

Physical-Based Algorithms for Interpolation and Extrapolation of Turbocharger Data Maps

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ABSTRACT

Data maps are easy to put in place and require very low calculation time. As a consequence they are often valued over fully physic-based models. This is particularly true when it is question of turbochargers. However, even if these maps are directly provided by the manufacturer, they usually do not cover the entire engine operating range and are poorly discretized. That's why before implementing them into any model they need to be interpolated and extrapolated.

This paper introduces a new interpolation/extrapolation method based on the idea of integrating more physics into the widespread Jensen and Kristensen's method [6]. It essentially relies on the turbo machinery equation analysis performed by Martin during his PhD thesis [9, 10, 11] and the interpolation and extrapolation strategies that he proposed. In most cases the new strategies presented in this paper rely on improvements of the models he proposed. However the major issue here is to associate to each model a robust algorithm to obtain the fully extrapolated data maps from the raw manufacturer's data.

The major outcomes of the study are: - new models for compression ratio and turbine flow rate extrapolations and - robust and easy to implement algorithms for interpolation and extrapolation to low rotational speeds. These address single stage turbocharging configurations with fixed geometry turbines (in the case of variable geometry turbines, taking each position as a different fixed geometry turbine allow to use the same algorithms).

KEY WORDS: Engine Control, 0D model, Turbocharger, Compressor, Turbine, Data Map, Extrapolation, Interpolation

CITATION: El Hadeif, J., Colin, G., Chamaillard, Y. and Talon, V., "Physical-Based Algorithms for Interpolation and Extrapolation of Turbocharger Data Maps," *SAE Int. J. Engines* 5(2):2012, doi:10.4271/2012-01-0434.

INTRODUCTION

Beyond the marketing that they represent, the reduction of fuel consumption and CO₂ emissions are mainly motivated by the always more drastic pollutant emission standards [5]. One of the most widespread ways investigated by car manufacturers, called downsizing, consists in reducing the engine displacement in order to increase the engine global efficiency. In this context, adding a turbocharger to the air path allows to maintain the equivalent performances. Another major issue for automotive industry is to reduce the development costs. A popular way to achieve this objective uses 0D modeling, either for simulation and/or model-based

control. For that matter, the latter is a very interesting way to deal with the increasing complication of engines technical definition.

In this case, characterizing the turbocharger behavior through the entire operating range becomes a cornerstone. In general practice, the supplier's data maps only cover the higher turbocharger speeds (at low speeds, the flow rate sensor accuracy is not sufficient). Unfortunately, the lowest speeds are predominant when it is question of normalized driving cycles. One must also take into account that these data maps, when supplied by the manufacturer, are poorly discretized: the few operating points provided do not contain much information which considerably complicates any

interpolation/extrapolation initiative. As a consequence, the development of a robust strategy to interpolate and extrapolate the manufacturer's data map can not only rely on mathematics but also physics in order to achieve accurate results.

An analysis of the general turbo machinery equations [10] has made possible to complete or develop new models for compression and expansion ratio as well as for compressor and turbine efficiencies. In this study, a particular attention has been paid to use model parameters whose trends are known through the whole operating range. Combined with efficient optimization algorithms, it makes easier the interpolation and extrapolation. This has been ensured both in the models development and the optimization techniques that has been used and demonstrated on more than ten turbocharger sets.

The paper is split into two sections dedicated to the compressor data maps (pressure ratio and isentropic efficiency) and to the turbine data maps (flow rate and isentropic efficiency). In each sub-section, the model is first developed then the details for extrapolation are provided and finally some results are presented. In each case, the reader will find a summary of the strategy in a dedicated sub-section.

MOTIVATIONS

As explained above, one of the principal interests for turbocharger data maps arise from engine 0D modeling [15], [11]. The compressor and turbine flow rate are usually read from the data maps $\{Q_{comp}, P_{ic}, \omega_t\}$ and $\{Q_{turb}, P_{it}, \omega_t\}$ in order to calculate the turbocharger rotational speed. In the case where frictions are disregarded [7], it is given by:

$$\dot{\omega}_t = \frac{1}{I} (C_{turb} - C_{comp}) \quad (1)$$

where ω_t is the turbocharger rotational speed, I is the polar moment of inertia, C_{comp} and C_{turb} are respectively the compressor and the turbine torque:

$$C_{comp} = \frac{Q_{comp} C_p (T_{inC} - T_{outC})}{\omega_t} \quad (2)$$

$$C_{turb} = \frac{Q_{turb} C_p (T_{inT} - T_{outT})}{\omega_t} \quad (3)$$

where Q_{comp} and Q_{turb} are the compressor and turbine flow rate, C_p is the specific heat at constant pressure and T_{inC} , T_{outC} , T_{inT} , T_{outT} are respectively the inlet and outlet temperature for the compressor and turbine (see Figure 1).

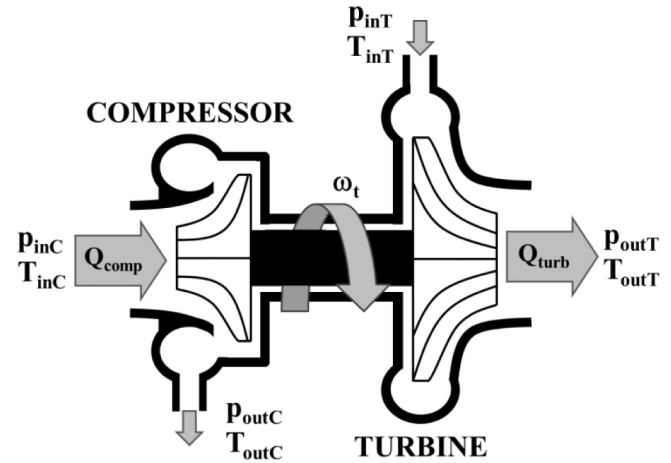


Figure 1. Turbocharger side scheme.

The temperatures are computed through algebraic relations which depend on the compressor and turbine efficiencies:

$$T_{outC} = T_{inC} \left(\frac{P_{ic}^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{comp}} + 1 \right) \quad (4)$$

$$T_{outT} = T_{inT} \left(1 - \eta_{turb} \left(1 - \left(\frac{1}{P_{it}} \right)^{\frac{\gamma-1}{\gamma}} \right) \right) \quad (5)$$

where P_{ic} is the total pressure ratio across the compressor (outlet pressure over inlet pressure), γ is the ratio of specific heats, P_{it} is the pressure ratio across the turbine (inlet total pressure over outlet static pressure), η_{comp} and η_{turb} are the compressor and turbine isentropic efficiency.

Usually the turbocharger manufacturer provides the four data maps f_i , $i = \{1, 2, 3, 4\}$ that are needed to run this model i.e.:

$$P_{ic} = f_1(\omega_t, Q_{comp}) \quad (6)$$

$$\eta_{comp} = f_2(\omega_t, Q_{comp}) \quad (7)$$

$$Q_{turb} = f_3(\omega_t, P_{it}) \quad (8)$$

$$\eta_{turb} = f_4(\omega_t, P_{it}) \quad (9)$$

Unfortunately, as suggested before, they only contain few points that do not cover the entire operating range that is needed to simulate a classical driving simulation. As a consequence, one must extrapolate and interpolate these four data maps.

The most popular way to tackle this, surely relies on the Jensen and Kristensen empirical mean value model published in 1991[6] and [1]. It can be widely found in literature [12] and is used in many commercial codes. This method presents the advantage to be really easy to implement. Unfortunately, since it does not rely on any physics but only local empirical observations, its robustness on extrapolated zone can be unsatisfactory [9] and [12]. This has motivated Martin to look for a physical way to extrapolate these data maps. The issue was to integrate physics into Jensen and Kristensen mean value model. Martin's work is the starting point of the research work presented here; some of the models presented here are direct improvement of his models.

COMPRESSOR

In this section, the issue is to present a new interpolation and extrapolation strategy for both the compressor pressure ratio and isentropic efficiency data maps. The first sub-section is dedicated to the pressure ratio and details successively the model, the extrapolation strategy and the results obtained during the study. The second sub-section concerns the isentropic efficiency and is split according to exactly the same scheme.

COMPRESSOR PRESSURE RATIO

This sub-section starts with a short presentation of the mostly used mean value model for compressor extrapolation. For more details, one can refer to [6] and [12]. It is at the origin of Martin's recent extrapolation techniques [9] and has been developed by Jensen and Kristensen.

Model

Jensen and Kristensen's work widely relies on Winkler PhD Thesis [18] and two dimensionless parameters that he introduced. They are known as the head parameter Ψ and the dimensionless flow rate Φ :

$$\Psi = \frac{c_p T_{inc} \left[(P_{ic})^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\frac{1}{2} U_c^2} \quad (10)$$

$$\Phi = \frac{Q_{comp}}{\rho_4 \pi D_c^2 U_c} \quad (11)$$

where ρ is the fluid density, D_c is the wheel diameter and U_c is the blade tip speed that can be written as:

$$U_c = \frac{\pi}{60} D_c \omega_t \quad (12)$$

Jensen and Kristensen extrapolation strategy then relies on a second expression of the head parameter Ψ which is easy to extrapolate and at the origin of the new model presented here:

$$\Psi = \frac{k_1 + k_2 M_{inl} + k_3 \Phi + k_4 M_{inl} \Phi}{k_5 + k_6 M_{inl} - \Phi} \quad (13)$$

where k_i , $i = \{1, 2, \dots, 6\}$ are constants to be identified with manufacturer's data and M_{inl} is the flow mach number defined as:

$$M_{inl} = \frac{U_c}{\sqrt{\gamma R T_{inc}}} \quad (14)$$

where R is the fluid gas constant.

Unfortunately, this model only gives good performance for interpolation at middle rotational speed (as suggested earlier the real issue is the extrapolation to low speeds). In fact, at high rotational speeds, the flow Mach number usually reaches such values that new fluid characteristics should be taken into consideration [6] (which would imply new k_i coefficients). It prevents the use of a unique set of coefficients for the whole speed range.

In order to tackle this lack of performances, Martin has proposed to treat each iso-speed separately [10]. A set of k_i coefficients must be found for each of them. The extrapolation then consists in extending their trend to low rotational speeds. This simple technique already presents good improvements but a more radical strategy has been completely developed here in order to go further on this path. In fact, if iso-speeds are considered separately, the initial model can be considerably simplified. At constant speed the flow Mach number M_{inl} become constant and the reduction trivial:

$$\Psi = \frac{A + B\Phi}{C - \Phi} \quad (15)$$

where A , B and C are the parameters of the new model to be identified.

Note that Eriksson has proposed a slightly variation of Jensen/Kristensen's model which suggested that it exists a second order relationship between the head parameter and the flow Mach number [3]:

$$\Psi = \frac{k_1 + k_2 M_{inl} + k_7 M_{inl}^2 + k_3 \Phi + k_4 M_{inl} \Phi + k_8 M_{inl}^2 \Phi}{k_5 + k_6 M_{inl} + k_9 M_{inl}^2 - \Phi} \quad (16)$$

The same development suggests that the relationship between our model parameters A , B and C and the turbocharger rotational speed might not be of the first order but of second order:

$$A = \alpha_A \cdot \omega_t^2 + \beta_A \cdot \omega_t + \gamma_A \quad (17)$$

$$B = \alpha_B \cdot \omega_t^2 + \beta_B \cdot \omega_t + \gamma_B \quad (18)$$

$$C = \alpha_C \cdot \omega_t^2 + \beta_C \cdot \omega_t + \gamma_C \quad (19)$$

It has been investigated on more than ten compressors that Eriksson's approach gives better results for the lowest iso-speed extrapolation. Because of these empirical observations, it has been decided to use second order functions to extrapolate A , B and C (see Figure 2).

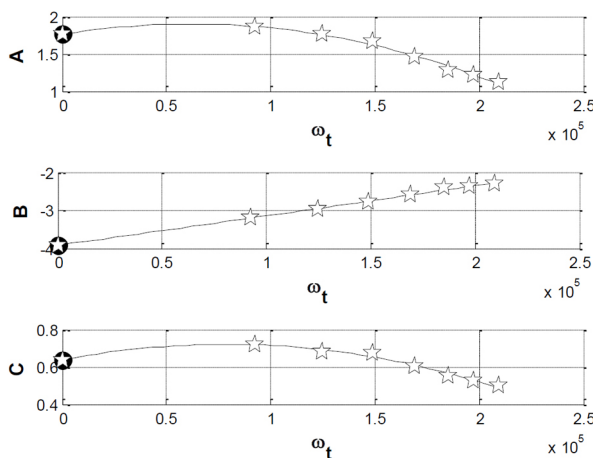


Figure 2. Model parameters A , B and C plotted versus turbocharger rotational speed ω_t . A second order polynomial is fitted (solid lines) through the reference points (white stars). The zero velocity point (black circled star) must be note down for later use.

This new model essentially presents two advantages:

- Treating each iso-speed separately allows taking account of the flow nature fluctuations through the whole operating range. Contrary to Jensen based model, an homogeneous fit is obtained for every rotational speeds.
- The new three parameters model is particularly adapted to the use of manufacturer's data maps. In fact, three parameters are ideal to be identified with 5 to 6 experimental data points that manufacturers usually provide for each iso-speed. Moreover, this identification can now be led with any simple gradient descent algorithm (with Jensen or Martin's model, local minima compelled to the use of global optimization algorithm).

Extrapolation

On Figure 2, one can see that the model does not perfectly fit the experimental data points provided by the manufacturer. This is obviously due to the hypotheses that have led to this

model but also to the measurement errors that manufacturers cannot overcome. The process investigated here to extrapolate the data to low rotational speeds presents two key steps:

- The y-intercept of each parameter is determined using a second order model as suggested by Eriksson (16) obtained with the classical Levenberg-Marquardt optimization algorithm [17],
- The interpolation is realized using a monotone piecewise cubic interpolation [4] based on cubic Hermite splines. They ensure to smooth the data map while preserving the monotonicity. In fact, it benefits from the flexibility offered by cubic spline as well as keeping the monotonicity guaranteed by the classical linear interpolation [1]. The interpolation itself keeps the model as close as possible to the supplier's data points.

This method has been applied on many compressor data maps (more than ten). The new model presents quite accurate results through the whole operating range as well as a very good robustness. Below are presented the full interpolated and extrapolated cartography obtained on a compressor (see Figure 3). The three lowest iso-speeds have been excluded from the data used for the model identification but are presented below to prove its validity.

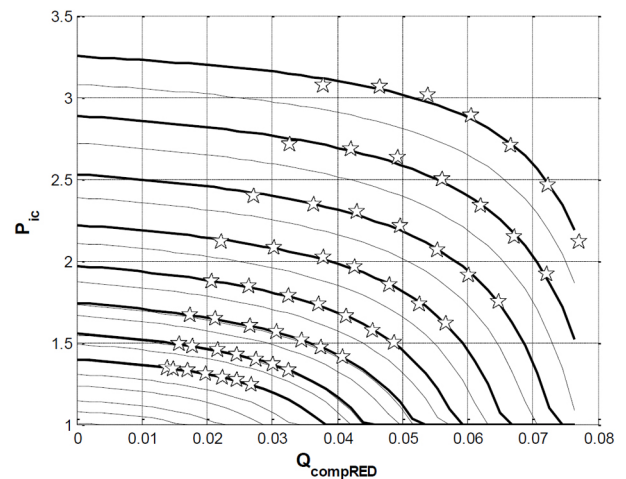


Figure 3. Compression ratio P_{ic} versus reduced mass flow rate $Q_{compRED}$. For each supplier's iso-speed, the pressure ratio is plotted (solid lines) and compared to the manufacturer's points (white stars). New iso-speeds, interpolated and extrapolated, are also presented (dash-dot lines).

Algorithm

The strategy can finally be summed up as below:

- 1-. Calculate Ψ for all the data points provided by the manufacturer (10).

2-. Treating each iso-speed separately, identify A , B and C according to the model defined in (15).

3-. Determine the best second order polynomials which fit A , B and C with respect to ω_t (17), (18) and (19).

4-. Determine the y-intercepts of the second order polynomial found at step 3, for A , B and C respectively, and add them to the values found at step 2 for zero rotational speed.

5-. Interpolate A , B and C through the entire operating range using monotone piecewise cubic interpolation.

6-. Determine Ψ through the whole operating range using the interpolated values of A , B and C and (15).

7-. Calculate P_{ic} through the entire operating range inverting (10).

Results

One major issue, when developing such a model, is to make sure that it is robust enough to be applied to a wide variety of experimental data. In this sub-section are summed up few results obtained on different compressors data maps available for this study:

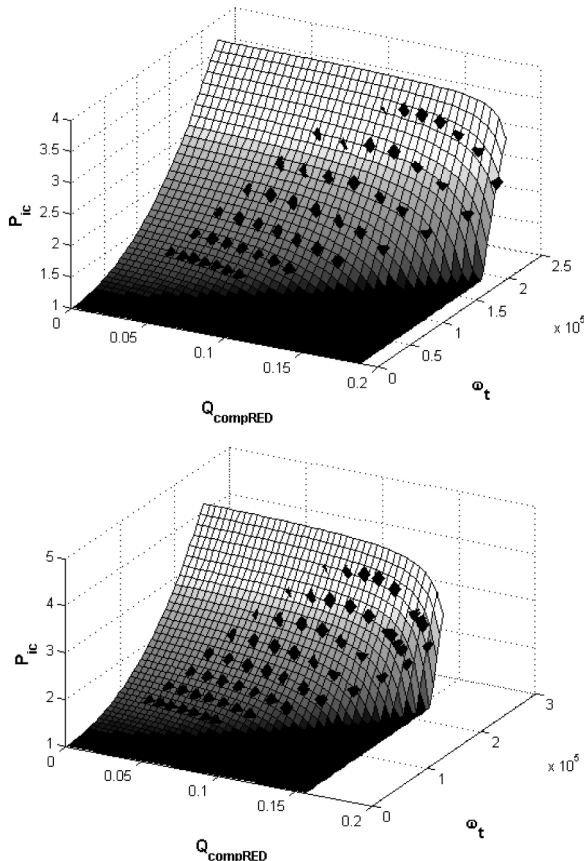


Figure 4. Extrapolated compression ratio data map. The complete compression ratio data map (colormap) is fitted without using the three smallest iso-speeds which are here plotted for validation only (black diamonds). Results are presented for three different compressors.

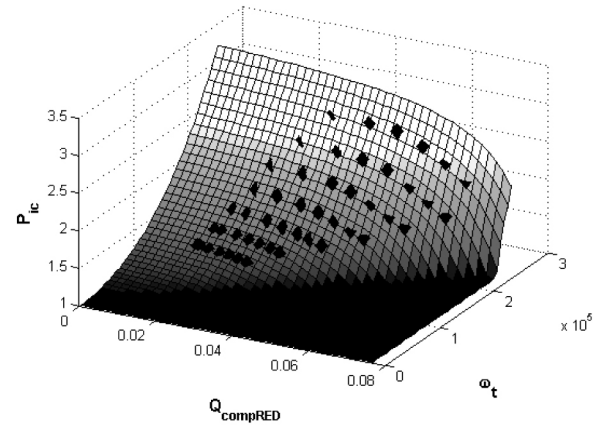


Figure 4 (cont.). Extrapolated compression ratio data map. The complete compression ratio data map (colormap) is fitted without using the three smallest iso-speeds which are here plotted for validation only (black diamonds). Results are presented for three different compressors.

COMPRESSOR ISENTROPIC EFFICIENCY

In this sub-section, the model used to interpolate and extrapolate the isentropic efficiency of the compressor is first presented. It can be divided into two intermediate steps: obtaining the specific enthalpy exchange and calculating the isentropic specific enthalpy exchange. Both sub-models are detailed below, as well as the calculation of the isentropic efficiency from these values.

Model

The isentropic efficiency of the compressor η_{comp} is defined as the ratio of the specific enthalpy exchange Δh and the isentropic specific exchange Δh_{is} [19], [20]:

$$\eta_{comp} = \frac{\Delta h_{is}}{\Delta h} \quad (20)$$

Some major improvements for the calculation of these physical quantities have been recently brought up by Martin [10]. In fact, starting from the Euler's basics equations, it has been proven that they can be calculated through simple linear relations.

Isentropic Specific Enthalpy Exchange Δh_{is}

Only little information can be found on the theoretical isentropic specific enthalpy exchange in the literature:

$$\Delta h_{is} = \left((P_{ic})^{\frac{\gamma-1}{\gamma}} - 1 \right) c_p T_{inc} \quad (21)$$

However, it is still possible to rely on the developments led by Jensen and Kristensen [6]. The definition of the head