

Optimal Control to Recover a Safe Situation from Low/High-Energy Situation in Approach

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ABSTRACT

The main study illustrated in this paper deals with the computation of commands which allow an aircraft to recover a nominal energy trajectory from a low/high energy state during the approach phase. The commands taken into account in this paper are the slat/flap aerodynamic control surfaces which allow the aircraft to maintain the best lift performance for low velocities during the approach phase. In this study, it is supposed that the aircraft maintains a known vertical trajectory, simplified by a constant ground slope, while no engines and airbrakes are used. A non-linear optimization approach is studied in this paper and two methods are tested: a) Hermite-Simpson, trapezoidal collocation methods, b) Sequential numerical integration method. These different methods are tested and simulation results are given for comparison, with different initial velocities permitting to change the initial energy state.

INTRODUCTION

The main study consists of computing the optimal commands which allow an aircraft to recover a nominal energy trajectory starting from an initial low/high energy state. From now, it is chosen that the main energy problematics can be divided in three major cases. The first case is related to the very high energy state (fig.1) which is corrected thanks to a new slat/flap configuration sequence, with in addition, a necessary use of airbrakes or a vertical trajectory modification. The second case is related to the very low energy state (fig.1) which is corrected thanks to a new slat/flap configuration sequence, with in addition, a necessary use of engines or a vertical trajectory modification. The third and last case is related to the moderate low/high energy state (fig.1) which can be corrected by a new slat/flap configuration sequence, and thus without to use neither engines nor airbrakes. During the

approach phase, one necessary operational step concerns the high lift configuration command. The slat/flap aerodynamic control surfaces are then extended in order to increase the lift performance of the aircraft for low velocities. The extension orders are generally based on specific velocities which are included between a maximum velocity (for structural aspect) and minimum velocity (for stalling aspect). These specific velocities are typically the ones giving the best lift performance of the aircraft at low velocity for a nominal approach scenario. However, if the aircraft is not established on the right vertical trajectory (too high, or too low) or even if the velocity is not the expected one (too fast, too slow), then the specific slat/flap extension velocities are not optimal anymore to recover a nominal energy trajectory. If no specific actions are done by the pilot, the aircraft will surely approach the runway with a high or low energy state. These energy states are the characteristics of the so-called non-stabilized approaches.

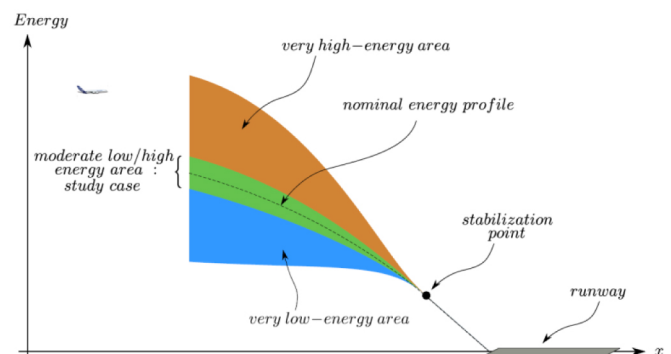


Figure 1. Low/high energy areas in approach

Only the third case is studied in this paper. The study case deals with the moderate low/high energy state which is corrected thanks to a new slat/flap configuration sequence

only. Furthermore, it is considered in this paper that airbrakes and engines are not used. Consequently, the optimization results become implicitly optimal in terms of fuel consumption and noise due to the engine and airbrake uses. This paper details in the second chapter the modeling of the aircraft behavior in an operational approach context taking into account the problematics of slat/flap changes. In this modeling, the transition dynamics of slat/flap are taken into account for more precision. The third chapter deals with the translation of the operational problem into a standard optimization problem. There are also given details to show how collocation and numerical integration methods can be used to solve this optimization problem as well as for implementation of the different constraints into the optimization process. The fourth and last chapter gives simulation results with the different optimization methods.

MODELING OF THE PROBLEMATICS

FORMULATION OF THE GLOBAL RETAINED MODEL

The longitudinal behavior of the aircraft has non-linear dynamics. The dynamical model can be expressed as follows:

$$\dot{X}(t) = f(X(t), W(t), U(t), t) \quad (1)$$

With $X(t)$ the state vector, $W(t)$ the outside perturbation vector, $U(t)$ the continuous command vector and t the explicit time. Before giving the whole expression of the state-space model, it is important to establish the aircraft performance model as it is detailed in the next subsection.

AIRCRAFT PERFORMANCE MODEL

By assuming small angle of attack and negligible engine tilt angle, the simplified thrust dynamic equation gives:

$$m\dot{V}_a + m\Delta A_a^x = F_x^a - \frac{1}{2}\rho S V_a^2 C_D - mg \sin \gamma_a \quad (2)$$

With the same assumptions, the simplified lift dynamic equation gives:

$$-mV_a\dot{\gamma}_a + m\Delta A_a^z = -\frac{1}{2}\rho S V_a^2 C_L + mg \cos \gamma_a \quad (3)$$

By assuming no wind gradient effects ($\Delta A_a^{x,z} = 0$) and by mixing both the [equation \(2\)](#) and [\(3\)](#), it is possible to write :

$$\frac{F_x^a}{mg} - \frac{1}{f} \left(\frac{V_a \dot{\gamma}_a}{g} + \cos \gamma_a \right) = \frac{\dot{V}_a}{g} + \sin \gamma_a \quad (4)$$

Where f represents the lift-to-drag ratio $\frac{C_L}{C_D}$. By assuming small flight path angle γ_a , the [equation \(4\)](#) can be reformulated as :

$$\frac{F_x^a}{mg} - \frac{1}{f} (N_z + 1) = \frac{\dot{V}_a}{g} + \gamma_a \triangleq \gamma_{Ta} \quad (5)$$

Where N_z represents the approximated vertical load factor perpendicular to the trajectory. The right member of the [equation \(5\)](#) is called the Total Flight Path Angle (Total FPA) representing the energy variation capacity of the aircraft with regard to its aerodynamic and engine thrust performances.

The aircraft performance model used in the study is then expressed in terms of Total FPA. It is assumed that the aircraft performance is the sum of:

- A nominal aircraft aerodynamic effect, with a certain slat/flap configuration j with an idle engine thrust and no airbrake effect: $\gamma_{Ta}^{CONF j}$.
- An extra engine thrust effect: γ_{Ta}^{ENG} ;
- An extra airbrake effect: $\gamma_{Ta}^{A/B}$
- An extra landing gear effect: $\gamma_{Ta}^{L/G}$.

Then, the global aircraft performance model can be written as the sum of all these effects, as follows:

$$\gamma_{Ta}^{A/C} = \gamma_{Ta}^{CONF j} + \gamma_{Ta}^{ENG} \cdot \delta_{ENG} + \gamma_{Ta}^{A/B} \cdot \delta_{A/B} + \gamma_{Ta}^{L/G} \cdot \delta_{L/G} \quad (6)$$

With δ_{ENG} and $\delta_{A/B}$ the associated continuous commands for engines and airbrakes respectively, and $\delta_{L/G}$ a discrete command for the landing gear which all evolve between 0 and 1 with a certain dynamics. In order to simplify the study, the landing gear command is included in the slat/flap configuration. It is then assumed that the landing gear is down only from the configuration 2. This simplification permits to rewrite the global aircraft performance model [\(6\)](#) as:

$$\gamma_{Ta}^{A/C} = \gamma_{Ta}^{CONF j, L/G} + \gamma_{Ta}^{ENG} \cdot \delta_{ENG} + \gamma_{Ta}^{A/B} \cdot \delta_{A/B} \quad (7)$$

Taking into consideration that the aircraft has four possible slat/flap configuration positions, it is then necessary to define five performance models including the clean configuration.

$\gamma_{Ta}^{CONF j, L/G}$ then has in the study five different models. For

the study, second-order polynomial functions in terms of CAS velocity are used to describe the five $\gamma_{Ta}^{CONF j,L/G}$ models. For $j \in \{0,1, \dots, 4\}$, a slat/flap configuration j gives a set of three coefficients a_2^j, a_1^j, a_0^j which describe the second-order polynomial function as follows :

$$\gamma_{Ta}^{CONF j,L/G} = a_2^j \cdot V_{CAS}^2 + a_1^j \cdot V_{CAS} + a_0^j \quad (8)$$

The [figure \(2\)](#) shows an example of shape of the used Total FPA-based performance models in the CAS velocity/Total FPA axes. It should be noted that these performance models are not representative of an existing aircraft.

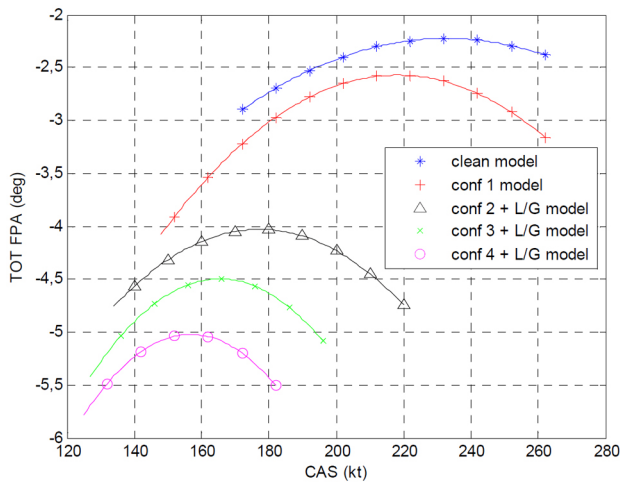


Figure 2. Total FPA-based performance model

AIRCRAFT CONTINUOUS COMMANDS AND PERTURBATIONS

In the global study, the continuous commands, as the engines and airbrakes have to be taken into account. However, this paper presents the case in which only the slat/flap configurations are used, then the continuous command vector becomes:

$$U(t) = \begin{pmatrix} \delta_{ENG} \\ \delta_{A/B} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (9)$$

The engines are then considered to maintain idle thrust level, and the airbrakes are fully retracted. Also for simplification, the perturbation vector is not taken into account at this stage of the study, consequently:

$$W(t) = \begin{pmatrix} V_w^x \\ V_w^z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (10)$$

STATE-SPACE MODEL OF THE AIRCRAFT BEHAVIOR DURING THE APPROACH PHASE

In order to simplify the study, the vertical path is assumed to be a constant ground slope ($\gamma_g = Cste$) and well maintained by the aircraft pitch control law. Consequently, the state vector $X(t)$ contains the aircraft airspeed V_a and the ground distance x^1 . In the study, it is assumed that the airspeed variation $\dot{V}_a$ is calculated thanks to the right member of the [equation \(5\)](#). The airspeed variation is then expressed as:

$$\dot{V}_a = g \cdot (\gamma_{Ta}^{A/C} - \gamma_a) \quad (11)$$

With $\gamma_{Ta}^{A/C}$ the global aircraft performance model defined in the previous section. The flight path angle γ_a can be expressed differently by using the equation linking the airspeed velocity V_a to the ground velocity V_g by use of the wind effect V_w :

$$\vec{V}_a + \vec{V}_w = \vec{V}_g \quad (12)$$

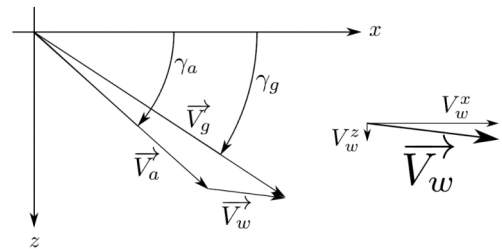


Figure 3. Link between airspeed, wind and ground speed

By projecting the [equation \(12\)](#) onto the vertical and horizontal earth axes as it is shown in [figure \(3\)](#), and assuming only longitudinal components, the following equations are obtained:

$$\begin{aligned} V_a \cdot \sin \gamma_a + V_w^z &= V_g \cdot \sin \gamma_g \\ V_a \cdot \cos \gamma_a + V_w^x &= V_g \cdot \cos \gamma_g \end{aligned} \quad (13)$$

¹The ground slope being constant, it is possible to estimate the altitude in function of the ground distance thanks to a geometrical rule.

By assuming little angles, the equations (13) become respectively:

$$\begin{aligned} V_a \cdot \gamma_a + V_w^z &= V_g \cdot \gamma_g \\ V_a + V_w^x &= V_g \end{aligned} \quad (14)$$

By replacing the expression of V_g of the second equation of (14) in the first equation of (14), it is possible to write:

$$\gamma_a = \frac{(V_a + V_w^x) \cdot \gamma_g - V_w^z}{V_a} \quad (15)$$

It is then possible to rewrite the equation (11) with (15) as:

$$\dot{V}_a = g \cdot \left(\gamma_{Ta}^{A/C} - \frac{(V_a + V_w^x) \cdot \gamma_g - V_w^z}{V_a} \right) \quad (16)$$

By introducing the global aircraft performance model of the equation (7), the equation (16) becomes:

$$\dot{V}_a = g \cdot \left(\gamma_{Ta}^{CONF j,L/G} + \gamma_{Ta}^{ENG} \cdot \delta_{ENG} + \gamma_{Ta}^{A/B} \cdot \delta_{A/B} - \frac{(V_a + V_w^x) \cdot \gamma_g - V_w^z}{V_a} \right) \quad (17)$$

By applying the simplifications concerning the wind, extra engine thrust and airbrake effects, the equation (17) can be written as:

$$\dot{V}_a = g \cdot (\gamma_{Ta}^{CONF j,L/G} - \gamma_g) \quad (18)$$

By using the polynomial form of the equation (8), the equation (18) can be rewritten as:

$$\dot{V}_a = g \cdot (a_2^j \cdot V_{CAS}^2 + a_1^j \cdot V_{CAS} + a_0^j - \gamma_g) \quad (19)$$

Because the $\gamma_{Ta}^{CONF j,L/G}$ models are function of the CAS velocity, it is necessary to include a function η which permits to convert the airspeed to CAS speed. This function is expressed as:

$$\eta = \eta(P_s, \rho, M) \quad (20)$$

Where P_s represents the static pressure, ρ the air density and M the Mach number. The equation (19) which expresses the airspeed variation can finally be rewritten as:

$$\dot{V}_a = g \cdot (a_2^j \cdot \eta^2 V_a^2 + a_1^j \cdot \eta \cdot V_a + a_0^j - \gamma_g) \quad (21)$$

This equation (21) is a first order non-linear differential equation (also called Riccati equation) representing the airspeed variation in terms of the flown ground slope γ_g and the current aircraft performance.

Concerning the second state of $\ddot{X}(t)$, the ground distance variation $\dot{x}$, it is possible to write :

$$\dot{x} = V_x = V_g \cos \gamma_g \quad (22)$$

By using the second equation of (14), assuming little angles, and removing the wind effects, the equation (22) becomes:

$$\dot{x} = V_a \quad (23)$$

Finally, the state-space model for a constant slat/flap configuration j , can be written as:

$$\begin{cases} \dot{X}(t) = \begin{pmatrix} \dot{V}_a \\ \dot{x} \end{pmatrix} = \begin{pmatrix} g \cdot (a_2^j \cdot \eta^2 V_a^2 + a_1^j \cdot \eta \cdot V_a + a_0^j - \gamma_g) \\ V_a \end{pmatrix} \\ U(t) = 0 \end{cases} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad (24)$$

Also, the transition from a slat/flap configuration to another one is taken into account in the study. That is why it is assumed a time linear transition from a model to another one during a slat/flap configuration transition.

For $t \in [t_{(j+1)c}, \dots, t_{(j+1)c} + T_{conf_{j+1}^{ext}}]$ in which a transition from a slat/flap configuration j to $j+1$ occurs, the state-space model for a slat/flap configuration transition can be written as:

$$\begin{cases} \dot{X}(t) = \begin{pmatrix} \dot{V}_a \\ \dot{x} \end{pmatrix} = \begin{pmatrix} g \cdot (a_2^{j \rightarrow j+1} \cdot \eta^2 V_a^2 + a_1^{j \rightarrow j+1} \cdot \eta \cdot V_a + a_0^{j \rightarrow j+1} - \gamma_g) \\ V_a \end{pmatrix} \\ U(t) = 0 \end{cases} = \begin{pmatrix} f_1^* \\ f_2^* \end{pmatrix} \quad (25)$$

With:

$$\begin{pmatrix} a_2^{j \rightarrow j+1} \\ a_1^{j \rightarrow j+1} \\ a_0^{j \rightarrow j+1} \end{pmatrix} = M^j \cdot \begin{pmatrix} a_2^j \\ a_1^j \\ a_0^j \end{pmatrix} + M^{j+1} \cdot \begin{pmatrix} a_2^{j+1} \\ a_1^{j+1} \\ a_0^{j+1} \end{pmatrix} \quad (26)$$

$$M^j = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}, m = \frac{-t}{T_{conf_{j+1}^{ext}}} + \frac{t_{(j+1)c} + T_{conf_{j+1}^{ext}}}{T_{conf_{j+1}^{ext}}} \quad (27)$$

$$M^{j+1} = \begin{pmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{pmatrix}, n = \frac{t}{T_{conf_{j+1}^{ext}}} - \frac{t_{(j+1)c}}{T_{conf_{j+1}^{ext}}} \quad (28)$$

Where t represents the current time, $t_{(j+1)c}$ the time at which the configuration $j + 1$ is commanded and $T_{conf_{j+1}^{ext}}$ the transition time from the configuration j to the configuration $j + 1$.

OPTIMIZATION-BASED SOLUTIONS FOR SLAT/FLAP COMMAND PROBLEM

FORMULATION OF A STANDARD OPTIMIZATION PROBLEM

Assuming a non-linear dynamical system defined by the following equation:

$$\dot{X}(t) = f(X(t), W(t), U(t), a, t) \quad (29)$$

Where $X(t)$ represents the state vector, $W(t)$ the perturbation vector, $U(t)$ the continuous command vector, a the optimization parameter vector, and t the explicit time. The initial and final conditions are given by:

$$\begin{aligned} \Psi_i &= \Psi(X(t_i), U(t_i), a, t_i) \\ \Psi_f &= \Psi(X(t_f), U(t_f), a, t_f) \end{aligned} \quad (30)$$

The set of constraints is given by:

$$g = g(X(t), U(t), a, t) \quad (31)$$

And the optimization criterion² to be minimized is expressed by:

$$J = J(X(t), U(t), a, t) \quad (32)$$

Functional Optimization

In the case where the dynamical system is represented by a model as follows:

$$\dot{X}(t) = f(X(t), W(t), U(t), t) \quad (33)$$

The standard optimization problem becomes a functional optimization problem where an optimal command $U^*(t)$ is searched in order to minimize a cost function J while satisfying a set of constraints g and the initial and final conditions Ψ .

Parametric Optimization³

In the case where the dynamical system is represented by a model as follows:

$$\dot{X}(t) = f(X(t), W(t), a, t) \quad (34)$$

The standard optimization problem becomes a parametric optimization problem where the optimal optimization parameter vector a^* is searched in order to minimize an optimization criterion J while satisfying a set of constraints g and the initial and final conditions Ψ .

As a result of the modeling presented in the previous sections, this kind of parametric optimization problem is used in the study, and detailed in the following sections.

CONSTRAINTS AND BOUNDS FOR NLP PROBLEM

To solve such a constrained non-linear optimization problem, several algorithms exist. The *Fmincon* solver from Matlab software, able to solve Non Linear Programming (NLP) problem, is chosen in the study.

The formulation of the constrained non-linear optimization problem is the following:

Find the optimal optimization parameter vector a^* which minimizes the optimization criterion $J(a)$ under the following constraints and bounds:

- Non-linear equality constraints, such as: $C_{eq}(a) = 0$;
- Non-linear inequality constraints, such as: $C_{ineq}(a) \leq 0$;
- Linear equality constraints, such as: $A_{eq} \cdot a = b_{eq}$;
- Linear inequality constraints, such as: $A_{ineq} \cdot a \leq b_{ineq}$;
- Optimization parameter vector bounds, such as: $low_b \leq a \leq high_b$;

The following subsections detail two different solutions used to solve the slat/flap command optimization problem.

²Also called cost function or objective function.

³It is possible to parameterize a continuous command vector from a functional problem in order to transform the problem in a parametrical one.