10	Arbitrary reference value

**[Used for consistency of integration with turbojet]

same conditions which are held fixed for the study. (Note that the number of significant digits for specific heat is carried for validation purposes.)

Fixed Parameters

In both the turbojet and wing section, there are certain variables that directly influence the performance of a system, but which are held fixed for a particular study. These parameters are detailed for the turbojet system³³ in [Table 2](#) and for the wing section in [Table 3](#)

The fixed parameter values for the turbojet ([Table 2](#)) and the wing section ([Table 3](#)) are first used for analysis of each system *separately*, then used collectively as an integrated system-of-systems. For the integration, one seeks a physically consistent and feasible “tie component” which is a design variable common to both systems. These design variables are outlined in the next section.

Design Variables

Key variables that dominate the performance of a system are designated as design variables. As such, they are useful in characterizing the entire system and are therefore of prime interest as surrogates for the actual system. We seek the following traits of design variables in the system identification process:

- 1). There exists a philosophy of integration before the design variables are selected,
 - a. In order to leverage the individual systems in integration, there must be a means of integrating/relating the systems together that is both meaningful and feasible,
- 2). The design variables selected must be amenable to advanced statistical analyses,
 - a. A valid relationship between input design variables and output responses must exist and must be quantifiable and statistically significant,

Table 4. Design variables for turbojet system.

Turbojet Design Variables	Range	Comment
Mach Number (M)	$1.5 < M < 2.5$	Upper limit "aggressive" in terms of current technology limits
Compressor Pressur Ratio (CPR)	$14 < CPR < 30$	Upper limit "aggressive" in terms of current technology limits
Turbine Inlet temperature (T_{turb} , K)	$1600 < T_{turb} < 2000$	Upper limit "aggressive" in terms of current technology limits

Table 5. Design variables for wing section system.

Wing Section Design Variables	Range	Comment
Mach Number (M)	$1.5 < M < 2.5$	Used as integration variable for both turbojet and wing section
Angle of attack (α , deg)	$0 < \alpha < 10$	Small angles to maintain attached oblique shock waves
Wedge half-angle (ϵ , deg)	$0 < \epsilon < 5$	Zero wedge half-angle is flat plate

3). The range of interest for the design variables is *useful* for the engineer.

a. The surrogate modules developed are relevant for a wide range of applications.

4). The range of applicability is *feasible*,

a. The performance of each model system obeys the physics of the actual system,

5). The surrogate models, although developed independently, may be integrated *posteriori*,

a. The engineer may use the models for each independent system in a modular fashion.

In both the turbojet and wing section, the design variables take on a potentially infinite number of values, but are typically constrained to certain ranges of interest for a particular study. These design variables are detailed for the turbojet system in [Table 4](#) and for the wing section in [Table 5](#).

With the operating conditions, the parameters determined, and the design variables selected and ranged, the analysis of each individual system is performed in preparation for the statistical analysis and integration.

System-Level Surrogate Development

The overall goal of the surrogate optimization is to replace the detailed analysis that must be performed for *every* new case studied with an expression that may be evaluated *independently* of the analysis. In so doing, the entire analysis of the turbojet is captured in a relatively simple expression (the surrogate), which is easily transferrable to any desired calculation environment (e.g. calculator, Excel, Fortran, MATLAB, etc.). Each influence or interaction of the design variable (input) is captured via a postulated regressed relationship to the desired output (response) of the form:

$$\text{Output} = f(\text{inputs}) \quad (9)$$

The author's experience, along with fellow statistical practitioners^{35,36,37}, indicates that a quartic surrogate model adequately captures most aerospace phenomena. Accordingly, quartic models are postulated for the wing, turbojet, and combined wing and turbojet systems. For the wing model, the formulation is presented as a function of three independent input design variables: Mach number (M), angle of attack (α), and wing wedge angle (ϵ). Generically, this is written for the output of coefficient of lift-to-drag ratio ($\frac{C_L}{C_D}$) for the wing model as

$$f_{\frac{C_L}{C_D}, \text{wing}} = f(M, \alpha, \epsilon) \quad (10)$$

Similarly, the generic formulation for the turbojet model is written as a function of the three independent design variables: Mach number (M), compressor pressure ratio (CPR), turbine inlet temperature (T_{Turb}). Generically, this is written for the output of thrust for the turbojet model as

$$f_{\text{Thrust}, \text{turbojet}} = f(M, CPR, T_{Turb}) \quad (11)$$

Lastly, the generic formulation for the combined wing and turbojet model is written as a function of the five independent design variables: Mach number (M), angle of attack (α), wing wedge angle (ϵ), compressor pressure ratio (CPR), and turbine inlet temperature (T_{Turb}). Generically, this is written for the output of lift-to-drag ratio for the combined wing and turbojet model as

$$f_{\frac{C_L}{C_D}, \text{wing+turbojet}} = f(M, \alpha, \epsilon, CPR, T_{Turb}) \quad (12)$$

The number of model parameters (p), including the intercept term, for a surrogate polynomial model of n th order accuracy

with k independent variables is given by the following equation³⁸

$$p = \frac{(d+k)!}{d!k!} \quad (13)$$

For the case of the lift-to-drag coefficient for the wing from Equation 9 and the turbojet thrust from Equation 10 that contain three independent variables (k) and the surrogate is proposed as quartic ($d=4$), we have the number of parameters p equal to 35. Likewise, for the case of the lift-to-drag coefficient for the combined wing and turbojet model from Equation 11 that contains five independent variables (k) and the surrogate is proposed as quartic ($d=4$), we have the number of parameters p equal to 126. These generic equations are too long to present in the body of the text and therefore only the parameter-dependent models preliminary and final models will be displayed in the appendix.

In general, a statistical hypothesis is stated for each surrogate statistical function. In this study, the same formulary was used for the regressor relationships for lift-to-drag ratio and thrust, and therefore the null and alternative hypotheses may be stated succinctly for a output responses of lift-to-drag ratio and thrust, respectively, as

$$H_o \left(\text{Null for } \frac{C_L}{C_D}, \text{wing} \right): \mu_M = \mu_\alpha = \mu_\epsilon = 0;$$

$$H_a(\text{Alternate}): \text{Otherwise.} \quad (14)$$

$$H_o(\text{Null for Thrust, turbojet}): \mu_M = \mu_{CPR} = \mu_{T_{Turb}} = 0;$$

$$H_a(\text{Alternate}): \text{Otherwise} \quad (15)$$

In a similar fashion, the null and alternative hypotheses for the output response of lift-to-drag ratio for the combined wing and turbojet model are presented as

$$H_o \left(\text{Null for } \frac{C_L}{C_D}, \text{wing} + \text{turbojet} \right): \mu_M = \mu_\alpha = \mu_\epsilon = \mu_{CPR} = \mu_{T_{Turb}} = 0;$$

$$H_a(\text{Alternate}): \text{Otherwise.} \quad (16)$$

The above null and alternate hypotheses may also be written in terms of the regression coefficients (β) as

$$H_o(\text{Null}): \beta_i = \beta_{ij} = \beta_{ii} = \beta_{ijk} = \beta_{iii} = \beta_{ijkl} = \beta_{iiii} = 0;$$

$$H_a(\text{Alternate}): \text{Otherwise.} \quad (17)$$

The null hypotheses in Equations 14, 15, 16 states that there is no influence of the independent regressors upon the mean expected response (μ), while the null hypothesis in Equation 17 equivalently states that the postulated regression coefficients are all equal to zero. On the other hand, the alternate hypotheses from Equations 14, 15, 16, 17 indicates that any/all of the hypotheses may be not likely to be true (otherwise).

For the three-input variable surrogate models, a designed experiment was developed for a quartic optimal response surface using 35 model points, 2 points for lack-of-fit test, no replicates, and 5 additional center points using Design Expert³⁹. The test matrixes are too large to present in the body of the text. For the lift-to-drag coefficient experimental design consistent with Equation 10, the partial table is presented as Table 6 (the full design is presented in Table A.1).

Table 6. Partial designed experiment test matrix for lift-to-drag coefficient model, wing (Equation 10).

Run #	Input Variables		
	Mach	Angle Attack	Wedge angle
	M	(α) deg	(ϵ) deg
1	1.67	1.00	2.00
2	2.50	1.12	2.00
3	1.50	1.93	2.00
⋮	⋮	⋮	⋮
9	1.92	6.46	2.28
10	2.40	3.22	2.36
11	2.06	4.21	2.42
12	1.51	5.82	2.59
⋮	⋮	⋮	⋮
31	2.05	7.00	4.49
32	1.50	7.00	4.54
⋮	⋮	⋮	⋮
42	2.50	7.00	5.00

For the thrust experimental design consistent with Equation 11, the partial table is presented as Table 7 (the full design is presented in Table A.2).

Lastly, for the five-input variable surrogate model, a designed experiment was developed for a quartic optimal response surface using 126 model points, 2 points for lack-of-fit test, no replicates, and 5 additional center points using Design Expert. For the lift-to-drag coefficient combined wing and turbojet model experimental design consistent with Equation 12, the partial table is presented as Table 8 (the full design is presented in Table A.3).

The modeled equations for the turbojet and wing section were simulated in MATLAB for the independent and combined systems. The outputs of the simulation were used to evaluate the postulated system regression equation, and only

Table 7. Partial designed experiment test matrix for thrust model, turbojet (Equation 11).

	Input Variables		
	<i>Mach</i>	<i>Comp P Ratio</i>	<i>Turb. Temp</i>
<i>Run #</i>	M	(CPR)	(T _{Turbin}) K
1	2.20	14.00	1600.00
2	1.50	16.72	1600.00
3	2.50	19.76	1600.00
⋮	⋮	⋮	⋮
9	2.36	26.48	1660.00
10	2.50	30.00	1660.00
11	2.30	17.44	1664.00
12	1.54	22.64	1668.00
⋮	⋮	⋮	⋮
31	1.70	14.00	1920.00
32	2.50	26.80	1926.00
⋮	⋮	⋮	⋮
42	2.47	30.00	2000.00

Table 8. Partial designed experiment test matrix for lift-to-drag coefficient model, wing+turbojet (Equation 12).

	Input Variables				
	<i>Mach</i>	<i>Comp P Ratio</i>	<i>Turb. Temp</i>	<i>Comp P Ratio</i>	<i>Turb. Temp</i>
<i>Run #</i>	M	(α) deg	(ϵ) deg	(CPR)	(T _{Turbin}) K
1	1.50	4.48	2.15	14.00	1600.00
2	2.46	1.00	3.35	14.00	1600.00
3	2.50	7.00	3.95	14.00	1600.00
⋮	⋮	⋮	⋮	⋮	⋮
9	1.76	2.05	3.10	16.16	1600.00
10	1.54	1.00	4.46	17.04	1600.00
11	1.70	5.65	5.00	18.00	1600.00
12	2.43	5.32	2.69	18.80	1600.00
⋮	⋮	⋮	⋮	⋮	⋮
31	2.50	1.57	3.44	22.57	1632.00
32	1.66	1.00	2.00	14.00	1640.00
⋮	⋮	⋮	⋮	⋮	⋮
133	2.47	30.00	2000.00	30.00	2000.00

statistically significant terms were retained according to the standard p-value rejection criteria of 0.05⁴⁰. This parsimonious surrogate model was further *a posteriori* transformed statistically as necessary to reduce the variance of the prediction equation. The result of this process is that the surrogate equation now best-represents the data with the fewest terms, lowest variance of estimates for confidence intervals of the calibration data, and lowest prediction intervals for the future estimated data. Results of this process are presented in the next section.

ANALYSIS & RESULTS

The process steps outlined in Figure 5 were followed to develop and test the surrogate models. In steps 1a and 1b, the wing and turbojet models developed in MATLAB were simulated for the designed test matrixes presented in Tables A.1, A.2, and A.3 for the wing, turbojet, and combined wing

plus turbojet statistical models, respectively. In step 2, the resulting calibration surrogate model was developed using advanced statistical techniques. In step 3, the number of terms in the calibrated surrogate model was reduced by eliminating statistically-insignificant terms and transforming the results to minimize the variance in the statistical model, both for the calibration data and for the future corroboration data. Finally, in steps 4 and 5, new corroboration data was developed and the parsimonious surrogate model was tested. The results of this process are quite lengthy, and therefore only summaries are presented herein.