

Assessing Dirlik's Fatigue Damage Estimation Method for Automotive Applications

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ABSTRACT

Fatigue analysis in the time domain using the rainflow cycle counting algorithm is considered the most accurate method for estimating damage. Dirlik's method has been found to be very accurate for damage estimation in the frequency domain. Previous studies have demonstrated the usefulness of Dirlik's method for ocean engineering and wind turbines but few have shown how well Dirlik performs in automotive applications. This study compares Dirlik's method with the rainflow cycle counting and with other frequency domain methods. The study analyzes measured data for an automotive component subjected to five test track load conditions. In addition, fourteen of Dirlik's original spectra and seven additional spectra which combine sine and random spectra are studied. It was found that Dirlik's method predicts more damage than the rainflow cycle counting method when applied to the original data used in creating the method. At the same time Dirlik's method underestimates damage for the automotive component by up to 30%. Still, when compared against other frequency domain methods, the Dirlik method is found to be the most accurate when compared to the rainflow cycle counting algorithm.

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INTRODUCTION

The physical world produces loads which can lead to premature failure in mechanical components. These loads vary in frequency and magnitude as well as in time. Most real world loads are continually changing randomly and a statistical approach is the only way to efficiently describe what is happening. Not all cyclic loads lead to damage of a component, however.

Matsuishi and Endo [1] discovered a method for counting load cycles that cause damage, called "rainflow" cycle counting, or RFC. The rainflow cycle counting method is the most accurate way to calculate the number of cycles for a given load level for the purpose of computing damages on complex load histories. RFC applies to both deterministically and randomly cyclic loads.

Rainflow cycle counting compares the range of one half cycle and the range for the next subsequent half cycle. A damaging cycle is counted if the first range is less than or equal to the next range. The range is recorded, the start of the first half cycle is removed and the process repeated until all cycles have been counted. Then each half cycle is paired with

half cycles of the same magnitude but opposite direction to create a count for one complete cycle. The user must choose the bin sizes for the load range and the mean load respectively. The full methodology has been described elsewhere [2, 3].

Random cyclic loading occurs frequently in nature. Waves in the ocean, wind passing over a body and road bumps are some of the more commonly known examples of external sources of random vibrations. Most vibration requirements today are random. For example, the majority of vibration loading conditions in MIL-STD-810 [4] today are random, not sinusoidal.

The rainflow cycle counting method relies on data previously collected over time to estimate damage, called time series data and this approach is called time domain analysis. On the other hand, random vibrations are not precisely described in terms of what has happened or will happen. Instead, random vibrations are described in terms of what happens on average at various frequencies. This approach is called the frequency domain analysis.

Various methods have been developed to estimate rainflow cycle counting damage by way of frequency domain methods. Approximate formulas have been proposed to calculate what the rainflow cycle count damage might be based on a narrowband assumption, such as the Wirsching-Light method [5] and the Ortiz-Chen method [6]. Others, like the Dirlik method [7], estimate the true rainflow cycle count distribution by assuming a probability density function. Full details on rainflow cycle counting and these frequency domain methods for damage estimation are described by Lee, Barkey, and Kang [8].

Comparisons of Dirlik's method to other frequency domain methods have been described earlier by Lee, Polhena and Kang [9] and found Dirlik's method to be the most accurate. Comparisons of damage estimates from Dirlik's method and rainflow cycle counting have been made for wind energy field data by Ragan and Manuel [10] who found that the Dirlik method performed poorly for conditions which have a very large periodic component. They also found that the method predicted less damage than corresponding rainflow cycle counted fatigue load estimates. Another study by Van Der Tempel [11] done for offshore structures also found that Dirlik on average predicted less damage than the rainflow cycle method. In addition Van Der Tempel [11] found that the frequency domain methods converge to similar results when applied to narrowband signals but found Dirlik's method most accurate for broadband load inputs. Benasciutti and Tovo [12] proposed a method that improves upon Dirlik's method but this will not be covered in this paper.

The objective of this paper is to calculate damages from time series measured data from an automotive component by Dirlik's method, by other frequency domain methods, and by conventional Rainflow cycle counting methods. Various created spectra, including a variety of Dirlik's original spectra, are analyzed also to help provide a better understanding of how the accuracy of Dirlik's method might be improved.

Throughout this paper, it is assumed that the structure is linear and subjected to stationary Gaussian loads of zero mean. An overview of fatigue damage calculations for the frequency domain and Dirlik's method will be given followed by an explanation on how to compare Dirlik's method and RFC. Then the paper will describe the calculation tools used, the results of data analyzed, and provide comparisons of the accuracy of Dirlik's and other frequency domain methods to Rainflow cycle counting.

CALCULATING FATIGUE DAMAGES

The most widely used equation for the stress based approach in fatigue analysis is the Basquin relation. The Basquin relation describes how many cycles N_f at a given stress amplitude S_a will lead to failure as shown in [equation 1](#).

$$S_a = S'_f (2N_f)^b \quad (1)$$

The slope of the S-N line on a log-log plot is b , called the fatigue strength exponent. The intercept on the load axis at $2N_f=1$ is called the fatigue strength coefficient S'_f . Stress can be expressed either as stress range S_r or stress amplitude S_a . The Basquin relation frequently is expressed in an alternate form using a material constant known as C and this form for stress is given below in [equation 2](#).

$$C_a = N_f S_a^{(m)} \quad (2)$$

The material coefficient C_a is given in [equation 3](#). The exponent m , called the fatigue slope factor, is given in [equation 4](#).

$$C_a = \frac{S'_f{}^{(m)}}{2} \quad (3)$$

$$m = -1/b \quad (4)$$

The Palmgren-Miner linear damage rule calculates damages of complex stress histories and is given in [equation 5](#).

$$D = \sum_{i=1}^N \frac{n_i}{N_{f,i}} = \frac{1}{C_a} \sum_{i=1}^N n_i \cdot S_{a,i}^m \quad (5)$$

The number of cycles for a given stress level is equal to n_i and failure occurs when $D \geq 1.0$

The result of RFC is a table giving the number of full cycles for various stress amplitudes and their corresponding mean stresses. The numbers of cycles n_i for each stress amplitude bin are summed and then the Palmgren-Miner damage is used to calculate total damage as shown in [figure 1](#).

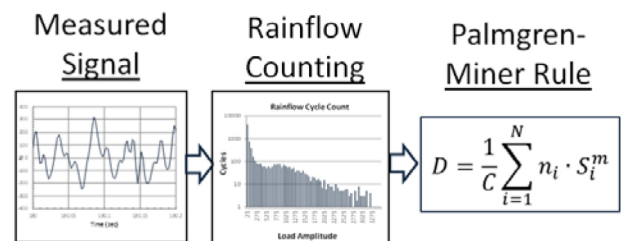


Figure 1. Process for time series damage estimation.

For small displacements, one can assume a linear stress-strain relationship, such that the stress level $S(t)$ is proportional to the relative displacement and the applied force. The proportionality constant relating stresses to forces can be included in the material constant C . The time series data described later in this paper were recorded in units of force and to be compatible with this data, all load calculations performed for this study are done in units of force (N). In addition, all calculations are performed in terms of load amplitude and not load range.

FREQUENCY DOMAIN METHODS

A random vibration spectrum is usually plotted in terms of load squared per hz versus frequency in hz. This plot is called the power spectral density, or PSD and it is customarily plotted in log-log format. The PSD is obtained by taking the fast Fourier transform, or FFT, of a complex vibratory stress or load time history. The FFT decomposes the tensile peaks and compressive valleys of the time history data into a frequency spectrum with varying spectral magnitudes.

The area under the PSD curve and the square root of that area (called the root mean square, or the rms value) are important parameters in the frequency domain analysis. The PSD is typically given with as few as two to as many as two dozen or more "breakpoints" at various frequency and amplitude values and connected by straight lines between on a log-log plot. The equation for a point along a one sided PSD segment W_x with a straight slope between breakpoints when plotted on log-log axes is given in equation 6.

$$W_x = W_1 \left(\frac{f_2}{f_1} \right)^k \quad (6)$$

The slope k between breakpoints is given in equation 7.

$$k = \frac{\log \frac{W_2}{W_1}}{\log \frac{f_2}{f_1}} \quad (7)$$

The equation to integrate the area under the PSD between breakpoints 1 and 2 is given in equation 8. The areas between each breakpoint are summed to obtain the total area and the square root of the area to obtain the rms value.

$$A_{1-2} = \int_1^2 W_1 \left(\frac{f}{f_1} \right)^k df \quad (8)$$

The spectral moments are used in the Dirlik Method. The spectral moment for a single sided PSD for a frequency range of 0 infinity is given in equation 9, where the order of the moment can be $j=0,1,2,\dots$

$$M_j = \int_0^\infty f^j W_x(f) df \quad (9)$$

The equation to calculate the moment between two breakpoints 1 and 2 is given in equation 10. The moments between each breakpoint are summed to give the total moment for order j .

$$M_j = \int_1^2 f^j W_1 \left(\frac{f}{f_1} \right)^k df \quad (10)$$

Note that the area under the PSD curve is also equal to the zeroth moment in the equation above, or M_0 . Also, equation 10 can be used to numerically integrate the spectral moments of a PSD to any order.

The irregularity factor γ is a measure of the bandwidth and is the ratio of the rate of expected zero mean up crossings, $E[0^+]$, to the rate of expected peaks, $E[P]$ where

$$E[0^+] = \sqrt{\frac{M_2}{M_0}} \quad (11)$$

$$E[P] = \sqrt{\frac{M_4}{M_2}} \quad (12)$$

For example, the measured data in Figure 2 has seven zero mean up crossings per half second and fourteen peaks per half second for an irregularity factor γ of 0.5.

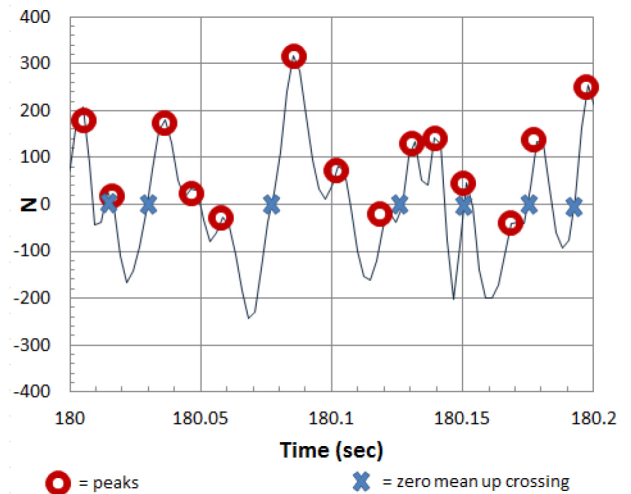


Figure 2. Zero mean up crossing and peak counting.

The irregularity factor γ is related to the spectral moments by equation 13.

$$\gamma = \frac{E[0^+]}{E[P]} = \sqrt{\frac{M_2^2}{M_0 \cdot M_4}} \quad (13)$$

A pure sine wave has an irregularity factor of 1.0. A broadband white noise input has an irregularity factor of 0.45.