

Understeer Concepts with Extensions to Four-Wheel Steer, Active Steer, and Time Transients

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ABSTRACT

An overview of existing and alternative forms of vehicle understeer/oversteer expressions is presented. New forms are derived consistent with conceptual extensions to the configurations of the vehicle's steering system, the driving mode - steady-state or transient, and the responses - path curvature or yaw velocity. Derivation of all understeer expressions is presented with a consistent use of the Ackermann reference case and the related "Ackermann vehicle" construct. The vehicle is otherwise represented in a traditional manner as a bicycle model operating in the linear range consistent with small angle approximations.

The vehicle's steering system is assumed to be more generally configured with four-wheel steer and active or steer-by-wire actuation at both axles. The actuation is assumed to allow the introduction of significant speed sensitivity to the effective overall steering ratios. Discussion is devoted to the significance of this speed sensitivity on test results and the determination of the mean reference steer angles. Expressions for understeer gradient associated with this generally configured vehicle are simplified for the more familiar case of a front-wheel steer vehicle with a passive steering system having a fixed overall steering ratio.

The driving mode can be either steady-state, as is traditionally assumed, or transient. The transients are assumed to be the result of steering transients or speed changes.

Understeer expressions are presented using traditional variables, included reference steer angle and included Ackermann steer angle, and less frequently used variables, axle reference sideslip angles and cornering compliances.

Context is given with a brief history of the development of understeer concepts, both traditional steady-state and transient, and usage of the terms "understeer" and "oversteer".

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INTRODUCTION

PURPOSE

There are four major purposes for this paper. The first two are to present a more general development of the concept of vehicle *understeer gradient* than in traditional approaches and to extend these concepts to be valid for vehicles with more generalized steering systems. The third is to use the same general approach, comparing the responses of a real vehicle to those of a reference "Ackermann vehicle", to define understeer relationships involving time transients. These would be logically related to the steady-state forms and also would be valid for vehicles with more generalized steering systems. And the fourth is to show the parallels between theory based on traditional relationships and theory associated with the *cornering compliance* concept [1]. These understeer expression, taken together, may provide a more

complete set of relationships under the heading "understeer". (Terminology and notation used in this paper are intended to closely match those in SAE J670 Vehicle Dynamics Terminology [2]. Alternative usage will be so noted. A term defined in the Terminology will be italicized the first time it is used.)

This development of understeer concepts may appeal to two different audiences for different reasons. Readers less familiar with the subject may appreciate the discussion of the history of the understeer/oversteer concept and the development of steady-state understeer expressions in both traditional and cornering compliance-based forms. Readers more familiar with the subject may appreciate the development of steady-state understeer expressions for applicability to vehicles with more generalized speed-sensitive steering systems (4WS, active steer, and steer-by-

wire), the development of understeer expressions involving time transients, and the interrelationships between the two.

The development and availability of transient understeer expressions may be more important than in the past due to the continuing introduction of active chassis systems. Besides stability control systems, production vehicles may now have active or by-wire steering systems, active *dampers*, active *stabilizer bars*, and active differentials. While many of these systems focus on modifying vehicle responses in the *nonlinear range*, they may also have significant effects in the *linear range* including relatively strong speed sensitivities. For instance, active dampers and stabilizer bars can have such effects by modifying *tire lateral load transfer distribution* as a function of vehicle speed and time.

Even though the transient understeer relations are valid only in the linear range, they may still prove to be helpful with these active/speed sensitive systems for tuning and development, performance characterization, and correlation to subjective assessments.

SCOPE

Vehicle types considered in this discussion are non-articulated passenger cars and light trucks with two *axles* and either front-wheel steer (FWS) or four-wheel steer. Either steering system could have a fixed mechanical linkage of constant or variable *overall steering ratio*, an active steering system, or a steer-by-wire system.

An active system is assumed to be comprised of a mechanical linkage with an additional actuator capable of changing displacements between the *steering wheel* and the *steering gear*. A steer-by-wire system is assumed to be comprised of the steering wheel and actuator(s) to steer the *road wheels*, either through a common linkage or independently. Actuators in either active or steer-by-wire systems are assumed to be controlled by a steering controller which could use *vehicle speed* as a control parameter. As a result, the active or steer-by-wire systems could be speed-sensitive systems and are assumed to be so in this paper. Control functions dependent on other motion variables such as *yaw velocity* or *lateral acceleration*, are treated as constituting part of the vehicle's inherent understeer/oversteer characteristics.

For generality and consistency, concept development assumes speed-sensitive active or steer-by-wire four-wheel steer unless otherwise noted. Since these general relationships for understeer gradient and included reference steer angle have a number of terms, they are simplified to the more familiar case of a FWS vehicle with fixed mechanical steering linkage having constant overall steering ratio.

Vehicle characterization follows traditional practices in most regards. The vehicle is assumed to be operating on a horizontal plane with kinematics associated with plane motion. It is characterized as a "bicycle model", compressed into its plane of symmetry, with the *vehicle reference point* at the mid-wheelbase point. Small angle approximations are

assumed and the Z-DOWN coordinate system from SAE J670 is used.

Common test methodologies for understeer gradient as well as key aspects of instrumentation and data processing are also covered. Because of the possibility of stronger speed effects on understeer gradient with speed sensitive steering, the significance of vehicle speed is covered in more depth than usual.

A key non-traditional aspect is the inclusion of time transients in new definitions of "transient understeer."

Steady-state and transient understeer concepts are presented first in a traditional form (**PART I**) and again in a form consistent with the cornering compliance concept (**Part II**).

A BRIEF HISTORY OF THE UNDERSTEER/OVERSTEER CONCEPT

This relatively brief history of the understeer/oversteer concept focuses primarily on the technological developments in chassis design and vehicle dynamics enabling the development of the concept and its first usage. These developments, in turn, rested on developments in related fields which are summarized briefly. This summary is based on three sources. The first, by Milliken and Whitcomb [3], is the first paper of five covering research in vehicle dynamics conducted by Cornell Aeronautical Laboratory (CAL) in the early 1950s. This paper presents an extensive history of vehicle dynamics at that time as influenced by developments in many fields, particularly aircraft stability and control. The second, by Segel [4], is a companion work by one of the CAL researchers and is a shorter summary. The third is a compendium of technical papers by Olley who worked at General Motors starting in 1930. These papers have been edited and annotated by Milliken and Milliken [5]. This history is a brief summary of these more in-depth histories as well as a limited summary of more recent work associated with concepts of transient understeer.

The earliest pertinent technology, from the late 18th century and early 19th century, is probably that credited to Ackermann. This is related to turning and steering geometry of four-wheeled vehicles and focuses on keeping carriage wheels from sliding sideways in turns (see **GENERAL CONCEPTS**, ACKERMANN CASE, "ACKERMANN VEHICLE"). In the late 18th century, there were pertinent developments in the analysis of the dynamics and stability linear system, such as Routh's stability criterion. The invention and development of airplanes brought renewed interest in control and stability issues. The Wright brothers correctly felt that control and stability issues were critical and invented coordinated wing warping and rudder control for turning. This allowed them to achieve controlled, powered flight before others. In the design of the Flyer, they opted for minimal stability, relying on the pilot for the remainder in return for more control. This design issue remained important in automotive design.

Chassis design in the early 20th century was cut-and-try in conjunction with subjective evaluation. There was adherence to the Ackermann paradigm that the wheels traveled within their own planes and good handling was thought to depend on correct steering geometry. While the term “understeer” was not in use, designers were aware of mechanisms which altered handling quality, primarily *roll steer*.

An early automotive dynamics problem was *shimmy*, which received much attention. In 1925, Broulhiet [6] published his research on this subject in which he proposed the concept of *slip angle*. As one would expect, there were soon tire tests involving imposed slip angle. These were performed on drums and were the first tire force and moment tests. The first results were published by German researchers in 1931 [7]. Also in 1931, Maurice Olley, of General Motors, requested that similar tests be performed by Goodyear and U. S. Rubber. These results appeared in 1935 [8] and 1939 [9]. After this, slip angle effects became understood as part of the mechanism of vehicle understeer¹.

Maurice Olley was hired in 1930 by General Motors to be a chassis engineer at Cadillac. Olley had worked at Rolls Royce in a number of engineering disciplines: machine tool design, machine design, engine design, and automotive chassis design. With Cadillac, Olley worked at the new GM Proving Ground in Milford and concerned himself with improving ride, steering (including shimmy), and handling and working in the disciplines of design, laboratory and road testing, and theory. He studied the new force and moment data and developed his understanding of understeer and oversteer effects. Milliken and Milliken's text [5] shows that he had a thorough understanding of understeer contributions including slip angle effects, roll steer, roll camber, compliance effects, and tire lateral load transfer. He also invented and developed skid pad tests and used the relative longitudinal positions of tire force resultant (*neutral steer line*) and center of gravity in a qualitative definition of understeer, neutral steer, and oversteer.

Olley's notes, originally internal GM reports, show his use of these terms starting in 1930. Usage is in the context of soft front springs, tire loading and tire properties (slip angle effects) and handling attributes more generally. The first published paper using these terms was presented by Olley to The Institution of Automobile Engineers in London 1938 [10]. Maurice Olley retired from Chevrolet in 1955 but returned in about 1960 to write his Notes on Suspension for internal GM usage, comprising much of Milliken and Milliken's text.

In the 1950s, the terms understeer, neutral steer, and oversteer were used more universally and appear in the first SAE Vehicle Dynamics Terminology, J670a, in about 1957.

This history should not imply that the researchers cited above and other automotive engineers limited their focus to

steady-state steering responses. Olley and his colleagues devoted considerable effort to understanding transient responses [5]. And it comprised a considerable portion of the CAL research [3].

A non-exhaustive review of the use of “dynamic understeer” and “transient understeer” shows an increase in usage in the recent years. Both terms are widely used in discussions of race car design and tuning but without rigorous definition, to the author's knowledge. In a 2007 paper Renfro, Semones, and Roberts [11] propose a definition and formulation for transient understeer. They also provide a short history of developments related to this concept, cited here. Olley [5] and Dixon [12] studied the relationship of yaw moment of inertia, wheelbase, and center of gravity longitudinal position on transient responses. Stonex [13] proposed a transient understeer concept defined by the ratio of Ackermann yaw velocity to actual yaw velocity, ω_{ZA}/ω_Z (terms from this paper, see SIDESLIP ANGLES, INCLUDED ACKERMANN STEER ANGLE and ACKERMANN CASE, “ACKERMANN VEHICLE” below). Blundell [14] also proposed this metric. And Ellis [15] also studied the significance of the difference between the Ackermann and actual yaw velocities.

Four other papers, not cited by Renfro, Semones, and Roberts are pertinent here. Bergman [16] defined understeer and oversteer in steady-state and transient conditions in a number of different scenarios. Garrott and Forkenbrock [17] proposed a plan to define transient oversteer. SAE Vehicle Dynamics Terminology [2] defines dynamic understeer/oversteer. And Sill and Ayalew use frequency domain analysis to define a transient understeer metric as it relates to on-center handling [18].

GENERAL CONCEPTS

OVERVIEW

Relationships for *path radius*, *path curvature*, yaw velocity, and lateral acceleration can be defined based on plane motion kinematics of rigid bodies developed in dynamics texts. These relationships are presented and used without proof. *Sideslip angles*, *reference steer angles*, and *axle reference sideslip angles* are defined. Finally, the Ackermann case and the concept of an “Ackermann vehicle” are presented.

Since the definitions of key angles in **GENERAL CONCEPTS** are closely related and can be easily confused, [Table A1](#), in [Appendix A](#), is provided as a reference for the reader. For brevity, the angle definitions are not rigorous. But, each is defined rigorously in the indicated section.

All figures have been drawn with color for maximum value with color publishing. Aside from reference lines and text in black, solid blue lines are used for the real vehicle case

¹ Today, it may be tempting to say that chassis engineers missed the obvious for a very long time. But, it seems plausible that slip angle effects were not a significant contribution to understeer on some cars of this era. Such cars would have had tires of the same construction, pressure, and loading on both axles and suspensions with similar roll camber and lateral force camber properties, specifically solid axles. For these cars, slip angle effects would have increased cornering compliances but would have had small effects on understeer.

while heavier dashed red lines are used for the Ackermann case (see ACKERMANN CASE, "ACKERMANN VEHICLE" below). Line weight and line type distinguish the two cases in grayscale. Figures 1 and 2 depict only the real vehicle case, Figure 5 depicts only the Ackermann vehicle case, and Figures 3, 4, 6, and 7 depict both.

SIDESLIP ANGLES, INCLUDED ACKERMANN STEER ANGLE

Figure 1 shows a very general diagram of the real vehicle bicycle model with the *horizontal velocity* vector V_H at the vehicle reference point and a path radius R to the vehicle reference point. The two bicycle model tires are represented by the shaded rectangles for some added realism. But, their respective *steer angles* are not shown since they are not required for subsequent definitions. Figure 1 is equally valid for a steady-state condition or for an instant in a transient.

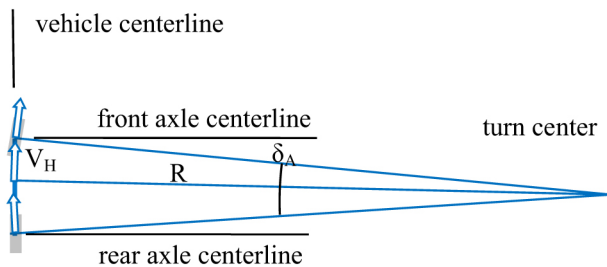


Figure 1. Turning geometry

In addition to V_H , velocity vectors are also shown at the axle centerlines. Plane motion kinematics dictates that the velocity vector of any point in the vehicle is perpendicular to the radius from the turn center to that point. Figure 1 is drawn consistent with this. As drawn, none of the three vectors is aligned with the *vehicle plane of symmetry*. The angles formed are called *sideslip angles*.

As a result of the small angle assumption, these sideslip angles are small and the path radius is much larger than the *wheelbase*. So, any radius from the turn center to a point in the vehicle is negligibly different than R and any velocity magnitude in the vehicle is negligibly different than V_H . Also, the lateral acceleration of any point in the vehicle is negligibly different than that of the vehicle reference point.

Figure 2 shows the vehicle of Figure 1 in more detail allowing a clearer depiction of the sideslip angles β , β_F and β_R between the vehicle plane of symmetry and each velocity vector. β is the *vehicle sideslip angle*, β_F is the *sideslip angle at the center of the front*

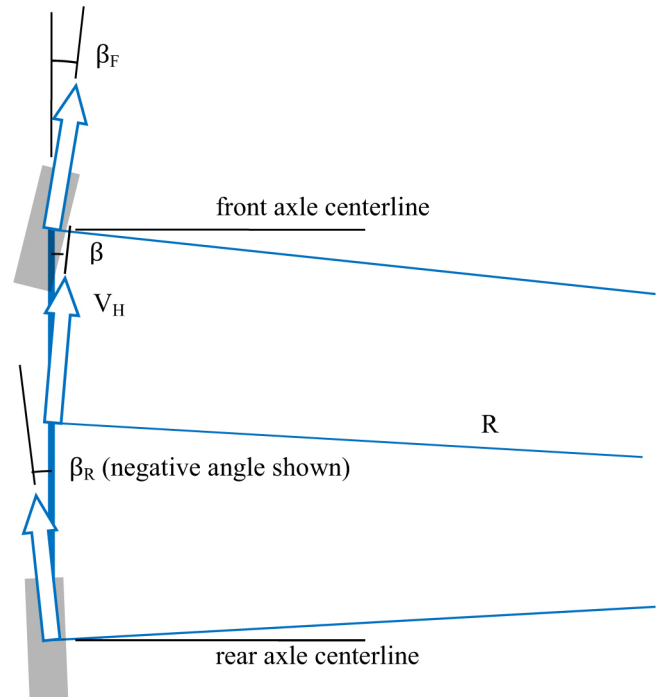


Figure 2. Sideslip Angles

axle and β_R is the *sideslip angle at the center of the rear axle*. The velocities are shown in realistic orientations to the vehicle plane of symmetry. With the SAE Z-DOWN coordinate system, β_R is a negative angle and is so marked. Again, the tires are represented by shaded rectangles, both with steer angles. A rear steer angle indicates the existence of four-wheel steer, as stated in Scope. As shown, the front steer angle is positive while the rear steer angle is negative.

Referring to Figure 1, the included angle at the turn center, δ_A , is the *included Ackermann steer angle* and is the sum of the magnitudes of β_F and β_R shown in Figure 2. Observing the signs of these sideslip angles, this angle is

$$\delta_A = \beta_F + (-\beta_R) \quad (1)$$

Small angle approximations dictate that the sine and tangent of a small angle equal the angle itself, expressed in radians. Using this, the path radius R is

$$R = \frac{L}{\delta_A} \quad (2)$$

The path curvature is the reciprocal of the path radius

$$\kappa = \frac{\delta_A}{L} \quad (3)$$

Since the horizontal velocity is the product of the path radius and yaw velocity, the yaw velocity is

$$\omega_z = \frac{v_H}{R} \quad (4)$$

Finally, the lateral acceleration of the vehicle reference point is

$$a_y = \frac{v_H^2}{R} \quad (5)$$

REFERENCE STEER ANGLES, INCLUDED REFERENCE STEER ANGLE

Assuming the general case of speed-dependent steering, each steerable *wheel* has a steer angle dependent on the *steering-wheel angle* and vehicle speed. In the absence of *suspension roll* and tire forces and moments (and thus suspension and steering deflections), these angles are called reference steer angles. The average of the reference steer angles of a pair of wheels on an axle is called the *mean reference steer angle*. In slight variance with the nomenclature in SAE J670, they will be represented by δ_{REF-F} and δ_{REF-R} . The mean reference steer angle for the wheels of a non-steered axle is zero.

Analogous to the included Ackermann steer angle, Eq. (1), the included angle associated with the mean reference steer angles is the *included reference steer angle*, δ_{REF}

$$\delta_{REF} = \delta_{REF-F} + (-\delta_{REF-R}) \quad (6)$$

Figure 3, similar to Figure 2, shows the mean reference steer angles. For added clarity, the velocity vector at the vehicle reference point, the sideslip angles, and the axle centerlines have been deleted. The mean reference steer angles are dashed (red) to represent the fact that they are associated with the Ackermann case and not real, physical angles but constructs from the steering-wheel angle and vehicle speed. As drawn, the front-axle mean reference steer angle is positive and the rear is negative.

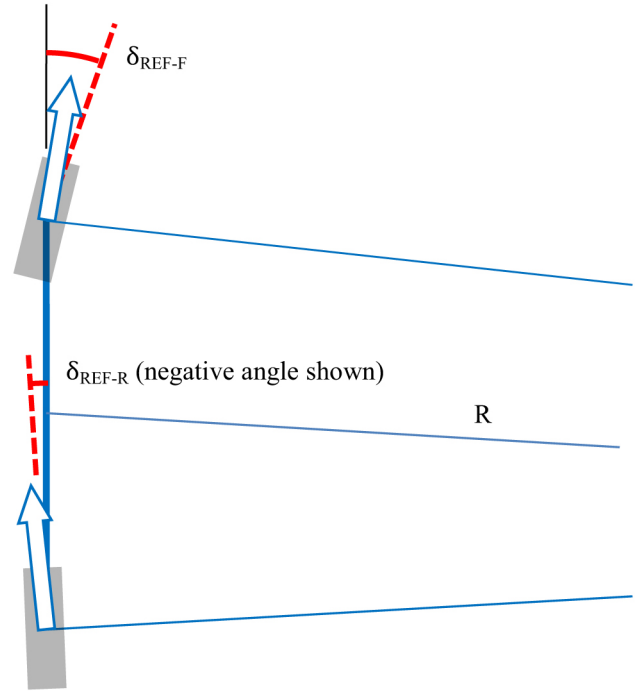


Figure 3. Mean reference steer angles

AXLE REFERENCE SIDESLIP ANGLES, CORNERING COMPLIANCE CONCEPT

The last angles to be defined are the axle reference sideslip angles. Analogous to the sideslip angles at front and rear axle centerlines (β_F and β_R), but with different references, the axle reference sideslip angles are the angles between the velocity vectors at the front and rear axle centerlines and the mean reference steer angles. Figure 4 shows these angles, σ_F and σ_R . Like Figure 3, Figure 4 has been pared down to its simplest form and the mean reference steer angles are again shown but no longer labeled. The axle reference sideslip angle for each axle is the difference between the sideslip angle and the mean reference steer angle. The front axle reference sideslip angle is

$$\sigma_F = \beta_F - \delta_{REF-F} \quad (7)$$

And the rear axle reference sideslip angle is

$$\sigma_R = \beta_R - \delta_{REF-R} \quad (8)$$

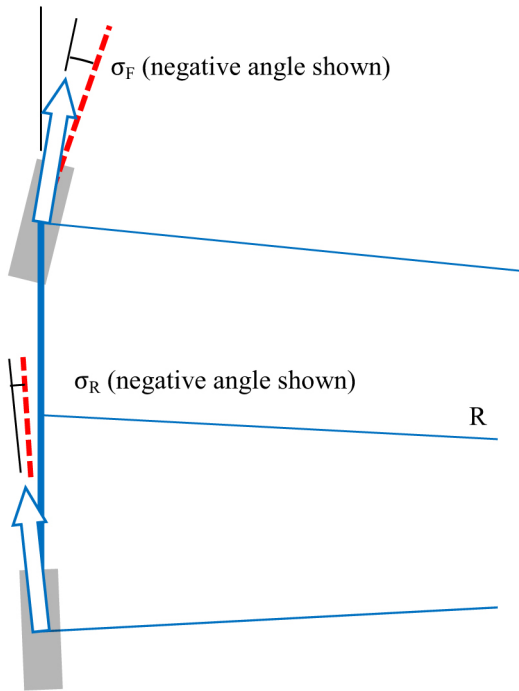


Figure 4. Axle reference sideslip angles

As drawn, the velocity vectors are oriented appropriately, relative to the mean reference steer angles, for a right hand turn. With the SAE Z-DOWN sign convention, both σ_F and σ_R are negative.

Cornering compliances are the rates of change of the magnitudes of σ_F and σ_R with lateral acceleration at a *steady-state equilibrium* condition. Since σ_F and σ_R are negative when lateral acceleration is positive, positive cornering compliances are

$$D_F = -\frac{d\sigma_F}{da_Y} \quad (9)$$

and

$$D_R = -\frac{d\sigma_R}{da_Y} \quad (10)$$

The cornering compliances in Eqs. (9) and (10) are signed for a right hand turn and would be negative in a left hand turn. But, common usage is to treat them as positive quantities for either turn direction.

For most vehicles, axle reference sideslip angle gradients approximate tire slip angle gradients. So, cornering

compliances are comprised largely of tire slip angle effects, along with smaller contributions from suspension and steering kinematic and compliance effects. The largest of these effects is typically the *compliance steer* of a steered axle.

Cornering compliance may be defined at any lateral acceleration. It is generally constant at low and moderate lateral accelerations and rises at higher lateral accelerations. This follows the trend of tire slip angle gradient. Without qualification, the term “cornering compliance” implies the low lateral acceleration, constant value.

ACKERMANN CASE, “ACKERMANN VEHICLE”

In vehicle dynamics and vehicle design, the terms *Ackermann geometry*² and Ackermann correction are associated with steering linkage design to minimize tire slip angles in low speed maneuvering. In low speed maneuvering or very low lateral acceleration cornering, a number of special conditions exist:

- Lateral acceleration is negligible. So, suspension roll and kinematic steer effects (roll steer) are negligible.
- With or without correct Ackermann geometry, net axle tire forces and moments are negligible so that compliance steer effects are also negligible.
- With no compliance steer effects, the average steer angle for a pair of wheels on an axle equals the mean reference steer angle for that axle.
- With negligible net axle forces and moments, any slip angles for a pair of tires on an axle are symmetric about the vehicle plane of symmetry. So, the sideslip angle at the center of an axle equals the average steer angle for the pair of wheels on the axle.
- For each axle, the sideslip angle, average steer angle, and mean reference steer angle are the same. And, with Eqs. (7) and (8), the axle reference sideslip angles are zero.

The core ideas above, especially that axle reference sideslip angles are zero at zero lateral acceleration, can be defined as a special case. And this special case of zero axle reference sideslip angles can be extended to higher lateral accelerations even though such a condition is not realizable in a real physical sense. In this paper, this is termed the Ackermann case and is used as a reference for the development of understeer concepts.

Figure 5 shows the Ackermann case. It is adapted from Figure 3 with the following changes:

² Recent scholarship by King-Hele [19] indicates that this steering mechanism should be credited to Erasmus Darwin, Charles' grandfather. Papers from The Royal Society indicate that, in about 1758, Darwin conceived of a new steering mechanism for carriages in which the wheels pivoted about axes at the ends of the front axle rather than having the entire axle steer about a single pivot at its center. Further, a four-bar linkage provided kinematic properties such that the front wheels turned about a common turn center on the rear axle centerline. Darwin did not patent this invention but did have two of his own carriages constructed incorporating the design. A handful of others were constructed, one of which was shown to The Royal Society, which endorsed the design in writing. But, Darwin's invention did not see widespread adoption and was reinvented about fifty years later in Bavaria by George Lankensperger. An acquaintance, Rudolph Ackermann, offered to patent the idea in England which he did in 1818. While crediting Lankensperger in the patent, Ackermann's name was in the title and so the basic ideas have come to be referred to with phrases incorporating his name.