

Stability Prediction of a UAV

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10/18/2011Ilhan Tuzcu
California State Univ.Kahtan Awni
ConsultantJavier Gonzalez-Rocha
California State Univ.

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ABSTRACT

Typical design challenges for Unmanned Aerial Vehicles (UAVs) require low aerodynamic drag and structural weight. Both of these requirements imply that these aircraft are considerably more flexible than conventional aircraft and their stability analyses are more complex since they require models unifying rigid body and elastic dynamics. This paper aims to built such a model for a generic UAV. The model is then used to address stability in terms of divergence and flutter.

INTRODUCTION

The use of Unmanned Aerial Vehicles (UAVs) in civilian and military applications has increased drastically in recent years. These applications include military intelligence, surveillance and reconnaissance missions, communication relaying, border patrol, search and rescue missions, and environmental monitoring. Design challenges typically are to increase endurance and range, as well as nonstructural weight (payload), which require increase in fuel efficiency and decrease in aerodynamic drag and structural weight. Low drag can be achieved by having a high aspect-ratio wing. Since UAVs are crewless they do not involve human lives, and hence, safety margins for them need not be as high as those for manned aircraft, which permit structural weight be low. Both high aspect-ratio wing and low structural weight imply that the resulting aircraft is considerably more flexible than the conventional aircraft. Hence, interaction between

elastic and rigid body dynamics of the aircraft is high and the separation between these dynamics no longer exists, implying that an approach unifying both of the dynamics must be used. This paper aims to derive an accurate model of a UAV accounting for both rigid body and elastic motions and to address stability in terms of flutter and divergence. The theory developed in [1] and [2] are best suited for these unconventional aircraft. Similar studies to the present paper have been done in [3], [4] and [5]. However, emphasis in the present study is placed on Vortex Lattice Method (VLM) that is used to accurately predict the distribution of aerodynamic loads on the wing.

There are several studies in the open literature that aim to address the dynamics of UAVs. The paper by Patil, Hodges and Cesnik [6] presents a nonlinear aeroelastic study on a complete aircraft model which is geometrically similar to High-Altitude-Long- Endurance (HALE) aircraft. The focus of the study seems to be on the effect of the static structural deflections on the aeroelastic behavior. A nonlinear model of a highly flexible flying-wing is developed in [7]. The equations of motion are derived in terms of rather abstract variables, namely intrinsic variables. Rigid body degrees of freedom include only the ones corresponding to longitudinal motion. Results are given for a typical high-aspect-ratio flying-wing configuration and include variations of trim solutions, and Phugoid and short-period modes with respect to payload, and nonlinear simulation of the wing in response to a flap deflection. Another important study related to the present paper is given in [8], which focuses on the nonlinear

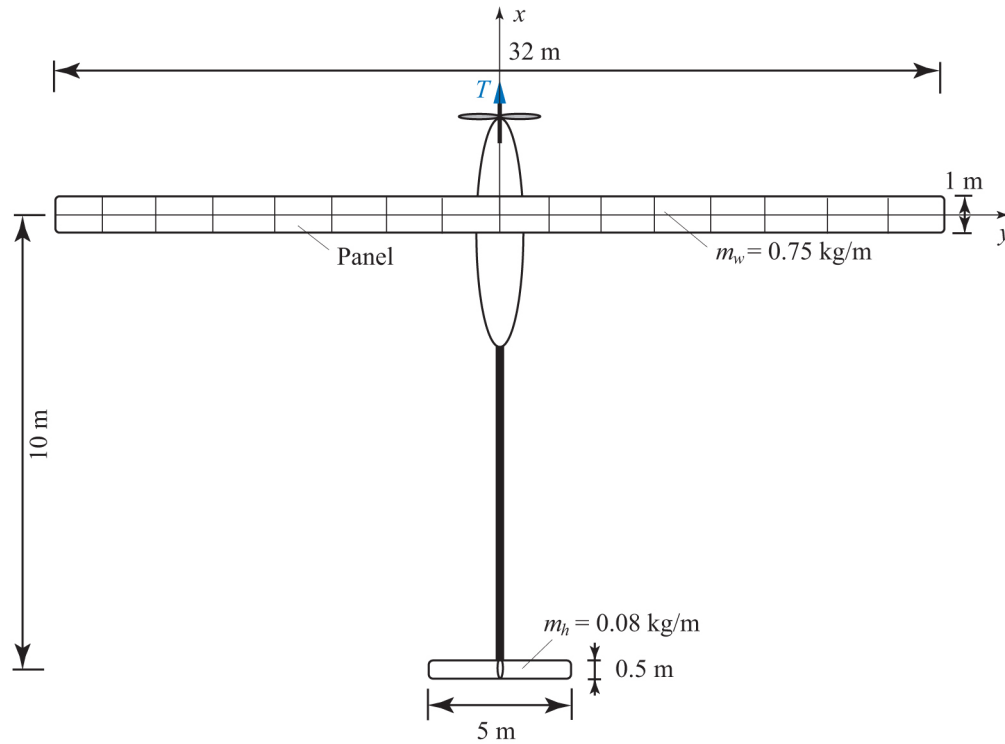


Figure 1. Generic UAV

response of a very flexible aircraft in flight. The results include the simulations of the aircraft in various flight conditions such as level gliding descent, low-pass-filtered square aileron input, rolling/gliding descent, and low-pass square elevator input gliding descent.

EQUATIONS OF MOTION

Wing is the most dominant aerodynamic and structural component of aircraft, and for this reason, we model only the wing as a flexible component and treat the remaining components, namely, fuselage, and vertical and horizontal stabilizers, as rigid. Furthermore, we assume that the wing is composed of a left- and a right-half parts both modeled as cantilever beams undergoing bending and torsion. We use the subscript r to denote the quantities associated with the right-half wing, l with the left-half wing, f with the fuselage, h with the horizontal stabilizer and, finally, v with the vertical stabilizer.

To describe the motion of the aircraft, we first attach a set of body axes xyz to the undeformed aircraft at a convenient point, as well as similar body axes to the left- and the right-half wing, namely, $x_r y_r z_r$ and $x_l y_l z_l$ with origins o_r and o_l , respectively (Fig. 1). The motion then can be expressed by three translations and three rotations of the body axes xyz and elastic deformation of the wing relative to the body axes. The explicit equations of motion can be derived using the Lagrangian equations of motion in quasico-ordinates, [1]:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \mathbf{v}} \right) + \tilde{\omega} \frac{\partial L}{\partial \mathbf{v}} &= \mathbf{F} \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \boldsymbol{\omega}} \right) + \tilde{v} \frac{\partial L}{\partial \mathbf{v}} + \tilde{\omega} \frac{\partial L}{\partial \boldsymbol{\omega}} &= \mathbf{M} \\ \frac{\partial}{\partial t} \left(\frac{\partial \hat{L}}{\partial \mathbf{v}_i} \right) - \frac{\partial \hat{L}}{\partial \mathbf{u}_i} + \frac{\partial \hat{\mathcal{F}}_i}{\partial \dot{\mathbf{u}}_i} + \mathcal{L}_i \mathbf{u}_i &= \hat{\mathbf{U}}_i \\ \frac{\partial}{\partial t} \left(\frac{\partial \hat{L}}{\partial \boldsymbol{\alpha}_i} \right) + \frac{\partial \hat{\mathcal{J}}_i}{\partial \dot{\boldsymbol{\psi}}_i} + \mathcal{H}_i \boldsymbol{\psi}_i &= \hat{\boldsymbol{\Psi}}_i, \quad i = r, l \end{aligned} \quad (1)$$

where $L = T - V$ is the Lagrangian for the whole aircraft, in which T is the kinetic energy and V is the potential energy, $\mathbf{v} = [U \ V \ W]^T$ and $\boldsymbol{\omega} = [P \ Q \ R]^T$ are the vectors of translational and angular velocities of xyz , C matrix of direction cosines between xyz and inertial axes XYZ , $\mathbf{R} = [X \ Y \ Z]^T$ position vector of the origin o of xyz relative to XYZ , E matrix relating Eulerian velocities to angular quasi-velocities, $\boldsymbol{\theta} = [\phi \ \theta \ \psi]^T$ symbolic vector of Eulerian angles between xyz and XYZ , $\mathbf{u}_i = [0 \ 0 \ w_i]^T$ and $\mathbf{v}_i = \mathbf{u}_i$ elastic bending displacement and velocity vectors, $\boldsymbol{\psi}_i = [\varphi_i \ 00]^T$ and $\boldsymbol{\alpha}_i$ elastic torsional displacement and velocity vectors, $\hat{L}$ Lagrangian density exclusive of strain energy, $\hat{\mathcal{F}}_i$ and $\hat{\mathcal{J}}_i$ Rayleigh's dissipation function densities, L_i and H_i matrices of stiffness differential operators, $\mathbf{F}$ and $\mathbf{M}$ resultant of gravity, aerodynamic, propulsion and control force and moment vectors acting on the whole aircraft in terms of body axes components, $\mathbf{U}$ and

Ψ_i resultant of gravity, aerodynamic, propulsion and control force and moment density vectors. The terms like $\tilde{\omega} \partial L / \partial \mathbf{v}$ indicate vector multiplication, $\omega \times \partial L / \partial \mathbf{v}$ where $\tilde{\omega}$ is a skew symmetric vector derived from ω :

$$\tilde{\omega} = \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} \quad (2)$$

As obvious in Eqs. (1), the equations of motion consist of ordinary differential equations for the rigid body translations and rotations of the aircraft as a whole, and the boundary-value problems for the elastic deformations of the wing.

The equations of motion for the flexible UAV described above in general does not admit a closed-form solution and the first approximation amounts to discretizing the distributed variables. To this end, we introduce the following expansions [1,2] to the equations:

$$\mathbf{u}_i(\mathbf{r}_i, t) = \mathbf{U}_i(\mathbf{r}_i) \mathbf{q}_i(t), \quad \psi_i(\mathbf{r}_i, t) = \Psi_i(\mathbf{r}_i) \xi_i(t), \quad i = r, l \quad (3)$$

where $\mathbf{r}_i(x_i, y_i, z_i)$ is a radius vector from o_i to a typical point on the wing, $\mathbf{U}_i(\mathbf{r}_i)$ and $\Psi_i(\mathbf{r}_i)$ are matrices of shape functions and $\mathbf{q}_i(t)$ and $\xi_i(t)$ are corresponding vectors of generalized coordinates. The shape functions for bending are the eigenfunctions of a uniform cantilever beam and for torsion the eigenfunctions of a uniform clamped-free shaft. Discrete versions of the equations of motion have been obtained in [1]:

$$\begin{aligned} \dot{\mathbf{R}} &= C^T \mathbf{v}, \quad \dot{\boldsymbol{\theta}} = E^{-1} \boldsymbol{\omega} \\ \dot{\mathbf{q}}_r &= \mathbf{s}_r, \quad \dot{\mathbf{q}}_l = \mathbf{s}_l, \quad \dot{\xi}_r = \boldsymbol{\eta}_r, \quad \dot{\xi}_l = \boldsymbol{\eta}_l \\ \dot{\mathbf{p}}_v &= -\tilde{\omega} \mathbf{p}_v + \mathbf{F} \\ \dot{\mathbf{p}}_\omega &= -\tilde{v} \mathbf{p}_v - \tilde{\omega} \mathbf{p}_\omega + \mathbf{M} \\ \dot{\mathbf{p}}_r &= \frac{\partial \mathcal{T}}{\partial \mathbf{q}_r} - K_r \mathbf{q}_r - D_r \mathbf{s}_r + \mathbf{Q}_r \\ \dot{\mathbf{p}}_l &= \frac{\partial \mathcal{T}}{\partial \mathbf{q}_l} - K_l \mathbf{q}_l - D_l \mathbf{s}_l + \mathbf{Q}_l, \\ \dot{\mathbf{p}}_r &= -K_r \xi_r - D_r \boldsymbol{\eta}_r + \boldsymbol{\Theta}_r \\ \dot{\mathbf{p}}_l &= -K_l \xi_l - D_l \boldsymbol{\eta}_l + \boldsymbol{\Theta}_l \end{aligned} \quad (4)$$

where $\mathbf{F}$ and $\mathbf{M}$ are the resultant of gravity, aerodynamic and propulsion force and moment vectors acting on the whole aircraft in terms of body axes components, K_i and $\mathbf{K}_i$ the stiffness matrices, D_i and $\mathbf{D}_i$ the structural damping matrices, $\mathbf{Q}_i$ and $\boldsymbol{\Theta}_i$ generalized force vectors and

$$\mathbf{p}_v = \frac{\partial \mathcal{T}}{\partial \mathbf{v}}, \quad \mathbf{p}_\omega = \frac{\partial \mathcal{T}}{\partial \boldsymbol{\omega}}; \quad \mathbf{p}_i = \frac{\partial \mathcal{T}}{\partial \mathbf{s}_i}, \quad \mathbf{p}_i = \frac{\partial \mathcal{T}}{\partial \boldsymbol{\eta}_i}, \quad i = r, l \quad (5)$$

the momenta vectors in which $\mathcal{T}$ is the kinetic energy. The total kinetic energy of the aircraft can be written as a summation of kinetic energies of the individual components as follows:

$$\mathcal{T} = \frac{1}{2} \sum_i \int \tilde{\mathbf{v}}_i^T \tilde{\mathbf{v}}_i dm_i, \quad i = r, l, f, h, v \quad (6)$$

where $\tilde{\mathbf{v}}_i$ is the velocity of a typical point in the component i and dm_i is the corresponding mass differential element. The velocity can be expressed as

$$\begin{aligned} \tilde{\mathbf{v}}_i(\mathbf{r}_i, t) &= C_i \mathbf{v} + \tilde{\chi}_i^T C_i \boldsymbol{\omega} + \mathbf{U}_i \mathbf{s}_i + \tilde{r}_i^T \Psi_i \boldsymbol{\eta}_i, \quad i = r, l \\ \tilde{\mathbf{v}}_i(\mathbf{r}_i, t) &= \mathbf{v} + \tilde{r}_i^T \boldsymbol{\omega}, \quad i = f, h, v \end{aligned} \quad (7)$$

where C_i is the matrix of direction cosines between $x_i y_i z_i$ and xyz and $\tilde{\chi}_i = \tilde{r}_i + \tilde{\mathbf{U}}_i \mathbf{q}_i$. Note that the velocities are in respective body axes. Inserting Eqs. (7) into Eq. (6) and carrying out the indicated operations, the total kinetic energy can be written in the compact form

$$\mathcal{T} = \frac{1}{2} \mathbf{V}^T \mathbf{M} \mathbf{V} \quad (8)$$

where $\mathbf{V} = [\mathbf{v}^T \quad \boldsymbol{\omega}^T \quad \mathbf{s}_r^T \quad \mathbf{s}_l^T \quad \boldsymbol{\eta}_r^T \quad \boldsymbol{\eta}_l^T]^T$ is the discrete system velocity vector and $\mathbf{M}$ is the system mass matrix, which can be expressed in the partitioned form given in Eq. (9) in which m is the aircraft total mass and

$$\mathbf{M} = \begin{bmatrix} mI & \tilde{S}^T & C_r^T \int \mathbf{U}_r dm_r & C_l^T \int \mathbf{U}_l dm_l & \\ & J & C_r^T \int \tilde{\chi}_r \mathbf{U}_r dm_r & C_l^T \int \tilde{\chi}_l \mathbf{U}_l dm_l & \dots \\ & & \int \mathbf{U}_r^T \mathbf{U}_r dm_r & 0 & \\ & & & \int \mathbf{U}_l^T \mathbf{U}_l dm_l & \\ & & \text{Symmetric} & & \\ & & C_r^T \int \tilde{r}_r^T \Psi_r dm_r & C_l^T \int \tilde{r}_l^T \Psi_l dm_l & \\ & & C_r^T \int \tilde{\chi}_r \tilde{r}_r^T \Psi_r dm_r & C_l^T \int \tilde{\chi}_l \tilde{r}_l^T \Psi_l dm_l & \\ & & \int \mathbf{U}_r^T \tilde{r}_r^T \Psi_r dm_r & 0 & \\ & & 0 & \int \mathbf{U}_l^T \tilde{r}_l^T \Psi_l dm_l & \\ & & \int \Psi_r^T \tilde{r}_r \tilde{r}_r^T \Psi_r dm_r & 0 & \\ & & & \int \Psi_l^T \tilde{r}_l \tilde{r}_l^T \Psi_l dm_l & \end{bmatrix} \quad (9)$$

$$\begin{aligned} \tilde{S} &= \sum_{i=r,l} \int C_i^T \tilde{\chi}_i C_i dm_i + \sum_{i=h,v} \int \tilde{r}_i dm_i \\ J &= \sum_{i=r,l} \int C_i^T \tilde{\chi}_i \tilde{\chi}_i^T C_i dm_i + \sum_{i=h,v} \int \tilde{r}_i \tilde{r}_i^T dm_i \end{aligned} \quad (10)$$

are the matrices of first and second moments of inertia of the deformed aircraft, respectively. Denoting the momenta vector