

Sensitivity of Collision Simulation Results to Initial Assumptions

Bradley Heinrichs, Brian Mac Giolla Ri and Ross Hunter
MEA Forensic Engineers & Scientists

ABSTRACT

PC-Crash simulations of staged collisions require dozens of parameters describing vehicle and impact parameters. The Collision Optimizer will vary initial speeds and impact parameters to obtain a best fit to a desired end state, but vehicle parameters are left unchanged. The present paper allows these other parameters to vary in thousands of combinations, re-optimizing the solution in each to find the relationships between the previously fixed parameters and the resulting impact speeds. The results show that tire friction and vehicle inertial properties have the most influence on impact speeds. Other parameters have little influence on the results.

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BACKGROUND

PC-Crash is a time-forward kinetic simulation of vehicle motions. Vehicles are modeled as rigid bodies which interact with the ground through a spring-damper suspension and tire model (Steffan and Moser, 1996). Impacts between vehicles are calculated using an impulse exchange model satisfying restitution and friction constraints at a single point of impact.

The goal of a typical PC-Crash simulation is to find an initial velocity and impact parameters that result in a prescribed set of vehicle motions. This requires the specification of many parameters. Each vehicle's initial position, orientation, and velocity are described by 12 parameters, though usually initial velocity is aligned with vehicle heading, angular velocities are zero, and the vehicle is on level surface, reducing this number to 4. Modeling vehicle motions requires a minimum of 26 parameters describing the vehicle. A variable number of additional parameters are required to describe vehicle braking and steering at each wheel; these additional parameters may be time-varying. Modeling a single impact requires 7 parameters.

The process of finding impact conditions that result in the desired vehicle motions can be facilitated by an automated optimization process. The quality of each solution is calculated based on the difference between the simulation results and the desired vehicle rest positions (Moser et al, 1998). The PC-Crash Optimizer varies the vehicle velocities and impact parameters in order to minimize this cost function, called the Optimizer Error.

Cliff and Moser (2001) reconstructed 20 staged collisions using PC-Crash, and reported both the difference between the simulated and actual rest positions (Optimizer Error) and the difference between the simulated and actual impact velocities (velocity error). While many simulations were performed in an effort to minimize Optimizer Error (and thereby hopefully the velocity error), a single set of assumptions were used for all parameters other than impact velocities, the point of impact location, and the coefficient of restitution. All parameters describing the vehicles and tire friction were held constant.

As one example of potential variation in the simulation inputs, Goudie et al (2000) reported skidding friction coefficients between 0.75 and 0.93 in repeated tests of three sets of tires on single vehicle on a single road. Given the range of friction values that can be obtained in repeated controlled tests, the effects of its variation in collision reconstruction should be assessed.

The sensitivity of PC-Crash to some of its inputs has been explored previously. MacInnis et al (1997) found that varying the yaw moment inertia by 30% changed the best-fit vehicle impact speeds by 3% for a single simulated collision. All other assumptions were held constant.

Moser et al (2003) presented a Monte Carlo analysis of two collisions. PC-Crash was used to calculate trajectories based on randomized initial conditions and vehicle parameters. The Optimizer Error was calculated to assess the quality of each simulation, but no optimization was performed for each input parameter set. Moser did not present

a complete list of parameters varied, or most of the input ranges used. Probability distributions for impact speeds and some impact parameters were presented based on simulations that provided the 500 lowest Optimizer Error values.

In a typical analysis, an accident reconstructionist will find a solution using best estimates for each input parameter, then will vary his assumptions and find a new best fit to the data, thereby generating a range of solutions. Recent additions to PC-Crash allow for automated setting of parameters, simulation, and recording of results. This allows the PC-Crash Optimizer to be run repeatedly while varying the assumptions, mirroring the reconstructionist's process. The sensitivity of the results to each assumption can therefore be determined. The goal of this study was to determine the relative influence of all input parameters on the impact speeds found in the reconstruction of twenty collisions previously analyzed by [Cliff and Moser \(2001\)](#).

METHODS

All twenty collisions previously analyzed by [Cliff and Moser \(2001\)](#) met the following criteria:

1. Vehicles were positioned at maximum engagement at $t=0$.
2. Vehicle rest positions were known.
3. An optimized solution had already been found.

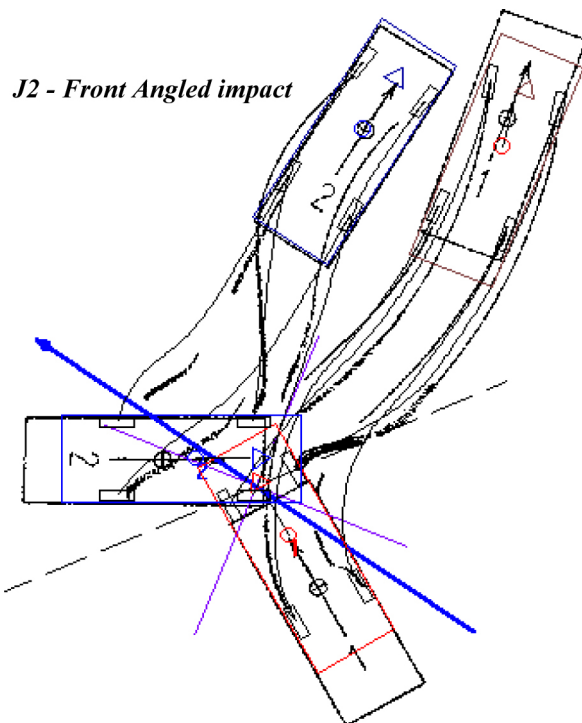


Figure 1.

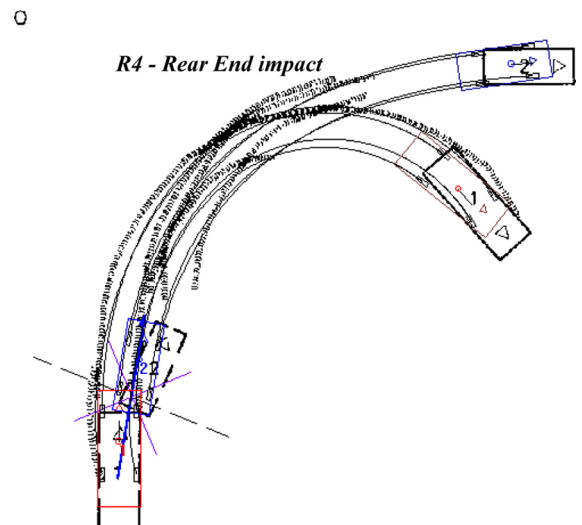
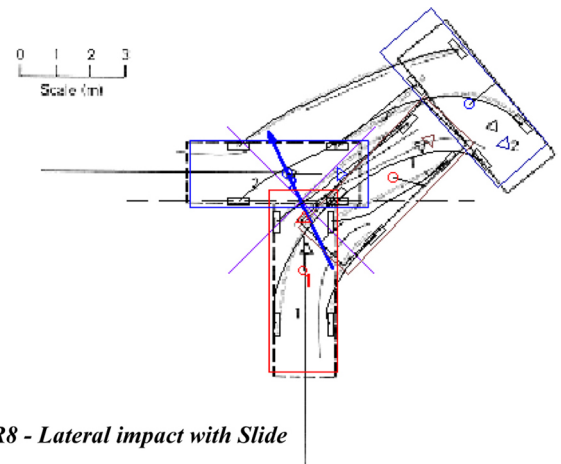
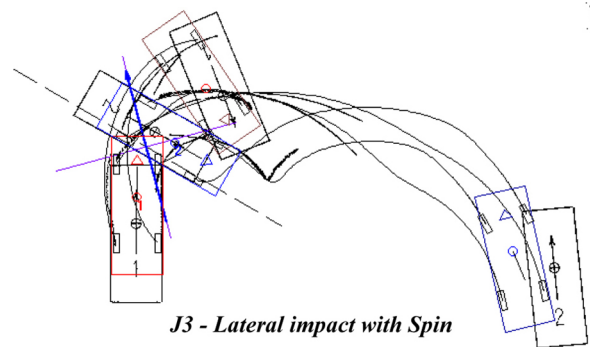


Figure 1 (cont.). Sample collisions from [Cliff and Moser \(2001\)](#). Front Angle impact (J2), Lateral Spin impact (J3), Lateral Slide impact (R8), and Rear End impact (R4). Vehicles shown at impact and rest, with simulated wheel trajectories, overlaid on measured rest positions and tire marks. Point of Impact shown at purple X with blue impulse vector.

VBA (Visual Basic for Applications) macros in Microsoft Excel 2003 were used to generate the input parameters, run the PC-Crash Optimizer, and compile the results of the sensitivity analysis through an entirely automated process. All randomization and data aggregation was done in Excel, using built-in functions.

The parameters describing basic vehicle geometry (length, width, height, wheelbase, overhangs, track widths) were assumed to be known to within 1% and were therefore held constant. Vehicle positions and orientations at impact were also held constant, as tire marks starting near the impact locations were generally available. The remaining parameters were allowed to vary according to the distributions listed in [Table 1](#). Note that impact parameters were varied by the Optimizer to find a best solution, and were therefore not part of the sensitivity analysis.

For each project file, the following steps were taken:

1. The assumptions listed in [Table 1](#) were read from the project file.
2. The simulation was reset.
3. The assumptions listed in [Table 1](#) were changed according to the random distributions listed in that table. For each initial value X , each new value X' was calculated using a uniform random variate $-1 < R < 1$ as follows:
 - a. For a percentage variation P , $X' = X(1+PR)$
 - b. For a fixed variation F , $X' = X + PR$
 - c. For a multiple variation M , $X' = XM^R$. The new assumptions were recorded.
4. The PC-Crash Optimizer was run, using the same optimization settings as initially used for that project.
5. The rest positions, orientations, impact velocities, and Optimizer Error for the optimized solution were recorded.
6. These steps were iterated 2000-3000 times.

The Optimizer did not find a good solution for all sets of assumptions. Cases where the Optimizer Error was high would be rejected by a human analyst. Only results with an Optimizer Error no more than that found by Cliff ([Table 2](#) and [Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21](#)) were used for further analysis.

Correlations between impact velocity and all assumptions were calculated. Optimizer Error was also plotted against the impact velocity error percentage for all trials. 90% confidence ellipses were calculated for the velocity error percentages.

Table 1. Assumptions with distributions

Assumption	Count	Range	Variation	Factor
Vehicle mass	1	$\pm 5\%$	Percentage	$P = 0.05$
CG to Front Axle	1	$\pm 5\%$	Percentage	$P = 0.05$
CG Height	1	$\pm 15\%$	Percentage	$P = 0.15$
Roll moment of inertia	1	$\pm 40\%$	Percentage	$P = 0.4$
Pitch moment of inertia	1	$\pm 30\%$	Percentage	$P = 0.4$
Yaw moment of inertia	1	$\pm 30\%$	Percentage	$P = 0.3$
Suspension Stiffness	4	Half to double	Multiple	$M = 2$
Suspension Damping	4	Half to double	Multiple	$M = 2$
Tire slip angle at peak force	4	$\pm 50\%$	Percentage	$P = 0.05$
Braking	4	Two thirds to three halves	Multiple	$M = 1.5$
Steer angle	1	$0.83X - 1.2X$	Multiple	$M = 1.2$
Tire-to-ground friction	1	± 0.15	Fixed	$F = 0.15$
Vehicle-to-vehicle friction	1	$\pm 50\%$	Percentage	$P = 0.5$
Coefficient of restitution for secondary impacts	1	Half to double	Multiple	$M = 2$

RESULTS

For all tests except R1 ([Table 2](#)), there were hundreds of simulations that had Optimizer Errors equal to or less than that found by Cliff. As only 6 simulations matched or bettered Cliff's Optimizer Error in test R1, the correlations calculated for this test are not useful. Tests J5 and R5 also had few simulations beating Cliff's Optimizer Error, indicating that his initial simulations were nearly optimal. In contrast, 80% of simulations for test R11 were better than Cliff.

The impact speeds for the subsets of simulations meeting the Optimizer Error cutoff had coefficients of variation between 5% and 11%. The exception is in the rear end collisions, where the stopped vehicles had a zero velocity for all simulations, and therefore had a zero coefficient of variation.

The highest correlations were between impact velocity and ground friction, in most cases accounting for 70%-90% of variance ([Table 2](#)). Vehicle mass, yaw moment of inertia and braking levels accounted for up to 51% of variance. The remaining parameters had weak correlation with optimized impact speed.

Yaw moment of inertia was most correlated with optimized impact speed for collisions involving post-impact rotation. Roll and pitch moments of inertia were only weakly correlated with impact speed, as were suspension parameters and tire slip angle.

The velocity error confidence ellipses were mostly of similar size ([Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21](#) and [Table 3](#)), indicating that the range of velocity errors percentages were similar across tests.

The nearly 45° angle to the major axis of the ellipses indicates that velocity errors (and hence the velocities) were correlated between the two vehicles.

Average velocity errors for the confidence ellipse centers for all vehicles across all tests (excluding the stationary vehicles) was -4%, the same result obtained by Cliff. Most confidence ellipses were centered near Cliff's simulation errors, except for tests J4, R9, and R10.

Table 3. 90% confidence ellipse parameters for optimized trials meeting Cliff's Optimizer Error. *a* and *b* are the major and minor axes of the confidence ellipse, with units of percentage velocity error. θ_a is the angle of the ellipse major axis. ΔV_1 and ΔV_2 are the center of the confidence ellipse.

Collision Type	Test	a	b	θ_a	ΔV_1	ΔV_2
Front Angle	J1	19%	6%	44°	-13%	-11%
	J2	18%	6%	43°	2%	-2%
	J6	19%	6%	41°	-5%	-9%
	J7	20%	6%	43°	-20%	-14%
	R11	25%	6%	36°	-33%	2%
	R12	20%	7%	39°	0%	9%
Lateral Spin	J3	22%	10%	49°	-4%	8%
	J4	24%	9%	50°	2%	-10%
	J5	23%	8%	53°	-5%	-6%
	R2	14%	9%	57°	-3%	-4%
	R6	14%	10%	24°	-2%	7%
	R7	14%	9%	32°	-5%	1%
Lateral Slide	McH	16%	3%	40°	4%	1%
	R1	28%	7%	28°	-2%	-7%
	R8	11%	7%	36°	-11%	-13%
	R9	16%	8%	25°	-8%	-5%
	R10	17%	9%	29°	-2%	-7%
Rear End	R3	15%	0%	0°	7%	0%
	R4	15%	0%	0°	0%	0%
	R5	11%	0%	0°	4%	0%
Average		18%	7%	39°	-4%	

DISCUSSION

The results show that two sets of impact speeds may give the same Optimizer Error when paired with different tire frictions. Vehicle inertia has a similar but smaller effect. Variations in all other vehicle parameters did not significantly affect the impact speeds.

The correlation between impact speeds and tire friction is expected, since impact speeds are expected to be roughly proportional to the post-impact speeds, which are in turn proportional to the square root of the tire friction. Impact speeds are also related to the mass ratio of the colliding vehicles. Since the tire friction coefficient is usually known with less precision than the vehicle masses, as reflected in the ranges used here, more of the uncertainty in impact speeds

will be due to uncertainty in the tire friction. If the correlation coefficients are normalized for the coefficients of variation in each assumption, the normalized correlations are similar for tire friction and mass.

Tire friction has a greater inherent variability than the vehicle mass or moments of inertia. In principle, a vehicle's inertial properties can be measured to within 0.1%. In contrast, repeated controlled skid tests will result in a range of at least $\pm 6\%$ (Goudie). Any simulation that is sensitive to the tire friction must have an inherent range in its results.

Some parameters that were insensitive here may be influential in other simulations. For example, cases with a rollover may be sensitive to roll moment of inertia and suspension parameters. In practice, if a simulation involves vehicle dynamics different than those occurring in these 20 collisions, then its sensitivity to each assumptions should be assessed.

Assumption variation ranges and the method of variation were chosen to maximize variation to illustrate sensitivity for this study. For example, the tire friction was selected from a uniform distribution with a total range of ± 0.3 around the current value. All of Goudie's 270 tests on three tires were within a range of ± 0.18 , and were normally distributed. The range of impact speeds in other analyses may be smaller than those presented here.

The fact that the velocity errors were mostly distributed around those found by Cliff is unsurprising, since Cliff's simulation inputs were the central values in the distributions used here. Most of the Optimizer Error vs. velocity error graphs show a flat bottom within the confidence ellipses, with steep walls around. This is also to be expected, since the graphs are essentially overlays of hundreds of individual graphs with each valley occurring at the optimal velocities for each set of input parameters. The individual graphs have much narrower valleys, but the aggregate is like a convolution of the individual graphs with the distribution of impact velocities.

The simulations performed here followed the method of [Cliff and Moser \(2001\)](#) for purposes of direct comparison. Including a constraint based on the vehicle deformation energy could reduce the observed sensitivity to tire friction. The Optimizer can include a target kinetic energy change in the form of an EES value for each vehicle (the Equivalent Energy Speed, which is closely related to the Equivalent Barrier Speed or EBS). Adding such a parameter would increase the Optimizer Error for solutions that match the required rest positions, but have kinetic energy changes that are too high or too low for the vehicle damage.

It may make sense to lump parameters for future sensitivity analysis. For example, mass and radii of gyration could be varied, rather than mass and moments of inertia. This would increase the influence of mass and decrease the influence of the radii. Similarly, suspension stiffness and damping constants could be replaced by natural frequencies and damping coefficients.

SUMMARY

The simulations of 20 staged collisions were found to be most sensitive to variations in tire friction, followed by variations in vehicle inertia. Since tire friction is an inherently variable quantity, it is important to assess the sensitivity of any collision reconstruction to this assumption. While the simulations tested here were not sensitive to large changes in other parameters, other simulations may be, and should be evaluated on a case-by-case basis if different dynamics are involved.

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Table 2. Results for each test and each vehicle. Coefficients of variation for each assumption and impact speed. Correlation coefficients for optimized impact velocity vs. each assumption. Intensity of correlation coefficient color varies with magnitude, from grey to bold red. All results are only for simulations with an Optimizer Error equal to or less than that obtained by Cliff. Total number of simulations and number of qualifying simulations given next to Cliff's Optimizer Error.

Test	Vehicle	SD/Mean	Vehicle 1	Vehicle 2	Global	Cutoff Optimizer Error	Total trials	Trials meeting cutoff
J1	1	7%	Mass	3%	3%	0.19	4.8% 2000	894
J2	2	7%	COG Position	3%	0.23	0.01	4.2% 2000	378
J6	1	6%	COG Height	8%	0.27	0.09	4.3% 2000	1229
J7	2	7%	Roll Moment of Inertia	23%	0.18	0.01	3.7% 2000	583
J7	1	9%	Pitch Moment of Inertia	17%	0.23	0.05	6.5% 3134	2523
R11	1	14%	Yaw Moment of Inertia	17%	0.08	0.05	7.1% 2728	922
R12	2	8%	COG Position	3%	0.25	0.01	9.0% 2000	621
J3	1	8%	COG Height	8%	0.10	0.04	8.6% 2000	176
J4	1	8%	Roll Moment of Inertia	23%	0.04	0.05	10.4% 1685	42
J5	1	8%	Pitch Moment of Inertia	17%	0.05	0.02	4.4% 2881	433
R2	1	6%	Yaw Moment of Inertia	17%	0.04	0.05	4.1% 3000	291
R6	1	7%	COG Position	3%	0.04	0.05	3.8% 2647	174
R7	2	5%	COG Height	8%	0.05	0.02	2.1% 2000	134
McH	1	5%	Roll Moment of Inertia	23%	0.03	0.10	3.4% 3000	6
R1	1	7%	Pitch Moment of Inertia	17%	0.02	0.03	1.8% 3000	282
R8	1	6%	Yaw Moment of Inertia	17%	0.02	0.03	2.4% 3000	113
R9	1	8%	COG Position	3%	0.02	0.03	3.4% 3000	153
R10	1	8%	COG Height	8%	0.02	0.03	2.5% 3000	1052
R3	1	6%	Roll Moment of Inertia	23%	0.02	0.03	3.7% 3000	295
R4	1	7%	Pitch Moment of Inertia	17%	0.02	0.03	2.2% 3000	67
R4	2	0%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	1	6%	COG Position	3%	0.02	0.03		
R5	2	0%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
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R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		
R5	2	0%	Pitch Moment of Inertia	17%	0.02	0.03		
R5	1	6%	Yaw Moment of Inertia	17%	0.02	0.03		
R5	2	0%	COG Position	3%	0.02	0.03		
R5	1	6%	COG Height	8%	0.02	0.03		
R5	1	6%	Roll Moment of Inertia	23%	0.02	0.03		

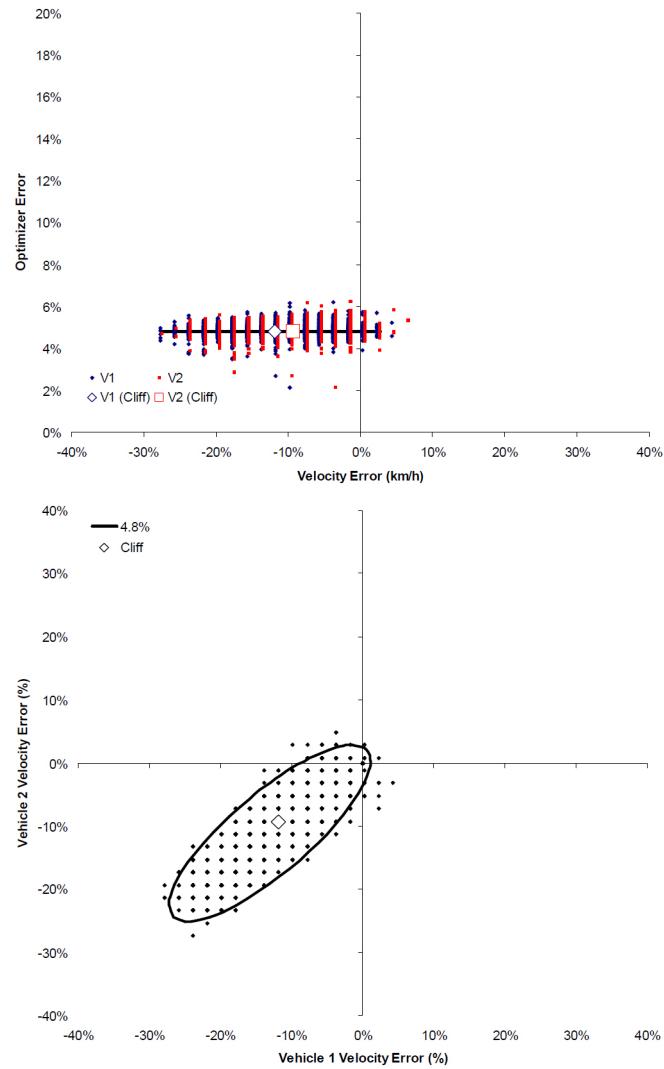


Figure 2. Test J1. Optimizer Error and velocity errors, with 90% confidence area for simulations with Cliff Optimizer Error or better (lines).

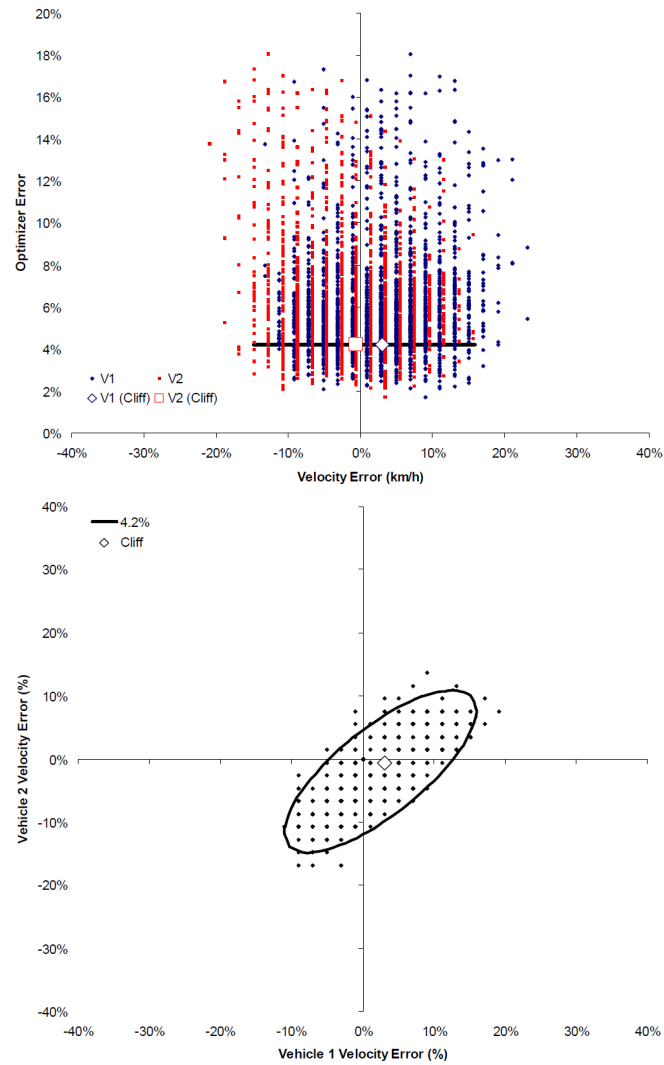


Figure 3. Test J2. Optimizer Error and velocity errors, with 90% confidence area for simulations with Cliff Optimizer Error or better (lines).