

Flutter Initiation Under Steady - State and Accelerated Free Stream Velocities

2011-01-2785

Published
10/18/2011

Harry H. Hilton

Copyright © 2011 SAE International

doi:[10.4271/2011-01-2785](https://doi.org/10.4271/2011-01-2785)

ABSTRACT

The motivation for the present investigation of damped motion through energy dissipation due to material properties and bifurcations is to better understand the fundamentals of flutter, with applications to elastic and viscoelastic composite flight and automotive vehicles. The traditional approach to determining flutter in a linear aeroelastic system, such as a wing, is to define flutter as the lowest velocity at which simple harmonic motion first occurs. This legacy definition is demonstrable using a Duncan-Ellis flutter engine that replicates the motions of a pitch-plunge wing. While simple harmonic motion is a useful rule of thumb definition, such a definition does not adequately and sufficiently describe the behavior of the wing before or after flutter. Further, the traditional definition could provide a misleading flutter velocity point. Damped motion is modeled with some modifications to the governing equations of an aeroelastic wing and the analysis reveals that the true flutter point requires an expanded definition. The modifications also reveal that bifurcation of the motion of a linear elastic small deformation wing is possible before any flutter point is reached. Material failures leading to ultimate velocities and structural lifetimes in addition to and independent of flutter velocities are also explored. The analysis and results have immediate applications to flexible composite aerospace lifting surfaces (UAVs, MAVs) and to automotive spoilers (rear wings). Proper analysis of light weight high strength composite lifting surfaces can significantly improve performances.

INTRODUCTION

The pervasive replacement of metals with fiber-matrix high polymer composites as materials of choice in primary load carrying flight vehicle and automotive structures necessitates reexamination of analyses and design protocols based on viscoelastic rather than legacy elastic behaviors. The

additional time dependent viscoelastic nature with accompanying creep, stress relaxation and material damping alters the phase relations between inertia, damping, structural and control forces thereby generating either more or less stable configurations, particularly since in self-excited systems.

Innovative new composites, such as Boeing Co.'s Pultruded Rod Stitched Efficient Unitized Structure PRSEUSTM [1, 2, 3, 4], effectively allow configurations with reduced residual stresses and abilities to carry tension in directions normal to composite panel surface. Additionally, the possibility of introducing the novel concept of designer materials, geometries, controls, etc., subject to constraints such as cost, stacking sequences, etc., [5, 6, 7] make the use of composite even more attractive. Such analyses based on inverse optimized approaches rather than using "of the shelf" materials and structural shapes has been made tractable with the imminent advent of the IBM-NCSA BlueWatersTM petascale computing system [8 - 9].

In flutter analyses, it is conventional to investigate the so-called $V-g$ diagrams [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], with g representing a virtual structural damping coefficient and V the flight velocity. It can be shown that when g reaches the value of zero at $V = V_F$, the lifting surface displacements reach a state of simple harmonic motion and V_F is defined as the flutter velocity.

In self-excited elastic lifting surface systems, second order differential equations in time model the interactions between inertia, internal forces (stresses) and aerodynamic pressures (lift). It can be readily demonstrated that in such linear systems, the displacements are all proportional to

$\exp[(\hat{d} + i\omega)t]$. This alternate protocol focuses on the evolution of $\hat{d}$ values as V increases to V_F and beyond at $V >$

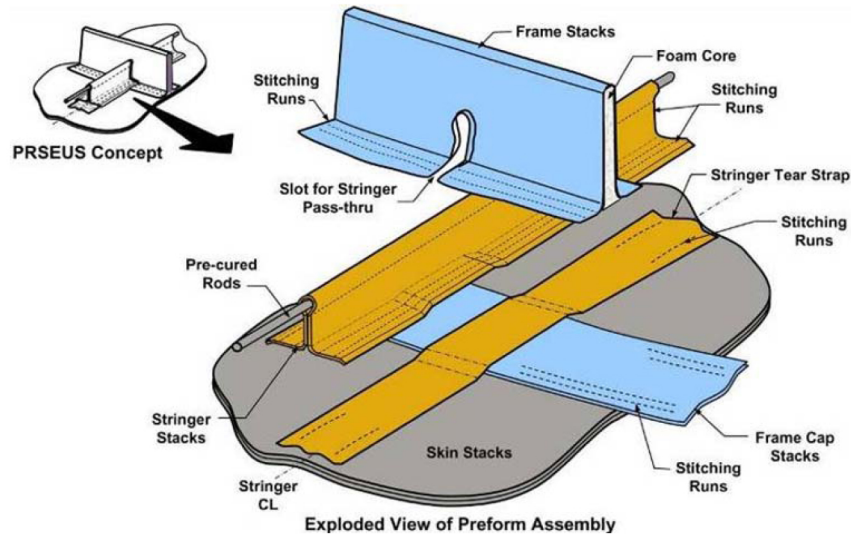


Figure 1. PRSEUS - Pultruded rod stitched efficient structure [1, 2, 3, 4].

V_F (Fig. 1) and the concept produces alternate $V-d$ diagrams [21, 22, 23, 24, 25, 26].

Unsteady aerodynamic theory analyses are generally based on simple harmonic motion (SHM) [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] and [27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] or Theodorsen's theory [38]. The authors' previous analyses indicate that the usual sole condition for simple harmonic motion (SHM), i.e. $d' = 0$, is insufficient in some cases to establish flutter boundaries. Additional constraints of

$$\frac{\partial \hat{d}}{\partial V} = 0 \quad \text{are thus required.}$$

Furthermore, bifurcations in d values at $V < V_F$, the flutter velocity, were also encountered. Also of interest to flight vehicles and automobiles are considerations of panel flutter, aero-servo-controls and excitations due to aerodynamic noise and the latter's attenuations and suppressions [39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52]. The analysis and results have immediate applications to aerospace lifting surfaces and to automotive spoilers (rear wings). Proper analysis of light weight high strength composite wings can significantly improve performance.

ANALYSIS

General Considerations - Steady-State Flight Velocities

Consider a flexible 2-D elastic lifting surface with Cartesian coordinates $\mathbf{x} = \{x_1, x_2, x_3\}$ and where x_2 is the span-wise coordinate and with w and θ the bending and torsional deflections. The general linear elastic and viscoelastic anisotropic isothermal non-homogeneous constitutive relation

expressions with the Einstein summation convention in force are [53, 54, 55]

general
isothermal
anisotropic
inhomogeneous
constitutive
relations $\Rightarrow$

$$\left\{ \begin{array}{l} \sigma_{ij}(\mathbf{x}, t) = \left\{ \begin{array}{l} E_{ijkl0}(\mathbf{x}) \epsilon_{kl}^E(\mathbf{x}, t) \quad \text{elastic} \\ \int_{-\infty}^t E_{ijkl}(\mathbf{x}, t-t') \frac{\partial \epsilon_{kl}(\mathbf{x}, t')}{\partial t'} dt' \quad \text{viscoelastic} \end{array} \right. \\ \epsilon_{ij}(\mathbf{x}, t) = \left\{ \begin{array}{l} C_{ijkl0}(\mathbf{x}) \sigma_{23}^E(\mathbf{x}, t) \quad \text{elastic} \\ \int_{-\infty}^t C_{ijkl}(\mathbf{x}, t-t') \frac{\partial \sigma_{kl}(\mathbf{x}, t')}{\partial t'} dt' \quad \text{viscoelastic} \end{array} \right. \end{array} \right.$$

(1)

with $\sigma_{ij}(\mathbf{x}, -\infty) = \epsilon_{ij}(\mathbf{x}, -\infty) = 0$. Each of the relaxation moduli $E_{ijkl}(\mathbf{x}, t)$ and creep compliance $C_{ijkl}(\mathbf{x}, t)$ is represented by a Prony series [56] of the form

$$E_{ijkl}(\mathbf{x}, t) = E_{ijkl\infty}(\mathbf{x}) + \sum_{n=1}^{N_{ijkl}} \underline{E_{ijkln}}(\mathbf{x}) \exp\left(-\frac{t}{\tau_{ijkln}(\mathbf{x})}\right) \quad (2)$$

$$C_{ijkl}(\mathbf{x}, t) = C_{ijkl0}(\mathbf{x}) + \sum_{n=1}^{N_{ijkl}} \underline{C_{ijkln}}(\mathbf{x}) \exp\left(-\frac{t}{\tau_{ijkln}(\mathbf{x})}\right) \quad (3)$$

and the ensemble of parameters $E_{ijkln}(\mathbf{x})$ or $C_{ijkln}(\mathbf{x})$ and $\tau_n(\mathbf{x})$ represents the material properties of a specific elastic or viscoelastic material. If thermal strains are present and the temperature has temporal dependence, then (1) longer posses convolution integral and the kernel functions become $E_{ijkl}(\mathbf{x}, t, t')$. etc.

The dynamical governing relations for lifting surfaces with 2-DOF (plunging bending and pitching torsion) are

$$\begin{aligned} \text{bending} \Rightarrow & \underbrace{m \frac{\partial^2 w(y, t)}{\partial t^2} - S_\alpha \frac{\partial^2 \theta(y, t)}{\partial t^2}}_{\text{inertia}} \\ & + \underbrace{c_w \frac{\partial w(y, t)}{\partial t}}_{\text{external damping}} + \underbrace{D_w^E \{w(y, t)\}}_{\text{elastic bending rigidity}} \\ & + \underbrace{\int_{-\infty}^t D_w(y, t-t') \left\{ \frac{\partial w(y, t')}{\partial t'} \right\} dt'}_{\text{viscoelastic bending rigidity}} = \underbrace{\Delta p_{CP}(\mathbf{x}, t)}_{\text{cabin pressure}} \\ & + L \underbrace{\left[y, t, w(y, t), \frac{\partial w(y, t)}{\partial t}, \frac{\partial^2 w(y, t)}{\partial t^2}, \theta(y, t), \frac{\partial \theta(y, t)}{\partial t}, \frac{\partial^2 \theta(y, t)}{\partial t^2} \right]}_{\text{lift}} \end{aligned} \quad (4)$$

$$\begin{aligned} \text{torsion} \Rightarrow & \underbrace{I_p \frac{\partial^2 \theta(y, t)}{\partial t^2} - S_\alpha \frac{\partial^2 w(y, t)}{\partial t^2}}_{\text{rotary inertia}} \\ & + \underbrace{c_\theta \frac{\partial \theta(y, t)}{\partial t}}_{\text{external damping}} - \underbrace{D_\theta^E \{\theta(t, t)\}}_{\text{elastic torsional rigidity}} \\ & - \underbrace{\int_{-\infty}^t D_\theta(y, t-t') \left\{ \frac{\partial \theta(y, t')}{\partial t'} \right\} dt'}_{\text{viscoelastic torsional rigidity}} = \underbrace{\int_0^t \Delta p_{CP}(\mathbf{x}, t) dx_1}_{\text{moment due to cabin pressure}} \\ & + M \underbrace{\left[y, t, w(y, t), \frac{\partial w(y, t)}{\partial t}, \frac{\partial^2 w(y, t)}{\partial t^2}, \theta(y, t), \frac{\partial \theta(y, t)}{\partial t}, \frac{\partial^2 \theta(y, t)}{\partial t^2} \right]}_{\text{aerodynamic moment}} \end{aligned} \quad (5)$$

The D_w and D_θ are y-differential operators and represent bending and torsional rigidities and are defined as

$$\begin{aligned} D_w = & \begin{cases} \text{elastic Euler –} \\ \text{Bernoulli beam bending} \\ \frac{\partial^2}{\partial y^2} \left(E_0 I(y) \frac{\partial^2}{\partial y^2} \left\{ \begin{matrix} \\ \end{matrix} \right\} \right) \\ \frac{\partial^2}{\partial y^2} \left(EI(y, t-t') \frac{\partial^2}{\partial y^2} \left\{ \begin{matrix} \\ \end{matrix} \right\} \right) \end{cases} \\ & \text{viscoelastic Euler –} \\ & \text{Bernoulli beam bending} \end{aligned} \quad (6)$$

and

$$\begin{aligned} D_\theta = & \begin{cases} \text{elastic torsion} \\ \frac{\partial}{\partial y} \left(G_0 J(y) \frac{\partial}{\partial y} \left\{ \begin{matrix} \\ \end{matrix} \right\} \right) \\ \frac{\partial}{\partial y} \left(G J(y, t-t') \frac{\partial}{\partial y} \left\{ \begin{matrix} \\ \end{matrix} \right\} \right) \end{cases} \\ & \text{viscoelastic torsion} \end{aligned} \quad (7)$$

These relations apply to wings, horizontal and vertical tails, etc., and to panels provided in the latter case they are expanded to include $x = x_1$ and $y = x_2$ coordinates. The free stream flow and chord direction is x_1 . while x_2 is the span wise direction.

The solution of Eqs. (4) and (5) is in the form of

$$\begin{Bmatrix} w(y, t) \\ \theta(y, t) \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} W_m(V, \hat{d}_m, \omega_m) F_m^w(y) \\ \Theta_m(V, \hat{d}_m, \omega_m) F_m^\theta(y) \end{Bmatrix} \exp \left[\left(\underbrace{\hat{d}_m + i \omega_m}_{= \lambda_m(V, \omega_m)} \right) t \right] \quad (8)$$

The $F(y)$ functions can be evaluated by separation of variables or by Galerkin's method for each m .

Specifically, the incompressible lift and aerodynamic moment of a flexible 2-D elastic wing flying at a variable velocity $V(t)$ is given by¹

$$\begin{aligned} L(\bar{\lambda}, y, t) = & A_1^* V(t) C(\bar{\lambda}) \left[\frac{\partial w(\bar{\lambda}, y, t)}{\partial t} + V(t) \theta(\bar{\lambda}, y, t) + A_2^* \frac{\partial \theta(\bar{\lambda}, y, t)}{\partial t} \right] \\ & + A_3^* \left[\frac{\partial^2 w(\bar{\lambda}, y, t)}{\partial t^2} + V \frac{\partial \theta(\bar{\lambda}, y, t)}{\partial t} - A_4^* \frac{\partial^2 \theta(\bar{\lambda}, y, t)}{\partial t^2} + \underbrace{\dot{V}(t) \theta(\bar{\lambda}, y, t)}_{\text{additional term due to } V(t)} \right] \end{aligned} \quad (9)$$

$$\begin{aligned} M(\bar{\lambda}, y, t) = & A_5^* V(t) C(\bar{\lambda}) \left[\frac{\partial w(\bar{\lambda}, y, t)}{\partial t} + V(t) \theta(\bar{\lambda}, y, t) + A_6^* \frac{\partial \theta(\bar{\lambda}, y, t)}{\partial t} \right] \\ & - A_7^* \left[\frac{\partial^2 w(\bar{\lambda}, y, t)}{\partial t^2} + V \frac{\partial \theta(\bar{\lambda}, y, t)}{\partial t} + A_8^* \frac{\partial^2 \theta(\bar{\lambda}, y, t)}{\partial t^2} + \underbrace{\dot{V}(t) \theta(\bar{\lambda}, y, t)}_{\text{additional term due to } V(t)} \right] \end{aligned} \quad (10)$$

where the A_i^* , ($i = 1, 2, \dots, 8$), are constants depending on geometry and aerodynamic properties of the lifting surface with the reduced frequency k and reduced $\bar{\lambda}$ defined as in terms of the half chord b as

$$k = \frac{b \omega}{V} \quad \text{and}$$

$$\bar{\lambda} = \bar{\lambda}(t) = \frac{b \lambda(t)}{V(t)}$$

$$\text{with} \quad \lambda(t) = \hat{d}[V(t), \omega] + i \omega \quad (11)$$

The Theodorsen function [38] modified for variable velocity and non-SHM [24], [25] is

$$C(\bar{\lambda}) = \frac{K_1(\bar{\lambda})}{K_1(\bar{\lambda}) + i K_0(\bar{\lambda})} \quad (12)$$

with $K_i(\bar{\lambda})$ being modified Bessel functions [57]. Eqs. (9) and (10) do not contain the trim angle of attack which in flight vehicles varies with time if $V(t)$. On the other hand, in automobiles the trim angle is fixed for all time regardless of how the velocity might behave.

It should be noted that the substitution of the aerodynamic forces and moments into the elastic governing relations, whether 2-D or multi-D and incompressible or compressible flows, does not alter the character or order of these PDEs and ODEs. Symbolically, for generalized multi-dimensional displacements $\mathbf{u}$ these relations can be written in matrix form as

$$\begin{aligned} [\mathcal{L}_E^1] \{ \mathbf{u}^E(y, t) \} = & \underbrace{[M(y)] \left\{ \frac{\partial^2 \mathbf{u}^E(y, t)}{\partial t^2} \right\} + [C(y)] \left\{ \frac{\partial \mathbf{u}^E(y, t)}{\partial t} \right\} + [D(y)] \{ \mathbf{u}^E(y, t) \}}_{\text{elastic flexible body contributions}} = \\ & \underbrace{\left\{ \mathbf{A}_r(y, t) + \mathbf{A}_0(y) \mathbf{u}^E(y, t) + \mathbf{A}_1(y) \frac{\partial \mathbf{u}^E(y, t)}{\partial t} + \mathbf{A}_2(y) \frac{\partial^2 \mathbf{u}^E(y, t)}{\partial t^2} \right\}}_{\text{flexible and rigid body aerodynamic contributions (independent of specific material properties)}} \end{aligned} \quad (13)$$

or

¹With particular applications to relatively slow moving very flexible vehicles, such as UAVs, MAVs, and to vehicles during landings and takeoffs and to cars with aero wings.

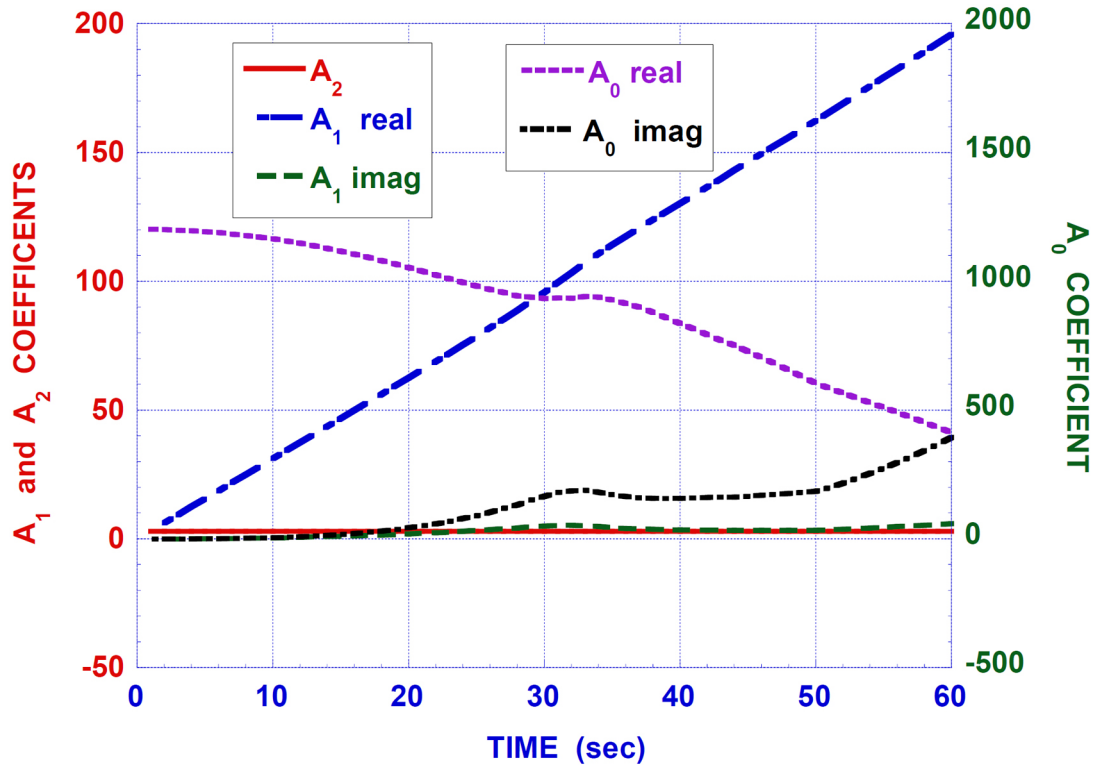


Figure 2. Aerodynamic coefficients of Eqs. (13), (14), and (16).

$$\begin{aligned}
 [\mathcal{L}_E]\{\mathbf{u}^E(y, t)\} &= [M(y) - A_2(y)] \left\{ \frac{\partial^2 \mathbf{u}^E(y, t)}{\partial t^2} \right\} + \\
 [C(y) - A_1(y)] \left\{ \frac{\partial \mathbf{u}^E(y, t)}{\partial t} \right\} + \\
 [D(y) - A_0(y)] \{\mathbf{u}^E(y, t)\} &= \underbrace{\{A_r(y, t)\}}_{\text{rigid body aerodynamics}}
 \end{aligned}
 \quad (14)$$

Eqs. (13) can be turned into ODEs as indicated above through the application of Galerkin's method. The generalized forces represented by the matrix A_r depend on the built-in and trim angles of attack $\alpha_r(y, t)$ and $\alpha_{TR}(y, t)$, and the angle of zero lift $\alpha_0(y)$ (Fig. 2). This figure displays typical values for the real and imaginary parts of the four coefficients as functions of the free stream velocity.

The above relations apply only to linear elastic materials. For linear viscoelastic media the superscript E is removed from the generalized displacements $\mathbf{u}$ and the D operators must be modified to read

viscoelastic contributions $\Rightarrow$

$$\begin{aligned}
 \int_{-\infty}^t [E(y, t - t')] [D] \left\{ \frac{\partial \mathbf{u}(y, t')}{\partial t'} \right\} dt' = \\
 \int_{-\infty}^t \left[\sum_{n=1}^N E_n(y) \exp\left(-\frac{t - t'}{\tau_n(y)}\right) \right] [D] \left\{ \frac{\partial \mathbf{u}(y, t')}{\partial t'} \right\} dt'
 \end{aligned}
 \quad (15)$$

The resulting integral partial differential relations (IPDEs) can be converted to PDEs by using the differential viscoelastic constitutive relations, resulting in

$$\begin{aligned}
 [\mathcal{L}_{VE}]\{\mathbf{u}(y, t)\} &= \\
 [P] \left([M(y) - A_2(y)] \left\{ \frac{\partial^2 \mathbf{u}(y, t)}{\partial t^2} \right\} + [C(y) - A_1(y)] \left\{ \frac{\partial \mathbf{u}(y, t)}{\partial t} \right\} \right) + \\
 [Q] \left([D] \{\mathbf{u}(y, t)\} \right) - [P] \left([A_0(y)] \{\mathbf{u}(y, t)\} \right) &= [P] \left(\{A_r(y, t)\} \right)
 \end{aligned}
 \quad (16)$$