

Physical Modeling of Shock Absorber Using Large Deflection Theory

Yuhang Chen, Konghui Guo, Yehai Yang and Ye Zhuang
Jilin University

ABSTRACT

In this paper, a shock absorber physical model is developed. Firstly, a rebound valve model which is based on its structure parameters is built through using the large deflection theory. The von Karman equations are introduced to discover the physical relationships between the load and the deflection of valve discs. An analytical solution of the von Karman equations is then deducted via perturbation method. Secondly, the flow equations and the pressure equations of the shock absorber operating are investigated. The relationship between fluid flow rate and pressure drop of rebound valve is analyzed based on the analytical solution of valve discs deflection. Thirdly, an inter-iterative process of flow rate and pressure drop is employed in order to adequately consider the influence of fluid flow on damping force. Finally, the physical model is validated by comparing the experimental data with the simulation output. The validation results show the physical model herein well matches the experiment curve. The large deflection theory is an efficient and accurate method to build rebound valves model of shock absorber.

CITATION: Chen, Y., Guo, K., Yang, Y. and Zhuang, Y., "Physical Modeling of Shock Absorber Using Large Deflection Theory," *SAE Int. J. Passeng. Cars - Mech. Syst.* 5(1):2012, doi:10.4271/2012-01-0520.

INTRODUCTION

Both the handling and ride comfort of a modern vehicle depend greatly on its shock absorbers. It is therefore obviously important to model the shock absorber properly in vehicle dynamics. However, the shock absorber is one of the most complex components to model in a vehicle due to its nonlinear characteristics. Besides, the measurement and observation on its internal operation are hard to implement.

A number of shock absorber models have been proposed by different authors. These proposed models can be basically divided into three categories: physical, nonparametric, and rheological models [1, 2].

Physical model, in which all the parameters have their physical meanings, is based on the shock absorber structure. In addition, the model equations are directly derived from the dynamics or other theories. Examples of physical models are found in [3, 4, 5, 6, 7, 8].

Nonparametric model follows the same meaning as black-box model, which may not depend on any physical structure. This modeling method builds a bridge between the inputs and outputs of the system by fitting and identifying the experimental data. The neural network based models as [9, 10] and other empirical or experimental methods as [11, 12] are located in this branch.

For rheological model, it abstracts the shock absorber as a set of dashpots, springs, friction and backlash elements. It to some extent reflects the physical characteristics via a simpler model than physical one does. Some representative references are [2] by Zacha et al., [13] by Besinger et al., and [14] by Lion and Loose.

It is difficult to say which method is the best. Nonparametric models are used commonly in control field due to its simple equations and rapid response, while physical models are more suitable to understand the shock absorber characteristics and to improve the design of shock absorber structure. The rheological method, however, is more like a tradeoff between physical and nonparametric models.

Physical modeling method, which is able to instruct shock absorber design, is investigated in this paper. The structure of a shock absorber can be improved and optimized via simulation of this method at an early development stage. In other words, if an entire analysis based on shock absorber structure can be employed before its prototype is made, then the structure parameters can be adopted and improved more efficiently and appropriately via simulation.

In this paper, an exact physical model of rebound valve, which is the key to the shock absorber physical modeling, is established based on the structure. Because the relationship between fluid flow rate and pressure drop of the rebound

valve determines the shock absorber performance directly and ultimately. Or rather, the precise solution of the rebound valve discs load with their deflection makes sense to model practically.

The large deflection theory for thin circular plate is a method of solving the relationship between valve discs load and deflection. The approach has been relatively well developed in the field of mechanics. However, only few works of shock absorber modeling introduced the large deflection theory through fitting the coefficient. This kind of application seems a nonparametric modeling rather than a physical one. The routine jobs of conducting experiments, fitting curves, and identifying parameters are needed, which is inefficient and time-consuming. Hence, it has a very limited capacity of improving a shock absorber structure.

Instead of being solved by simply fitting the coefficient, an analytical method of solving large deflection equations is developed in this paper, in order to build an entire physical model of rebound valve. An analytical solution of the large deflection equations is obtained via perturbation method. An inter-iterative process of flow rate and pressure drop is involved subsequently in order to adequately consider the influence of fluid flow on damping force. The entire shock absorber physical model is then completed by linking to other hydraulic dynamics formulas.

The structure of this paper is as follows: In section 2, the von Karman expression for the rebound valve is developed and solved via perturbation method. An analytical expression of distributed load with deflection of the valve is achieved. In section 3, the entire shock absorber physical model is completed. The developed model is validated in section 4. The conclusions are reached in section 5.

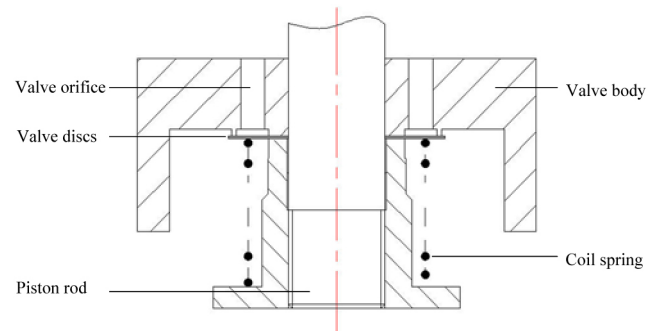
REBOUND VALVE MODELING VIA LARGE DEFLECTION THEORY

In this section, the physical model of the rebound valve is developed. The other three valves, the compression valve, the compression intake valve and the rebound intake valve are modeled in a simplified way. This is because the structures and operating principles of all these three valves are quite simple. The rebound and compression intake valves also contribute very little to the damping force, so they can be simplified as proportional check valves with large flow rate gradients. The compression valve can be seen as an orifice, since its valve discs work only as a check valve and operate without an open valve pressure. The main damping force of the shock absorber is generated by rebound valve.

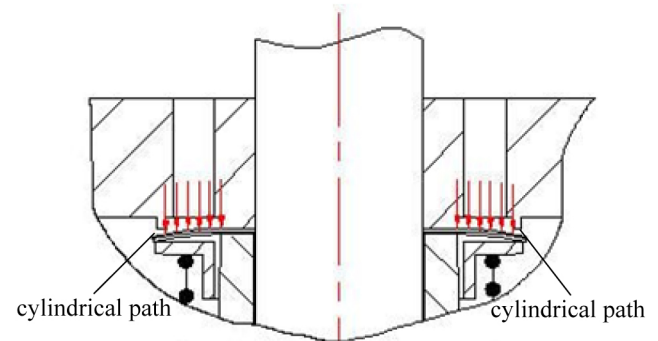
The large deflection theory is introduced to build the rebound valve model. This method mainly has two advantages in shock absorber valve modeling. First of all, it shows a high modeling precision compared with other methods such as small deflection theory, since the definition of the large deflection is more suitable for the valve disc deflection. The other reason is the model equation deduction, though, are quite complicated, the modeling process can be

quickly and easily, by virtue of the direct substitution of the appropriate parameters into these developed solution equations. However, it is very complicated and time-consuming to finish a FEM model even using FEM commercial software.

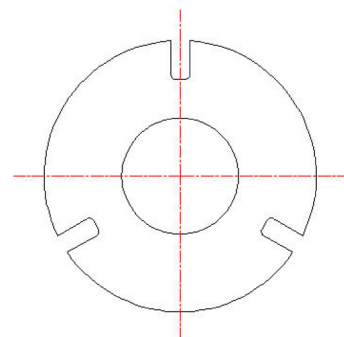
1. VALVE DISCS FORCE ANALYSIS



(a). Structure of rebound valve



(b). Cylindrical path during valve open



(c). Valve disc with notches

Figure 1. Configuration of rebound valve

Figure 1 shows the configuration of a shock absorber rebound valve. The two valve discs inner ends are fixed and outer ends are exerted a preload by a coil spring. There are three notches in the valve disc next to the piston. If the forces on the valve discs are less than the preload, the rebound valve is still closed and the fluid can only pass through these notches. Until a specified pressure greater than the preload, the valve will be propelled to deflect by the fluid. A

cylindrical path is then shaped, the fluid passes through the rebound valve using both the cylindrical path and the notches.

The rebound valve discs are exerted by three loads: the preload by coil spring, the uniform pressure at the inner part and the descending pressure at the margin by flowing fluid. The deflection of the valve discs is determined by these three loads together. To reduce the calculated difficulties, two equivalent simplifications are employed: the equivalent valve open pressure P_{open} instead of preload force; and the equivalent uniform pressure of the two loads by flowing fluid. The final resultant force on the valve discs is the P_{open} subtracted from the equivalent uniform load. Figure 2 illustrates the process of calculating the equivalent uniform pressure of the uniform and the descending pressure. The extent of the uniform pressure is L , while the descending pressure is L_1 . According to the rule that the bending moment toward discs inner ends fixed point are equal in equivalent process, the uniform equivalent pressure range $L+x$ is then calculated.

$$L+x = \sqrt{L^2 + \frac{L_1^2}{3}} + L_1 L \quad (1)$$

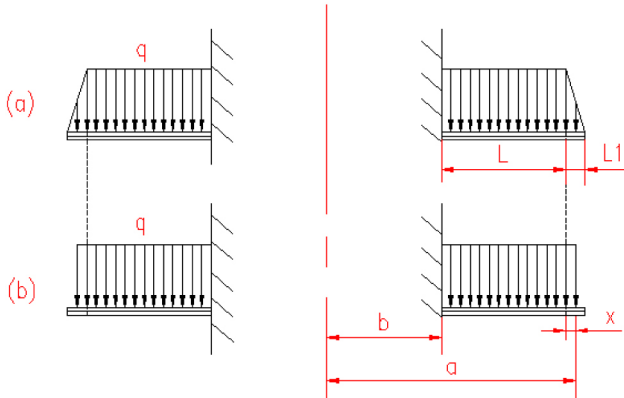


Figure 2. Force equivalent process of valve discs

2. EQUATIONS ESTABLISHMENT

For the axisymmetric valve discs which are exerted by uniform distributed pressure, the von Karman equations are established to determine the deflection of the discs [15, 16, 17, 18]:

$$\begin{cases} D \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} r \frac{d\omega}{dr} = \frac{1}{r} \frac{d}{dr} (r N_r \frac{d\omega}{dr}) + q(r) \\ r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} (r^2 N_r) + \frac{Eh}{2} (\frac{d\omega}{dr})^2 = 0 \\ N_\theta = \frac{d}{dr} (r N_r) \end{cases} \quad (2)$$

$$D = \frac{Eh^3}{12(1-\mu^2)}$$

where r -radial position; ω -the valve discs deflection; q -uniform pressure; h -the valve disc thickness; μ -Poisson's ratio; E -Young's modulus; N_r , N_θ -the radial and tangential membrane stress.

Introduce the following dimensionless variables:

$$y = \frac{r^2}{a^2} \quad (b \leq r \leq a) \quad (3)$$

$$W = \sqrt{3(1-\mu^2)} \frac{\omega}{h} \quad (4)$$

$$\varphi(y) = y \frac{dW}{dy} \quad (5)$$

$$Q = \frac{3}{4} (1-\mu^2) \sqrt{3(1-\mu^2)} \frac{qa^4}{Eh^4} \quad (6)$$

$$S(y) = 3(1-\mu^2) \frac{a^2 N_r}{Eh^3} y \quad (7)$$

$$\eta = \frac{b^2}{a^2} \quad (8)$$

Then, the dimensionless differential equations are determined:

$$\begin{cases} y^2 \frac{d^2 \varphi(y)}{dy^2} = \varphi(y) S(y) + Q(1-\eta y) y^2 \\ y^2 \frac{d^2 S(y)}{dy^2} = -\frac{1}{2} \varphi^2(y) \end{cases} \quad (9)$$

Due to its fixed inner end and free outer end, the boundary conditions are

$$\begin{cases} \varphi(y) = 0, \quad S(y) = \frac{2\eta}{1+\mu} \frac{dS(y)}{dy} & (y = \eta) \\ \varphi(y) = \frac{2}{1-\mu} \frac{d\varphi(y)}{dy}, \quad S(y) = 0 & (y = 1) \end{cases} \quad (10)$$

3. EQUATION SOLUTION BY PERTURBATION METHOD

The set (9) represents the nonlinear differential equations regarding $\varphi(y)$ and $S(y)$. The perturbation method is

introduced to find the solutions. The valve discs maximum deflection is chosen as the perturbation parameter [19, 20], i.e.

$$W_m = - \int_{\eta}^1 \frac{1}{\xi} \varphi(\xi) d\xi \quad (11)$$

Developing each dimensionless variable to the ascending power series of W_m ,

$$\varphi(y) = \sum_{i=1}^{\infty} \overline{\varphi}_i(y) W_m^{2i-1} \quad (12)$$

$$S(y) = \sum_{i=1}^{\infty} \overline{S}_i(y) W_m^{2i} \quad (13)$$

$$Q = \sum_{i=1}^{\infty} \beta_i W_m^{2i-1} \quad (14)$$

where the $\overline{\varphi}_i$, $\overline{S}_i$, β_i are undetermined coefficients. Substituting from the same power terms of W_m into (9), a new set of differential equations are established. Each undetermined coefficient is calculated by combining with the boundary conditions.

Twice-perturbed process is used in this paper, since for the von Karman equations for axisymmetric circular plate, the enough precision can be assured with the second order perturbation. Besides, higher order perturbation causes the much more complicated solutions [21].

For the linear power series of W_m , the simplified equation and boundary conditions are

$$\begin{cases} y^2 \frac{d\overline{\varphi}_1}{dy^2} = \beta_1 (1 - \frac{\eta}{y}) y^2 \\ - \int_{\eta}^1 \frac{1}{y} \overline{\varphi}_1(y) dy = 1 \end{cases} \quad (15)$$

$$\begin{cases} \overline{\varphi}_1(\eta) = 0 \\ \overline{\varphi}_1(1)(\mu - 1) + 2 \frac{d\overline{\varphi}_1}{dy} \Big|_{y=1} = 0 \end{cases} \quad (16)$$

$\overline{\varphi}_1$ and β_1 are obtained by substituting the integral results of equation (15) into equation (16).

$$\overline{\varphi}_1(y) = \frac{\beta_1 y^2}{2} + \eta \beta_1 y - \eta \beta_1 y \ln(y) + G_1 y + G_2 \quad (17)$$

$$\beta_1 = - \frac{4}{4K_1 + 8\eta + 4\eta^2 \cdot \ln(\eta) - 9\eta^2 - 4K_1\eta - 4K_2 \ln(\eta) + 1} \quad (18)$$

where G_1 , G_2 are integral constants, K_1 , K_2 are valve discs constant coefficients.

For the quadratic power series of W_m , the simplified equation and boundary conditions are

$$y^2 \frac{d\overline{S}_1}{dy^2} = - \frac{1}{2} \overline{\varphi}_1 y^2 \quad (19)$$

$$\begin{cases} \overline{S}_1(\eta) = \frac{2\eta}{1+\mu} \frac{d\overline{S}_1}{dy} \Big|_{y=1} \\ \overline{S}_1(1) = 0 \end{cases} \quad (20)$$

$\overline{S}_1$ is derived by substituting the integral results of equation (19) into equation (20).

$$\begin{aligned} S_1(y) = & - \frac{\beta_1^2}{2} \left[\frac{y^4}{48} - K_2^2 \ln(y) - \ln^2(y) (K_2 \eta y - \frac{\eta^2 y^2}{2}) \right. \\ & + y^2 \left(\frac{K_1^2}{2} + \frac{5K_1\eta}{2} + \frac{15\eta^2}{4} + \frac{K_2}{2} \right) + y^3 \left(\frac{K_1}{6} + \frac{11\eta}{36} \right) \\ & - \ln(y) \left(\frac{5\eta^2 y^2}{2} + \frac{\eta y^3}{6} + K_1 \eta y^2 - 4K_2 \eta y - 2K_1 K_2 y \right) \\ & \left. - y(2K_1 K_2 + 4K_2 \eta) \right] + \frac{\beta_1^2}{2} G_3 y + \frac{\beta_1^2}{2} G_4 \end{aligned} \quad (21)$$

where G_3 , G_4 are integral constants.

For the cubic power series of W_m , the simplified equation and boundary conditions are

$$\begin{cases} y^2 \frac{d\overline{\varphi}_2}{dy^2} = \overline{\varphi}_1 \overline{S}_1 + \beta_2 (1 - \frac{\eta}{y}) y^2 \\ - \int_{\eta}^1 \frac{1}{y} \overline{\varphi}_2(y) dy = 0 \end{cases} \quad (22)$$

$$\begin{cases} \overline{\varphi}_2(\eta) = 0 \\ \overline{\varphi}_2(1)(\mu - 1) + 2 \frac{d\overline{\varphi}_2}{dy} \Big|_{y=1} = 0 \end{cases} \quad (23)$$

$\overline{\varphi}_2$ and β_2 are obtained by substituting the integral results of equation (22) into equation (23).