

# A Forward-Looking Stochastic Fleet Assessment Model for Analyzing the Impact of Uncertainties on Light-Duty Vehicles Fuel Use and Emissions

Parisa Bastani  
University of Cambridge-MIT

John B. Heywood  
MIT

Chris Hope  
University of Cambridge

## ABSTRACT

Transport policy research seeks to predict and substantially reduce the future transport-related greenhouse gas emissions and fuel consumption to prevent negative climate change impacts and protect the environment. However, making such predictions is made difficult due to the uncertainties associated with the anticipated developments of the technology and fuel situation in road transportation, which determine the total fuel use and emissions of the future light-duty vehicle fleet. These include uncertainties in the performance of future vehicles, fuels' emissions, availability of alternative fuels, demand, as well as market deployment of new technologies and fuels. This paper develops a methodology that quantifies the impact of uncertainty on the U.S. transport-related fuel use and emissions by introducing a stochastic technology and fleet assessment model that takes detailed technological and demand inputs. This model stochastically calculates the probability density functions for fuel use and emissions over time by propagating and calculating the effect of input uncertainties throughout fleet calculations. The fleet turn over is tracked based on the calendar year, vehicle model year, the market penetration rate of advanced technologies and fuels, and the scrappage rate of vehicles on the road. Full life-cycle emissions of fuels are projected and tracked in this model. Three carefully chosen illustrative examples are assessed using the developed methodology in this paper. The results show the probability distribution of the total light-duty vehicle fleet fuel use and emissions out to year 2050, including the mean, mode, standard deviation, spread, and confidence interval of these outputs in the near to long term. The major contributors to the fleet fuel use and emissions are also identified and ranked. These include vehicle scrappage rate, new vehicle sales, emphasis on reducing fuel consumption, and fuel consumption of naturally aspirated spark ignition vehicles. The major contributing variables and uncertainties to fuel use and emissions change over the time period 2020 to 2050. The findings in this paper indicate that uncertainties in the future fleet fuel use and emissions are significant and need to be taken into consideration when choosing amongst alternative fuel and emissions reduction pathways, in the light of their possible consequences.

**Keywords:** Stochastic modelling, decision theory, numerical decision making, transport GHG emissions and fuel use, light-duty vehicle fleet, uncertainty, STEP

**CITATION:** Bastani, P., Heywood, J. and Hope, C., "A Forward-Looking Stochastic Fleet Assessment Model for Analyzing the Impact of Uncertainties on Light-Duty Vehicles Fuel Use and Emissions," *SAE Int. J. Engines* 5(2):2012, doi: 10.4271/2012-01-0647.

## 1. INTRODUCTION

Light-duty vehicles are responsible for about 45% of the total U.S. and 10% of the world oil consumption [1]. There

are about 250 million light-duty vehicles (LDV) in the U.S., which consists of about 55% cars and 45% pick-up trucks, SUVs, and vans. The total fuel consumption from LDVs was about 504 billion litre in 2009. LDVs produced 1.108 million

metric tons of CO<sub>2</sub> emissions in 2009, which account for 19% of the total U.S. GHG emissions [1]. Road-transport fuel consumption and emissions have become an important issue on the nation's policy agenda with ever increasing concerns about impacts of climate change and energy security issues.

The U.S. Corporate Average Fuel Economy (CAFE) program has enforced fuel economy standards on light duty vehicles for more than forty years. The new regulations have mandated a significant increase in fuel economy for an average vehicle to 34.1 miles per gallon (MPG) by 2016. Stricter regulations are also under consideration for the mid and longer term. The automotive industry is expected to play an important role in reducing the total fleet fuel use through technology improvements. While most automakers claim to be taking action to reduce the carbon footprint and energy consumption of their products, whether voluntarily or under regulatory pressure, the anticipated result of these actions is unknown due to a number of uncertainties. Such uncertainties include performance of future vehicles, emissions and availability of alternative fuels, demand, and market deployment of new technologies.

In an effort to get a better picture of what the future of road transportation might look like, various vehicle fleet models have been developed and employed in scenario analysis. Specific deterministic scenarios, however, are unable to identify the range of possible outcomes that may result from choosing a particular emissions reduction pathway, as well as the associated likelihood of each outcome. This is a critical shortfall given the inherent real-world uncertainties in vehicle technology and fuel development over time. Assessing the impact of these uncertainties is further made difficult given the interactions amongst variables and that each variable affects the outputs in a different direction. Therefore, merely averaging and summing up those effects, without proper stochastic modelling of the vehicle fleet risks misrepresenting the likely future impacts of policy choices. Accordingly, this paper develops a stochastic fleet model that quantifies the impact of uncertainty on the U.S. light-duty vehicle fleet fuel use and GHG emissions. This model provides a mathematical framework to help decision makers choose among a set of alternatives, given not only their "average" outcome but the full spectrum of possible consequences given the uncertainties in the inputs. In some fields, such as climate change, the distinction between deterministic scenario analysis and stochastic modeling has been clearly established: major analyses used in IPCC assessments are based on probabilistic studies [3, 4]. However, in the transport sector the distinction remains somewhat blurry, and scenario analysis is sometimes considered as an alternative for dealing with uncertainties. Scenario analysis, however, identifies what the "average" outcome would be given a set of deterministic inputs. Simply changing the average scenario thus does not provide one with a range of possible outputs because each scenario has a different set of underlying

assumptions and is thus equivalent to taking different pathways in the real-world, let's say to reduce emissions. Further, deterministic scenarios cannot quantify the likelihood associated with the possible range of outcomes. In contrast, the aim of this paper is to provide a methodology that quantifies the uncertainties in the outputs: it provides the decision maker with a more complete picture indicating the range of possible outcomes and the probability associated with each outcome, given a chosen fuel and emissions reduction pathway.

Although scenario analysis can be useful, it does not tell decision makers what the range of possible outcomes and their associated likelihood would be if a certain pathway is taken. This has a significant practical importance as policy makers are restricted to choose one pathway that best addresses their objectives, and thus would need to understand the risk profile of the outcomes and the chances of hitting a certain target, under a consistent set of assumptions and given the input uncertainties. This paper introduces a stochastic technology and fleet assessment framework, STEP (Stochastic Transport Emissions Policy Model), which determines the integrated impact of uncertainty on the light-duty vehicle fleet fuel use and emissions, as well as the major contributing variables and their relative importance in determining the outcomes out to year 2050. The significance of the results in the context of emissions mitigation and transport policy planning are discussed.

## 2. LITERATURE REVIEW

This work builds on the vehicle fleet modelling literature in transport research and stochastic modelling techniques in the climate change literature. This study makes a methodological contribution in transport research literature and provides a policy tool to help decision makers in the transport sector analyze the future of light-duty vehicle fleet given real-world uncertainties

### 2.1. U.S. VEHICLE FLEET MODELS

A number of fleet models have been designed in the past few years to explore the future of road transport in the U.S. using scenario analysis. Some of the prominent studies include the MIT fleet model by Bandivadekar et al, LEVERS model by Yang et al, fleet scenarios by Greene and Plotkin 2011 study, NEMS model by Morrow et al, and the CGE-MARKAL Hybrid Model by Schafer and Jacoby [5,6,7,8,9,10]. The studies by Morrow, and Schafer et al are pursued in a macro-economic context, whereas the MIT model, LEVERS, and Plotkin et al are based on bottom-up technology and fuel models developed to examine in detail the fleet turn over, demand, and technology improvements and deployment, and their impact on the total fuel consumption and emissions in the next couple of decades. A number of models with engineering and technological details have also been developed by the Argonne National Laboratory, including the GREET, Autonomie, PSAT, GC

Tool, and VISION[11, 12]. These models have been drawn upon in various analyses pursued by the U.S. Department of Energy, U.S. Department of Transport, automotive industry, and academic researchers to better understand the impact of proposed CAFE standards on the future fleet and the automakers.

## 2.2. STOCHASTIC MODELLING

Stochastic modelling is widely used in climate change research. A number of integrated assessment models (IAM) have been developed worldwide to investigate the impact of uncertainty in climate change policy making. These include DICE by Nordhaus (1994), PAGE (1993) by Hope et al, ICAM by Dowlatabadi and Morgan (1993, 1995), SLICE by Kelly and Kolstad (1997a), AS/ExM by Lempert, MIT (1994), and FUND by Tol et al (1995) [13,14,15,16,17,18,19,20,21]. Some of the prominent IAMs have been cited in assessments for stochastic cost benefit analysis of various climate policies. Examples include the PAGE2002 mode, which was used in the Stern Review and the Working Group II report of the IPCC Fourth Assessment [3, 4]. IAMs are considered the only credible method for cost benefit analysis of climate change mitigation policies given the uncertainties in climate change science and economics [22]. These models inform policy makers of the range of possible outcomes of their decisions using stochastic modelling and numerical decision making.

## 2.3. REGULATIONS MODELLING

The U.S. government uses a number of models to support the rule making process for the CAFE program [23]. Some of these models draw upon the detailed technological simulations developed by Argonne National Laboratory, and others include the U.S. DOT CAFE model [12, 24]. These models are used to investigate, using scenario analysis, the impact of the proposed fuel economy standards on the fleet fuel use and emissions reduction in the short and long term.

## 3. MODEL

This section describes the methodology and structure of the STEP model.

### 3.1. METHODOLOGY

The model uses similar techniques to those developed for Integrated Assessment Modelling to provide a forward looking numerical framework to explore the impact of uncertainty on the future U.S. light-duty vehicle fleet fuel use and emissions. Numerical modelling is used to help decision making by calculating outputs based on the user's degree of belief about the likely values of the input variables. A probability distribution is thus assigned to input variables based on "up-to-date knowledge" in the engineering and LDV markets [25]. This model stochastically calculates the probability density functions for GHG emissions and fuel use outputs over time by propagating and calculating the effects

of input uncertainties throughout fleet calculations. Figure 1 shows an overview of the Stochastic Transport Emissions Policy Model stochastic (STEP). The fleet turn over is tracked based on the calendar year, vehicle model year, the market penetration rate of advanced technologies and fuels, and scrappage rate of vehicles on the road. The fuel economy of each powertrain as well as the Well-to-Tank (WTT) and Tank-to-Wheel (TTW) efficiencies of conventional and alternative fuels are tracked to account for the full life-cycle GHG emissions of fuels. Vehicle weight reduction impact is further taken into account through powertrain fuel consumption improvements over time. The main features of the STEP model are described in section 3.2 and the equations details can be found in the Appendix.

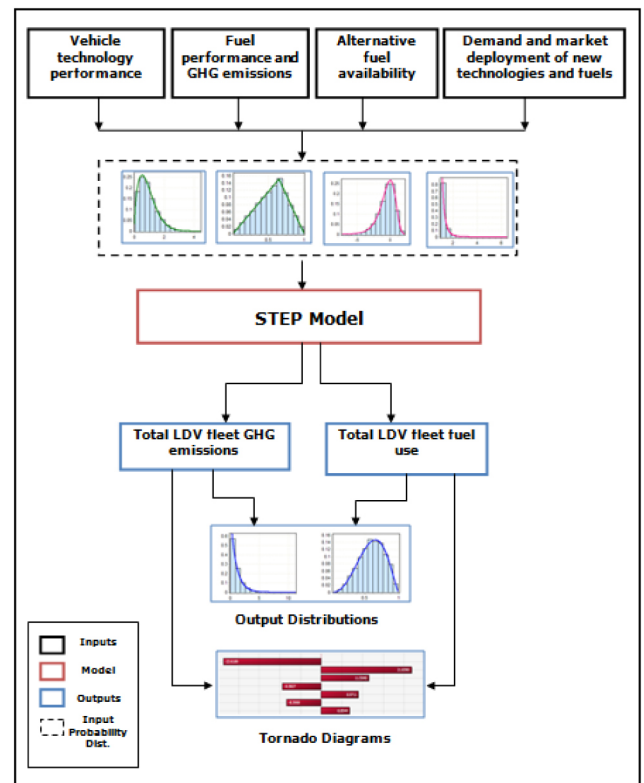


Figure 1. STEP Model Overview

The basic calculation logic that is used to compute the fleet fuel use and emissions here follows the MIT fleet calculations logic [5] which allows sensible and robust tracking of bottom-up details that characterize the fleet. Data is drawn from various sources, including the Transportation Energy Data Book (TEDB) [1], EPA Light-Duty Automotive Technology and Fuel Economy Trends report [26], and the U.S. Department of Transportation report (compiled by NHTSA for CAFE compliance) [27], to calculate fleet fuel use and life-cycle GHG emissions.

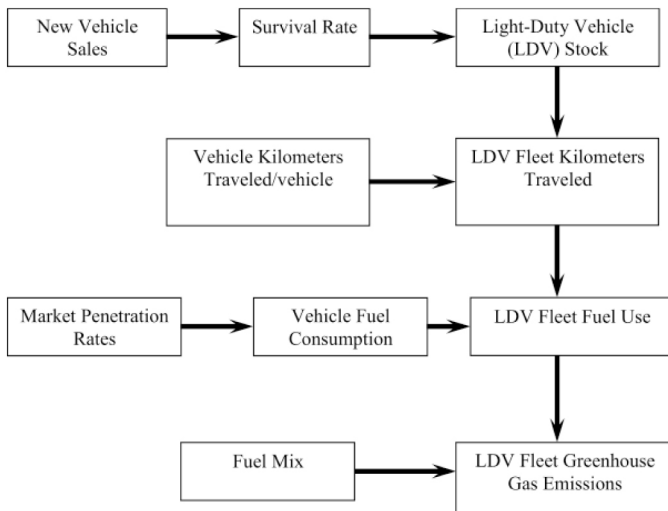


**Figure 2. Fuel use and GHG emissions calculations logic**

As Figure 1 shows the uncertainties in the inputs are captured as a set of probability distributions and fed into the STEP model. The model then uses Latin Hypercube Sampling (an advanced form of Monte Carlo simulation) to calculate the outputs, fuel use and emissions, as a stochastic distribution over time out to year 2050. Typically the model is run 10,000 times. Values are randomly sampled from input distributions in each run and the outputs are calculated based on the aggregated result of these runs. The most influential contributors to the outputs are also identified and ranked using multivariate linear regression and displayed as tornado diagrams. These analyses thus identify what drives the uncertainties in the outputs to inform priorities in transport policy making under uncertainty.

This methodology adopts a decision theory framework. Decision theory is designed to distinguish between a set of alternatives, where each alternative faces uncertain states of the world that can be represented by probability distributions [25]. Subjective probability is used to estimate the underlying uncertainty in the inputs based on expert assessments. These probabilities are subjective because they depend on the experts' judgement, which is likely to vary based on the information they each have available [25]. These subjective assessments will be subject to representativeness, availability and anchoring effects identified by Tversky and Kahneman leading to predictable biases [28]. For instance, anchoring leads to a bias in the uncertainty range by the experts, which are likely to be stated as narrower than can be justified by the experts' knowledge [28,29,30,31]. To reduce such biases, a range of different sources were consulted to determine the probability distribution in the inputs represented in this paper, and probability elicitation techniques were followed using direct probability assessment techniques to obtain probability estimates while minimizing bias and overconfidence [32, 33]. These techniques, which are well-established in climate change modelling [34], involve interactions with prominent

experts in the field. First, experts are briefed on why the study is conducted, then a clear understanding is reached on what the quantities mean and their units of measure. The experts are then asked to estimate the upper and lower bounds for each parameter, to minimize anchoring and overconfidence biases. The interviewer then proposes more extreme values and asks the experts whether there is a reason for such values to occur. Then, if there is a sensible explanation, the expert is asked to extend the bounds. The rest of the distribution is then similarly completed in consultation with the expert [32]. Figure 3 illustrates the logic of the methodology for determining the inputs for STEP. A survey of US road transport literature indicate a number of different underlying assumptions used in exploring technological and fuel development, demand, and market deployment rates. We suggest that the wide range in these differing assumptions is indicative of the underlying uncertainty in the variables. Further, a number of engineering and vehicle simulations and market behavioural models are used here to inform the uncertainty in the input variables. Finally, ranked and weighted screening of inputs is used to inform preliminary judgements on the possible ranges of each input variable. The probability elicitation techniques, mentioned earlier [32, 33] are then used with experts in the field to determine a realistic set of input distributions, discussed in the authors' 2011 study [35]. Model results have been compared with predictions from other recent studies in the authors' companion paper [35]. The historical trends, 1980-2010, agree with the data. Projections, 2010-2030, are comparable in trends and magnitude with the several other studies.

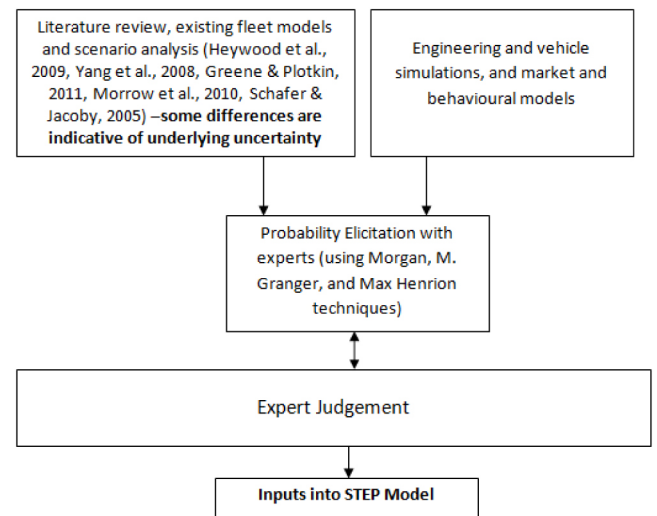


**Figure 3. Process for determining the inputs probability distributions**

This methodology therefore contributes to closing the gap between a snap shot analysis of future road transport using specific deterministic scenarios, and a more complete picture of the future vehicle fleet characteristics given uncertainties.

using a stochastic model that shows the probability distribution of fuel consumption and emissions over time out to year 2050. Moreover, using the tornado diagrams in this paper, one can answer questions such as: what is the sensitivity of fuel use and emissions to their major contributors, given technological and fuel uncertainties in the near to long term? Which factors are dominant in determining the outputs and how do they change over time? Given technological and behavioural uncertainties, is a 5% change in vehicle kilometres travelled more or less important than a 15% change in average fuel economy standards? And many more questions with important practical implications.

## 3.2. MODEL SET-UP

The model assumptions and some of the key equations are described in this section. The model is set up such that the ranges for most of the inputs are specified at year 2030 and data are interpolated and extrapolated before and after that date. 2030 was chosen since it is not too far out in time that useful judgements cannot be made, but it is also far enough in time such that significant changes can be reasonably expected. The uncertainty in the inputs is propagated throughout the data points in the model out to year 2050 as described in the following sections. Input distributions are represented as triangular distributions, defined by a maximum, most likely, and minimum value. Refer to the Appendix for the complete list of equations, parameters, and the symbols used to represent the input and output variables. The equations referred to in this section are described in full in [Appendix A](#).

### 3.2.1. Vehicle Stock

The share of vehicles in the market is captured as percentage of sales in each year. The share of conventional NA-SI engines in the market is not specified directly but is calculated as the residual not covered by the percentage of sales of other types of powertrains. An exponential growth function is used to interpolate and extrapolate sales shares, total vehicle sales, and percentage of cars (versus light trucks) from present to 2030 and out to year 2050, as illustrated in [Eqn 1](#) below: where TSL is the total vehicle sales, which is an input to the model. The extrapolated values are then checked to ensure the changes are reasonable over time. Year 2009 is used as the base year here. The uncertainty in the sales shares, total sales, and percentage of cars is propagated over time using similar equations to [Eqn 1](#), where both the mode and the range are extrapolated out to year 2050. These numbers are not to be taken as forecasts but as conditional projections into the future.

$$TSL(i) = TSL(2009) \times e^{[(\ln(TSL(2030)/TSL(2009)))/21] \times (i-2009)}$$

(Eqn 1)

Market segmentation is also tracked, to allow for a shift between cars and light trucks over time using [equation 4](#), where PCAR represents the percentage of cars (versus light trucks) in the sales share in year  $i$ . The future scrappage rate

input is then used to keep track of the number of cars removed in each calendar year. This is then used to calculate the on road vehicle stock, using the number of surviving cars based on model year (from 1960 onwards) and the calendar year. Refer to [equations 5,6,7,8,9,10](#) described in the Appendix. The same sets of calculations are performed for all segments of the LDV market (cars, SUVs, and OLTs), and the details of each segment are kept track of separately until the very last calculations to compute the total fuel use and GHG emissions. This in turn allows segment specific analyses.

### 3.2.2. Vehicle Kilometres Travelled and Fuel Consumption

VKT is calculated using annual growth rates, which vary in different periods of time, and is based on the vehicle's model year. It is assumed that VKT of vehicles decline with age, in other words, older cars travel less. [Equations 12](#) and [13](#) describe how VKT is kept track of over time, based on the vehicle model year. The uncertainty in the VKT variable is propagated over time through the growth rate as shown in [Eqn 13](#).

$$VKT(i, MY_h) = (1 + VKT_{gr}) \times VKT(i-1, MY_h)$$

(Eqn 13)

Fuel consumption is the inverse of the well-known measure of fuel economy and is measured in Litres/100km. Fuel consumption of Cars, SUVs, and OLTs, and different powertrains are kept track of separately. A correction factor has been used to take into account the difference between laboratory fuel economy measurements and highway driving conditions, as explained in section 3.4. Fuel consumption is calculated based on a measure called Emphasis on Reducing Fuel Consumption (ERFC), which takes a value between 0 and 1, and is defined as the percentage of the actual FC reduction realized to total FC reduction possible with constant size and performance [[5](#), [36](#)]. ERFC of cars and light trucks in year 2030 are specified as inputs to the model and are described using an exponential relation over time, as indicated in [Eqn 15](#). The uncertainty in the future vehicles' fuel consumption is thus propagated through ERFC, as described in [Eqn 15](#), as well as their relative fuel consumption over time. The relative fuel consumption of each powertrain (relative to a NA-SI conventional engine in 2008) is then used to calculate the vehicle fuel use over time. The relative fuel consumption is a function of ERFC as described earlier. The relative fuel consumption of each powertrain at 100% ERFC is also described over time with an exponential curve similar to that described in [Eqn 14](#). The relative fuel consumption of different powertrains in 2030 at 100% ERFC level is also input into the model. The actual fuel consumption of each powertrain in each year is then calculated based on the ERFC in that year as well as the relative fuel consumption of that powertrain in the same year, multiplied by the fuel consumption of the base vehicle (2008 NA-SI). ERFC is used here to account for weight savings and

changes in the performance of vehicles over time. The relative fuel consumption for PHEV used in this paper is for the miles these vehicles travel on gasoline engine. Their electricity consumption is then calculated using their electricity use per kilometres travelled on the battery, treated the same way as BEVs. The utility factor is used to calculate the number of kilometres PHEVs travel on gasoline versus electricity. For FCVs, we keep track of the energy consumption per 100 km, which is then used to calculate the total FCV life-cycle energy use based on its associated VKT.

### 3.2.3. Fuel Use and Life-Cycle Emissions

The fuel consumption for each powertrain and each calendar year is kept track of separately and then summed up in the last step to calculate the total fuel use in billion litre gasoline equivalent per year. The total fuel consumption is calculated using equations 19 and 23, and includes all liquid based fuels. This excludes electricity primary fuel use; life-cycle energy use from electricity is included in the energy graphs discussed in section 4.5. The volume of alternative fuels is calculated based on the blend percentage. It is assumed that total available tarsand will be split equally in replacing diesel and gasoline. Total fuel consumption is reported in billion litre gasoline equivalent and is adjusted for different fuel energy densities. The total energy consumption is calculated using fuel volumes and fuel energy densities based on Eqn 24, which also includes the electricity total life-cycle energy consumption.

$$EC(i) = \sum [FU_a(i) \times FD_a] + FU_E(i) \times FD_3 \quad (Eqn 24)$$

WTW coefficients are described using an exponential curve over time shown in equation 25, as it is assumed here that the WTW emissions will improve over time due to better raw material processing and more efficient use on the vehicle. The GHG emissions are then calculated using life-cycle WTW emissions and the fuel use based on equation 26, and summed up to represent the total life-cycle emissions from the fleet in Mt CO<sub>2</sub> equivalent per year. For electricity, the WTW emissions represent the grid average emissions. This is calculated, as illustrated in Eqn 26, based on the percentage of electricity that comes from renewable clean sources (which is an input to the model) as well as the WTW of conventional electricity (coal and natural gas based), which is also an input to the model. The uncertainty in the life-cycle emissions are thus propagated through WTW coefficients and alternative fuel use over time following equations 25 and 26. The impact of vehicle recycling is not included here, which is on the order of about 10%, with significant material recycling currently about 85%, and expected to rise[5].

## 3.3. ILLUSTRATIVE EXAMPLES

Three illustrative examples are carefully chosen here to demonstrate the methodology. The inputs are first ranked based on their level of uncertainty and then grouped in very

low, low, medium, and high uncertainty categories. The rankings are chosen to be representative and sensible but are for demonstration purposes only to allow readers to understand the methodology without getting into detailed consideration of why a certain range and mode is used for each input. Refer to the authors' 2011 study which covers those details for a realistic technology and fuel development pathway [35]. Table 1 below summarizes the ranking chosen for these examples. The inputs are represented by triangular distributions and the min, mode, and max values are shown in the table below. Refer to the Appendix for the complete list of input parameters.

The measure used in this model is the inverse of fuel economy, which is the vehicle fuel consumption in L/100km. Vehicle fuel use is calculated from ERFC (Emphasis on Reducing Fuel Consumption) and powertrain's relative fuel consumption. ERFC measures the degree to which weight savings and technological improvements are used to reduce vehicle fuel use. ERFC is defined as the actual fuel consumption reduction realized, divided by the fuel consumption reduction achievable if size and performance are kept constant, and is reported as a percentage [23]. At 100% ERFC all the technological improvements are used to reduce fuel consumption while power and weight are kept constant. At 0% ERFC, the fuel consumption stays the same because all technological improvements have been offset by performance gains (power and faster acceleration time, and added weight). Though not explicitly stated, weight reduction is taken into account in these calculations through its coupling with ERFC.

**Table 1. Ranked Input List for example A**

| Parameter                               | Min    | Mode   | Max    | Mean   | STD   | Uncertainty Level | Values in 2010 |
|-----------------------------------------|--------|--------|--------|--------|-------|-------------------|----------------|
| Total light vehicles Sales in 2030      |        |        |        |        |       |                   |                |
| [*000]                                  | 16,149 | 18,403 | 20,657 | 18,403 | 920   | Low               | 11,500         |
| Future Scrappage Rate(2011+)            | 60%    | 80%    | 100%   | 80%    | 8%    | Medium            | 80%            |
| %Sales HEV in 2030                      | 5%     | 10%    | 15%    | 10%    | 2%    | High              | 3%             |
| % Sales PHEV in 2030                    | 3%     | 5%     | 7%     | 5%     | 1%    | High              | 0%             |
| %Sales BEV in 2030                      | 2%     | 4%     | 6%     | 4%     | 1%    | High              | 0%             |
| VKT-Annual-Growth(2020-2030)            | 0.13%  | 0.25%  | 0.37%  | 0.25%  | 0.05% | High              | N/A            |
| ERFC Cars                               | 60%    | 80%    | 100%   | 80%    | 8%    | Medium            | 50%            |
| %blend cellulosic ethanol in 2030       | 7%     | 14%    | 21%    | 14%    | 3%    | High              | 0%             |
| %electricity from clean sources in 2030 | 38%    | 50%    | 62%    | 50%    | 5%    | Medium            | 29%            |
| Cellulosic Ethanol WTW in 2030          | 4      | 8      | 12     | 8      | 2     | High              | 10             |
| Gasoline WTW in 2030                    | 86     | 92     | 98     | 92     | 2     | VeryLow           | 92             |
| FC-r NA-SI cars in 2030                 | 0.53   | 0.70   | 0.87   | 0.7    | 0.07  | Medium            | 1.00           |
| FC-r Turbo LT in 2030                   | 0.46   | 0.61   | 0.76   | 0.609  | 0.06  | Medium            | 0.83           |

Then three illustrative examples are simulated using these ranked categories, with very low, low, medium, and high categories representing the spread (standard deviation/mean) set to 2.5%, 5%,10%,15% respectively in example A.; to 5%, 10%,20%,30% in example B; and to 10%, 20%40%,60% in example C. These illustrative examples are chosen as simplified examples where the mode is kept constant and the spread is doubled to give readers a better sense of how the integrated impact of uncertainty on the outputs changes as the level of uncertainty is increased, with a consistent set of inputs. These examples also confirm the internal validity of the results. The simulation results for these examples are presented in the next section.