

Parametric Importance Analysis and Design Optimization of a Torque Converter Model Using Sensitivity Information

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ABSTRACT

Torque converters are used as coupling devices in automobile powertrains involving automatic transmissions. Efficient modeling of torque converters capturing various modes of operation is important for powertrain design and simulation, (Hroval and Tobler 1, Ishihara and Emori 2) optimization and control applications. Models of torque converters are available in various commercial simulation packages, Hadi et. al. 3. The information about the effect of model parameters on torque converter performance is valuable for any design operation. In this paper, a symbolic sensitivity analysis of a torque converter model will be presented. Direct differentiation (Serban and Freeman 4) is used to generate the sensitivity equations which results in equations in symbolic form. By solving the sensitivity equations, the effect of a perturbation of the model parameters on the behavior of the system is determined. A parametric importance analysis is performed on the model: the model parameters are arranged according to their effect on the amount loss of energy during the operation of the torque converter. The radii of the pump, turbine and stator, the density of the hydraulic fluid and the exit angle of the vanes of the stator were found to have the most significant effects on the model. Using the sensitivity information, a design optimization problem is defined and solved to obtain a set of parameter values that minimizes the energy lost during the torque converter operation.

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INTRODUCTION

A very basic requirement of an automobile transmission system is the ability to accommodate discrepancies between the angular velocities of the engine flywheel and the drive shaft connected to the transmission differentials. For manual transmission systems a mechanical friction clutch is usually used to address this requirement by allowing the engine to be physically disengaged from the rest of the transmission. For automatic transmission systems a device known as the torque converter is often used as a coupling device. A torque converter allows the engine to rotate even when the wheels

are slowing down by means of fluid coupling. Structurally it has three main components.

- A pump element, which is rigidly attached to the engine flywheel and under forward flow mode, pumps hydraulic fluid radially outward due to its vaned construction.
- A turbine element which is connected to the transmission and also has a vaned construction, so that the hydraulic fluid can impart momentum to the turbine.
- A stator element which is connected to the fixed body of the transmission through a one way clutch and has blades to

deflect the fluid from the turbine, to the pump, thereby increasing the flow efficiency.

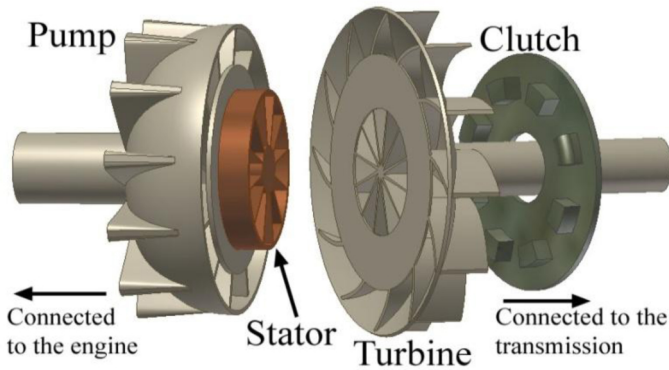


Figure 1. Axial assembly of the components of a torque converter

For this study we have worked with the model of a torque converter which captures the dynamics of components and the hydraulic fluid, (Hrovat and Tobler 1 and Hadi et. al. 3). In this paper we use symbolic differentiation to generate sensitivity equations from the governing equations of the model. By solving the sensitivity equations and the system equations simultaneously we obtain the state variables and the corresponding sensitivities as a function of time. Using the sensitivity information, a parametric importance analysis is performed to identify the most important parameters of the model. A design optimization problem is set up using the sensitivity information. The expression for the power loss during the operation is considered as the objective function of the optimization problem. The results section outlines the most important parameters and the optimized set of values for the parameters which minimizes the energy lost during the operation of the torque converter.

MODEL DESCRIPTION

A schematic diagram showing the motion of a fluid particle inside a typical torque converter is shown in Figure 2. The operation of a torque converter can be clearly understood by following the motion of a fluid particle around the three components: the pump (or the impeller), the turbine, and the stator.

Normally the rotational speed of the pump element is greater than that of the turbine element. The ratio of the angular velocities of the pump and the turbine is known as the speed ratio. When the speed ratio is less than unity it is known as the forward mode of operation. In the forward mode the impeller pumps hydraulic fluid radially outward through its vanes thus imparting momentum to the fluid. This momentum is transferred to the turbine when the fluid flows along the vanes of the turbine. There are two stages of operation in the forward mode. When the speed ratio is considerably less than one, the one way clutch holds the stator in place and the stator bends the flow of fluid, aligning it with the pump vanes. This way the turbine torque obtained

is more than the pump torque and torque multiplication is achieved. As the speed ratio approaches unity, at one point of time, the stator torque becomes zero and the stator overruns. At this point the pump and the turbine torques are approximately equal and this is known as the coupling point.

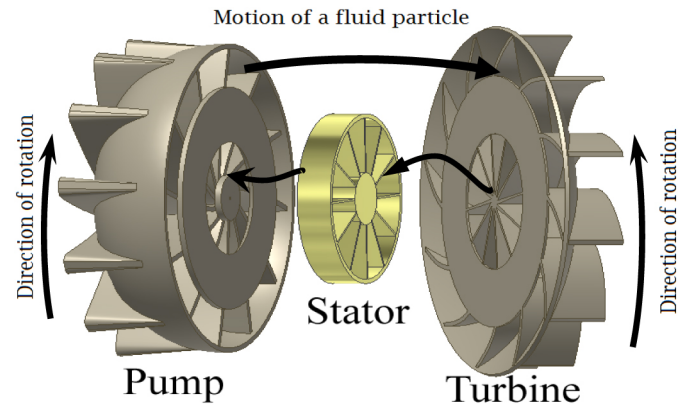


Figure 2. Schematic diagram of the motion of a fluid particle inside a torque converter

When the speed ratio is more than one, the torque converter works in the reverse flow mode. In this mode of operation the flow reverses, the stator overruns, and the turbine drives the pump. The reverse flow mode corresponds to the operation of engine braking.

The equations presented in this paper correspond to the forward mode of operation. In forward mode the elements that contribute to the energy of the system are the pump, the turbine, the stator and the hydraulic fluid. Since there are four components contributing to the energy of the system, there are four differential variables leading to four differential equations.

The torque balance formulation leads to the first three differential equations (Hrovat 1).

$$I_p \dot{\omega}_p + \rho S_p \dot{Q} = \rho \left(-\omega_p R_p^2 - R_p \frac{Q}{A} \tan \alpha_p + \omega_s R_s^2 + R_s \frac{Q}{A} \tan \alpha_s \right) Q - T_p \quad (1)$$

$$I_t \dot{\omega}_t + \rho S_t \dot{Q} = \rho \left(-\omega_t R_t^2 - R_t \frac{Q}{A} \tan \alpha_t + \omega_p R_p^2 + R_p \frac{Q}{A} \tan \alpha_p \right) Q - T_t \quad (2)$$

$$I_s \dot{\omega}_s + \rho S_s \dot{Q} = \rho \left(-\omega_s R_s^2 - R_s \frac{Q}{A} \tan \alpha_s + \omega_t R_t^2 + R_t \frac{Q}{A} \tan \alpha_t \right) Q - T_s \quad (3)$$

The fourth equation corresponds to the balance of energy for the system.

$$\begin{aligned} & \rho \left(S_p \dot{\omega}_p^2 + S_t \dot{\omega}_t^2 + S_s \dot{\omega}_s^2 \right) + \frac{\rho L_f}{A} \dot{Q} = \\ & \rho \left(R_p^2 \omega_p^2 + R_t^2 \omega_t^2 + R_s^2 \omega_s^2 - R_s^2 \omega_p \omega_s - R_p^2 \omega_t \omega_p - R_t^2 \omega_s \omega_t \right) \\ & + \omega_p \frac{Q}{A} \rho \left(R_p \tan \alpha_p - R_s \tan \alpha_s \right) \\ & + \omega_t \frac{Q}{A} \rho \left(R_t \tan \alpha_t - R_p \tan \alpha_p \right) \\ & + \omega_s \frac{Q}{A} \rho \left(R_s \tan \alpha_s - R_t \tan \alpha_t \right) - \frac{P_{sh} + P_f}{Q} \end{aligned} \quad (4)$$

In the above equations ω_p , ω_t , ω_s and Q are the state variables. The first three state variables represent the angular velocities of the pump, turbine and the stator element and Q represent the volumetric flow-rate of the hydraulic fluid in the system. A complete list of state variables and the parameters of the model are given in [tables 1](#) and [2](#) respectively. [Table 2](#) is presented in [appendix A](#).

Table 1. State Variables

Name	Description
Q	Flow rate of hydraulic fluid
ω_s	Angular speed of the stator
ω_p	Angular speed of the pump
ω_t	Angular speed of the turbine
ω_r	Speed Ratio
T_r	Torque Ratio
η	Efficiency of the torque converter

The terms P_{sh} and P_f represent the losses due to shock and fluid friction respectively. They are expressed in terms of the fluid velocity and the geometry of the inner construction of the torque converter.

$$\begin{aligned} P_{sh} &= \frac{\rho |Q| C_{sh}}{2} \left[\left(R_s (\omega_s - \omega_p) + \frac{Q}{A} (\tan \alpha_s - \tan \beta_p) \right)^2 \right. \\ & \quad + \left(R_p (\omega_p - \omega_t) + \frac{Q}{A} (\tan \alpha_p - \tan \beta_t) \right)^2 \\ & \quad \left. + \left(R_t (\omega_t - \omega_s) + \frac{Q}{A} (\tan \alpha_t - \tan \beta_s) \right)^2 \right] \\ P_f &= \frac{\rho |Q| C_f Q^2}{2 A^2} \left(3 + (\tan \alpha_p)^2 + (\tan \alpha_t)^2 + (\tan \alpha_s)^2 \right) \end{aligned} \quad (5)$$

In [equation \(5\)](#), C_{sh} and C_f are the shock and friction loss factors and are constants. The angles α and β represent the exit and entry angles of the vanes. The subscripts s, p and t are used to denote the stator, pump and the turbine respectively.

We also define a few related quantities to help us represent the dynamics of the system. Speed ratio is the ratio of the angular speed of the turbine to that of the pump. For forward mode of operation this quantity is always less than unity.

$$\omega_r = \omega_t / \omega_p \quad (6)$$

Torque ratio is the ratio of the turbine torque to the pump torque.

$$T_r = |T_t / T_p| \quad (7)$$

In the forward mode of operation before the coupling occurs, the torque at the turbine is greater than that acting on the pump. Thus at this stage the torque ratio stays more than unity and corresponds to the phenomenon of torque multiplication. When the system reaches the coupling point, the torque ratio becomes close to unity and the pump and turbine torques become approximately equal to one another.

Efficiency is the ratio between the output power and the input power of the torque converter. It is expressed in terms of the torque and angular velocities of the pump and the turbine.

$$\eta = \left((T_t \omega_t) / (T_p \omega_p) \right) \quad (8)$$

We also evaluate the capacity factor defined by the following expression.

$$K_c = \omega_p / \sqrt{T_p} \quad (9)$$

The capacity factor is a measure of the ability of the torque converter to absorb and transmit torque, Wong 7. During the operation of the torque converter, the capacity factor reaches its minimum value at stall conditions and increases with an increase of the speed ratio. For proper matching, the capacity factors of the torque converter and the engine should have similar ranges of values.

In the forward mode of operation, when the speed ratio is considerably less than unity, the one-way clutch holds the stator in place and prevents it from rotating. As a result, the quantity $\omega_s(t)$ is always zero at this stage. This reduces [equation \(3\)](#) to an algebraic equation and the total number of differential equations becomes three.

The stator torque required to hold the stator in place decreases as the turbine speed catches up with that of the

pump. At the coupling point, when the stator torque becomes close to zero, the stator starts rotating. From this point onwards the system is governed by four differential equations, given by equations (1), (2), (3), (4).

MODEL SIMULATION

For this study we have specified the pump and the turbine torques, to correspond to the acceleration of the system. Figure 3 and Figure 4 show the pump and turbine torques used as inputs for this simulation. As a result of the specified torques, the angular speed of the turbine side, which is connected to the transmission, is expected to increase

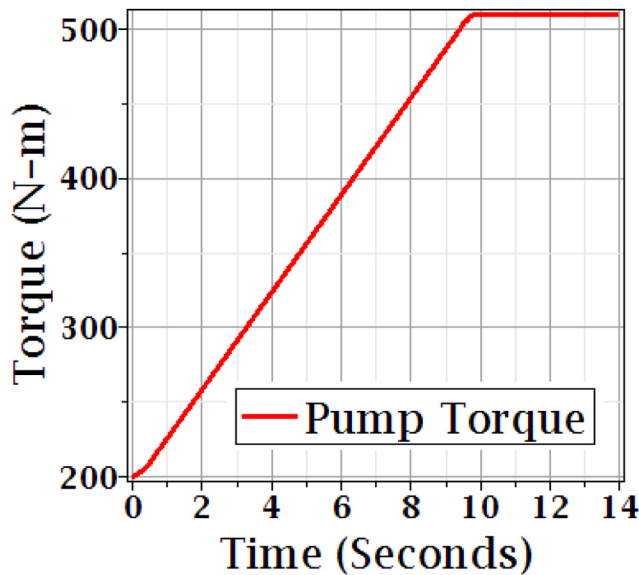


Figure 3. Specified pump torque

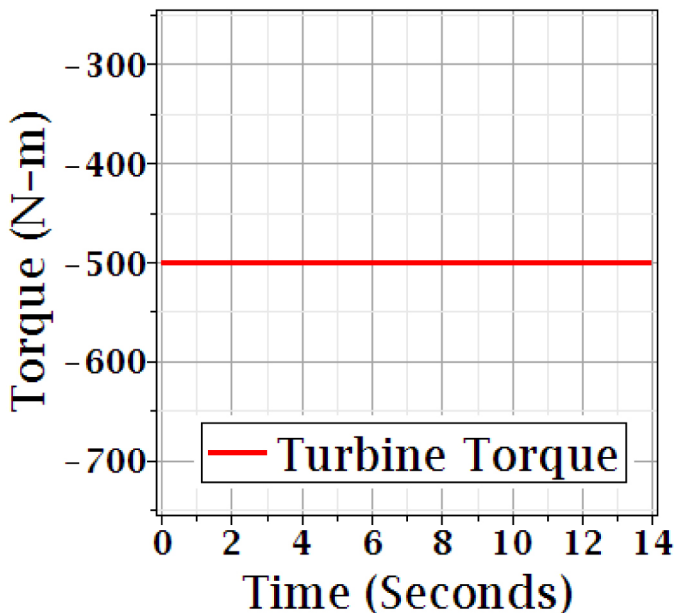


Figure 4. Specified turbine torque

To simulate the model, we have used Maple™, a powerful computational software package, as the working platform. Since there are two distinct stages of operation for the system, we simulate one stage at a time. We start the simulation by using the initial conditions shown in Table 3. At this point the system is governed by three differential equations. The stator torque is calculated using equation (3) with ω_s set to zero and is used to detect the point where the coupling takes place, which occurs when T_s goes to zero.

At the coupling point the stator starts rotating and the stator torque becomes zero. To simulate operation above the coupling point, we consider all four differential equations (1), (2), (3), (4) and set T_s to zero. Also the final values of the state variables in the first stage are used as the initial conditions for this stage.

For this study we start the simulation with speed ratio at zero and stop when the stator speed gets to $\omega_s = 20$ rad/s. The assumption is that at this point the lock-up clutch becomes activated and locks the relative angular velocity of the two sides of the torque converter and transforms it into a mechanical transmission. The reasons behind this particular choice of ω_s , at which the lock-up clutch becomes activated, will be presented in a subsequent chapter.

Table 3. Initial conditions for the system

States	Initial Values	Unit
$Q(0)$	0.13	m^3/s
$\omega_s(0)$	0	rad/s
$\omega_p(0)$	280	rad/s
$\omega_t(0)$	0	rad/s

SENSITIVITY EQUATIONS

In this study we have used direct differentiation (Serban and Freeman 4) to generate the sensitivity equations. For systems governed by a set of non-linear 1st order differential equations the equations can be written in matrix notation as

$$\mathbf{M}\dot{\mathbf{q}} = \mathbf{F} \quad (10)$$

For this system, $\mathbf{q} = [\omega_p \ \omega_t \ \omega_s \ Q]^T$, the mass matrix $\mathbf{M}$ is a matrix 4×4 matrix and the force vector $\mathbf{F}$ is a 4×1 matrix. Starting with the assumption that the mass matrix $\mathbf{M}$ and the force vector $\mathbf{F}$ are functions of time t , the state vector $\mathbf{q}$ and a set of model parameters represented by $\mathbf{p}$, we can write equation (10) as

$$\mathbf{M}(\mathbf{q}, \mathbf{p}, t) \dot{\mathbf{q}}(\mathbf{p}, t) = \mathbf{F}(\mathbf{q}, \mathbf{p}, t) \quad (11)$$

Differentiating equation (11) with respect to a single parameter b , where $b \in p$, we get

$$(\mathbf{M}_b + \mathbf{P})\dot{\mathbf{q}} + \mathbf{M}\dot{\mathbf{q}}_b = \mathbf{F}_b + \mathbf{F}_q \mathbf{q}_b$$

$$\text{where } \mathbf{P}_{ij} = \sum_{k=1}^n \frac{\partial m_{ij}}{\partial \mathbf{q}_k} (\mathbf{q}_k)_b$$
(12)

By combining equation (12) with equation (11) we form the equation:

$$\begin{bmatrix} \mathbf{M}_b^{4 \times 4} + \mathbf{P}^{4 \times 4} & \mathbf{M}^{4 \times 4} \\ \mathbf{M}^{4 \times 4} & \mathbf{0}^{4 \times 4} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}^{4 \times 1} \\ \dot{\mathbf{q}}_b^{4 \times 1} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_b + \mathbf{F}_q \mathbf{q}_b \\ \mathbf{F} \end{Bmatrix}^{8 \times 1}$$
(13)

By numerically solving equation (13) the state variables of the system and the corresponding sensitivities can be calculated as functions of time. Further details of the solution procedure are provided in the subsequent sections. The expressions of the combined set of equations are presented in appendix B.

IMPORTANCE ANALYSIS

The model parameters, i.e. the dimensions and the geometric properties of the components and the physical and hydrodynamic properties of the hydraulic fluid, play a major role in determining the dynamics of a torque converter. For various analyses it is important to understand which of these parameters have more influence on the dynamics than others. In other words it is necessary to determine which of the parameters are most important for the model. Using sensitivity analysis it is possible to develop a relative order of importance among the parameters.

OBJECTIVE FUNCTION

To perform importance analysis on a system, we need to specify the criterion with respect to which the effects of the parameters would be measured and compared. The result of an importance analysis is very much dependent on this criterion or what is known as the objective function. A general objective function can be expressed in terms of the set of model parameters $\mathbf{p}$ and the state variables $\mathbf{q}$ and can be written in the general form:

$$\lambda = \int_{\tau_1}^{\tau_2} f(\mathbf{q}, \mathbf{p}, t) dt$$
(14)

By differentiating equation (14) with respect to a single model parameter b , we obtain

$$\lambda_b = \frac{\partial \lambda}{\partial b} = f(\mathbf{p}, \tau_2) \frac{\partial \tau_2}{\partial b} - f(\mathbf{p}, \tau_1) \frac{\partial \tau_1}{\partial b} + \int_{\tau_1}^{\tau_2} \left(\frac{\partial f}{\partial b} + \frac{\partial f}{\partial \mathbf{q}} \mathbf{q}_b \right) dt$$
(15)

In the above equation, λ_b is the sensitivity of the objective function with respect to an arbitrary model parameter b . It is also known as the absolute sensitivity of the objective function f with respect to the parameter. The quantity $\mathbf{q}_b$ is the sensitivity of the state variables $\mathbf{q}$ with respect to the parameter b . These quantities are obtained by solving the set of differential equations in eq.(13).

The first two terms of equation (15) are required only if the limits of integrations are dependent on the parameter in question. Since for most of the time, the limits are independent of the model parameters, these terms don't figure in the expressions. However, for certain situations these assumptions might not be practical and due consideration must be given to these terms to ensure accurate computation of the sensitivity information.

Depending on the effect that a particular parameter has on the objective function, the value of the absolute sensitivity would be different and this piece of information can be used to arrange the parameters in their order of importance.

For this study, we have chosen the total loss of energy as the objective function for the importance analysis. The energy lost during the process can be written in terms of the input torque T_p , the angular velocity ω_p and the efficiency of the torque converter η as

$$E_{loss} = \int_0^{\tau_{lockup}} Power(t) dt$$

$$Power(t) = T_p(t) \omega_p(t) (1 - \eta(t))$$
(16)

The upper limit of the integration τ_{lockup} is the time when the lock-up clutch joins the two sides of the torque converter into a single piece and eliminates the dynamic effects of the torque converter. Using (15) we can obtain the expression of the sensitivity of the quantity E_{loss} with respect to an arbitrary model parameter as

$$E_{loss_b} = \frac{\partial E_{loss}}{\partial b} = Power(\tau_{lockup}) \times \frac{\partial \tau_{lockup}}{\partial b} + \int_0^{\tau_{lockup}} \left(\frac{\partial T_p}{\partial b} \omega_p (1 - \eta) + T_p \frac{\partial \omega_p}{\partial b} (1 - \eta) - T_p \omega_p \frac{\partial \eta}{\partial b} \right) dt$$
(17)

The first term of equation (17) is particularly troublesome due to the fact that although the time of lock-up (τ_{lockup}) is dependent on the model parameters, it is not easily expressed as an explicit function of any of them. It is dependent on the termination criterion mentioned in the last section and is only determined from the numerical simulation.

Thus the only way to calculate the term $\frac{\partial \tau_{lockup}}{\partial b}$ is to approximate the derivative with a finite difference approach

and simulate the model repeatedly with perturbed values of the parameter, something that may undermine the benefits of the direct differentiation approach. However, numerical comparison shows that the magnitude of this term is small compared to that of the second term and can be safely ignored for the purpose of analysis. Results confirming this statement are presented in a subsequent section.

NORMALIZATION

The absolute sensitivity of the objective function with respect to an arbitrary model parameter is a measure of the change in the objective function for a unit change in the parameter value, assuming a first order approximation. However this piece of information, in this form, is not very suitable to determine which parameter has more effect on the objective function. Since different parameters have different units, the significance of the perturbation of one unit is completely different for two separate parameters. To overcome this problem we normalize the absolute sensitivity by multiplying it with the nominal value of the parameter and dividing it by the nominal value of the objective function. This gives us what is known as the relative sensitivity of the objective function and is defined as (Smith *et al.* 5)

$$\mu = \frac{\partial E_{\text{loss}}}{\partial b} \left| \frac{b^*}{E_{\text{loss}}(b^*)} \right| \quad (18)$$

In equation (18) the symbol b^* denotes the nominal value of the parameter. The quantity thus obtained, i.e. the normalized sensitivity μ , represents the fractional change in the objective function for a fractional change in the value of the parameter. It is dimensionless and suitable for the purpose of comparison.

It is to be noted that while performing the normalization, it is important that absolute values are used, to ensure that the sense of direction of the changes are preserved in the relative sensitivity information.

DESIGN OPTIMIZATION

Using the sensitivity information a design optimization problem is set up to tune the model parameters of the torque converter for a better design. To improve the design of the torque converter, the main criterion is to minimize the energy loss during the operation, however, it is also a general observation that when efficiency is increased the torque capacity of a torque converter usually goes down. As a result, a more efficient torque converter often becomes severely restricted in terms of usability. Therefore, the proper optimization philosophy should focus on decreasing the area under the power lost curve and increasing the area under the torque capacity curve, at the same time. To achieve that, we combine the two separate criteria into a single objective function using the expression given below.

$$\lambda = \frac{a_1}{T_p(0)\omega_p(0)} \left(\int_0^{\tau_{\text{coupling}}} T_p(t)\omega_p(t)(1-\eta(t))dt \right) - \frac{a_2}{K_c(0)} \left(\int_0^{\tau_{\text{coupling}}} K_c(t)dt \right) \quad (19)$$

In equation (19), the two integrals represent the two separate criteria that are being combined into a single objective function. Since the equation features quantities with different units and physical significance, reference values of the relevant quantities are used to make the objective function non-dimensional. The constants a_1 and a_2 are the weight coefficients that determine the relative importance given to the individual criterion. It is to be noted that the outcome of the optimization is expected to be strongly dependent on the values of these weights.

By using direct differentiation and assuming minimal dependence of the term τ_{coupling} on the model parameters, as discussed in the previous section, we obtain the expression for the sensitivity of the objective function. This expression is to be used to determine the sensitivity of the objective function during the optimization iterations.

$$\frac{\partial \lambda}{\partial b} \approx \frac{a_1}{T_p(0)\omega_p(0)} \int_0^{\tau_{\text{lockup}}} \Gamma dt - \frac{a_2}{K_c(0)} \int_0^{\tau_{\text{lockup}}} \frac{\partial K_c}{\partial b} dt$$

$$\Gamma = \left(\frac{\partial T_p}{\partial b} \omega_p (1-\eta) + T_p \frac{\partial \omega_p}{\partial b} (1-\eta) - T_p \omega_p \frac{\partial \eta}{\partial b} \right) \quad (20)$$

In this study we want to give equal importance to the two aspects of optimization. The individual parts of the objective function are rendered non-dimensional by dividing them by their initial values. During the simulation these non-dimensional components vary within different ranges. The values of a_1 and a_2 are chosen to scale these ranges and make them comparable to one another. Also for proper scaling, it is important to ensure that $a_1 + a_2 = 1$. Using the ranges for the components of the objective function for the original system, the weights were determined to be

$$a_1 = 0.47 \text{ and } a_2 = 0.53 \quad (21)$$

Importance analysis reveals the list of model parameters which have more influence on the performance of the torque converter than others. For the purpose of the design optimization we choose the most important parameters and consider other parameters to be constants, since they are not expected to have a great influence on the objective function.

We have used gradient-based steepest decent method to adjust the parameters. The parameter values were restricted to