

Improving the Simulation of the Acoustic Performance of Complex Silencers for ICE by a Multi-Dimensional Non-Linear Approach

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ABSTRACT

In this paper a three-dimensional time-domain CFD approach has been employed to predict and analyze the acoustic attenuation performance of complex perforated muffler geometries, where strong 3D effects limit the validity of the use of one-dimensional models. A pressure pulse has been imposed at the inlet to excite the wave motion, while unsteady flow computation have been performed to acquire the time histories of the pressures upstream and downstream of the silencer. Pressures in the time domain have been then transformed to acoustic pressures in the frequency domain, to predict the transmission loss. In order to achieve reliable predictions of the acoustic attenuation, especially above the plane wave cut-off frequency, an advanced non-reflecting boundary condition to model the anechoic termination, based on the NSCBC characteristic theory, has been implemented in a multidimensional open-source CFD code and it has been used together with a time dependent compressible Navier Stokes equation solver. The original local one dimensional inviscid relations (LODI) in Cartesian coordinates presented in [1] have been extended to local coordinates and have been applied for unsteady calculations in a multistage time stepping scheme. Numerical results show that the agreement between simulations and experiments strongly improves at high frequency when the novel boundary condition is applied. A comparative study with a 1D non-linear model for unsteady compressible flows is shown on four different configuration of reverse flow chambers and on a single-plug perforated muffler for internal combustion engine applications. Advantages and drawbacks of the proposed approach are discussed.

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INTRODUCTION

Silencers are widely employed to reduce the intake and exhaust noise in internal combustion engines because of their high acoustic attenuation and low pressure drop. The approaches most commonly used for design purposes in silencer simulation are based on linear acoustic models or on non-linear gas dynamic models. In the first case, under the hypothesis of small pressure perturbations in the computational domain, the transmission loss of a silencer is calculated by frequency domain techniques based on the fourth-pole transfer matrix method [2]. This approach is very fast but sometimes it might not be really predictive for complex geometries, where non zero mean gas flow is present and with significant spatial and temporal gas

temperature gradients. Also, it requires an experimental assessment of the impedance of the elements used for the linear domain discretization. Non linear gas dynamic models are able to simulate the wave motion, because they are based on time domain techniques and they can provide reliable results even with non-zero mean flow of the gas. Non linear one-dimensional time domain approaches can predict the acoustic performance of complex silencers, such as perforated tube reactive silencers and reverse chambers. Much work has been done in this field: advanced techniques for the segmentation of the geometry have allowed for achieve reliable results in the case of simple and multi-pass resonators, with perforated ducts, with and without sound absorbent material [3, 4]. The limit of one-dimensional non-linear codes is still in the assumption of planar wave

propagation, that does not seem to work when the contribution of the higher order modes is not negligible. This assumption has been removed instead by quasi-3D approaches, that are still developed in one-dimensional frameworks: a multi-dimensional geometry is modeled using a concatenation of junction or flow-split volumes in quasi-3D space [4] or by a network of 0D elements of a given characteristic length and the governing equations are solved along the three main directions by a staggered explicit solver [5].

When a more detailed description of the wave propagation within the silencer (with and without gas flow velocity) is required, multi-dimensional simulation represents the most reliable methodology, as it will be discussed further. At the same time, it is important to note that the anechoic termination and the outlet end of the system to test represents a crucial aspect to be considered both in experiments and simulations. The main purpose of an anechoic termination is to not include reflections, downstream of the silencer, in the pressure time histories, that are recorded to be converted to the frequency domain for the determination of the Transmission Loss (TL) of the silencer. The pressure time histories that are Fourier transformed must be only of waves traveling from the inlet to the outlet of the system. In the measurements, the accuracy penalty due to the imperfect anechoic terminations is still under study [6]. In the simulations, the accuracy penalty is easier to estimate and it leads to significant errors. This aspect is more or less apparent depending on the non-linear approach followed to simulate the silencer. In one-dimensional simulation, the numerical reflection coming from the outlet to the inner domain is very low when compared to the error introduced by the assumption of one-dimensional flow. The same holds for quasi-3D approaches [5], where the result is mainly influenced by the number of 0D volumes used for domain discretization rather than by the kind of non-reflecting boundary model. In multidimensional non-linear acoustic simulation, where the complete form of the Navier-Stokes equations is solved, the nature and the kind of implementation of the boundary conditions becomes very important. Boundaries may cause artificial wave reflections, which may include both physical waves and numerical waves, in particular when the non-reflective scheme does not account for full inter-field coupling. Artificial wave reflections from the boundary may appear as high frequency noise in the instantaneous pressure and velocity traces calculated by the code and they may lead to wrong predictions of the transmission loss, especially for frequencies above 1500 Hz. To try to overcome this problem, the multi-dimensional domain of the silencer was coupled in [7] with a long outlet pipe and with the anechoic termination modeled in one dimension, to allow the reflected waves from the inlet and outlet boundaries to arrive at the monitoring points at a time after which all the necessary acoustic data were recorded. Similarly, in [8] silencers were modeled with long inlet and outlet pipes in three dimensions. In this latter

case, the impact of the increase of the pipe length on the computational time required by the simulation may be very high, because it may lead to a significant increase of the computational grid size. Cutting off the computational domain that is not of primary interest (long inlet and outlet pipes) lead to smaller amount of computational cells, but makes the accuracy of unsteady flow calculations strongly rely on the treatment of the boundary conditions.

The aim of the study is to implement a fully predictive three-dimensional CFD time-domain approach to evaluate the transmission loss of complex silencers with zero and non-zero mean flow velocity of the gas, to test its reliability and to determine its limitations. In order to do this, a non-reflecting boundary condition based on the Navier Stokes Characteristic Boundary Conditions (NSCBC) theory for outlet terminations has been implemented in a multidimensional open-source CFD code [9] and it has been tested together with a time dependent compressible fully parallel Navier Stokes equation solver. The original local one dimensional inviscid relations (LODI) in Cartesian coordinates presented in [1] have been extended to local coordinates and have been solved by a multistage time stepping scheme. The paper is organized as follows: the implementation of the boundary condition is discussed first. Then, simulations to predict the acoustic attenuation performance of five different muffler configurations are presented. The case setup and the simulation parameters used have been reported and predictions of the multi-dimensional code have been compared with those coming from 1d simulation code, developed by the authors [10]. Finally, conclusions are given. The method adopted to measure the acoustic performance of a muffler is discussed in [Appendix A](#). The basic features of the 1d solver used are reported in [Appendix B](#).

NON REFLECTING B.C. FOR CFD COMPUTATION: STATE OF THE ART

The transmission loss has been chosen as a suitable magnitude representative of the frequency response of a given muffler. It is defined in terms of a logarithmic ratio between the acoustic power incident on the muffler and the acoustic power transmitted by the muffler.

$$TL(f) = 10 \log_{10} \left[\frac{A_i p_{in}^2(f)}{A_o p_{tr}^2(f)} \right] \quad [\text{dB}] \quad (1)$$

p_{in} is the acoustic pressure of incident wave at the inlet of the silencer, p_{tr} is the acoustic pressure of the transmitted wave at the outlet of the silencer; A_i and A_o are the cross-sectional areas where pressures are measured. The calculation of the incident Sound Pressure Levels (SPL) for the calculation of the Transmission Loss requires therefore the decomposition of waves, in order to discard spurious reflections from both the inlet open end and the silencer

itself. Moreover, an anechoic termination is needed at the outlet pipe to measure the transmitted pressure p_{tr} , so that the measured downstream pressure history is not affected by unwanted reflections from the outlet. The evaluation of the power of the incident waves, which appear in the calculation of the Transmission Loss, is based on the two sensor method, that is described in [Appendix A](#).

In most of the numerical codes, non-reflecting boundary conditions for compressible flow attempt to set the instantaneous value at the boundary (e.g., static pressure at an outlet boundary) as consistent as possible with the value of the incident pressure wave, to create a tendency towards the undisturbed pressure value p_∞ , defined over an ideal surface located at an arbitrary distance L_∞ from the boundary. The larger the value of L_∞ , the further the boundary condition will deviate from p_∞ . On the other hand, the smaller is the value of L_∞ , the more reflective the boundary tends to be. An alternative implementation of a non-reflecting boundary condition, that is very common in CFD codes [11, 12], makes direct use of the characteristic analysis of Thompson [13] to solve the simplified governing equations for inviscid flows along the normal direction to the outlet surface. As will be shown further in the paper (the boundary condition will be named waveTransmissive), non-reflection will be obtained by setting the amplitude of the characteristic wave reflected by the boundary to zero.

In both cases, pressure waves incident on the boundary result to be reflected in an unphysical manner, leading to local errors. This is unacceptable if predictions of Transmission Loss in the silencer has to be performed. A non-reflecting boundary condition must permit waves to pass through the boundaries without spurious reflections. A common method to reduce reflections from the outlet boundary consists of perform Fourier transformation of solution variables at the non-reflecting boundary; the solution is rearranged as a sum of terms corresponding to different frequencies, and their contributions are calculated independently. This implementation usually applies to steady compressible flows and with structured meshes to correctly handle the corners and performs quite well for steady flow problems. The most advanced implementation of non-reflecting boundary conditions is based on the NSCBC theory of Poinso [1], where to obtain the primitive flow quantities on the pressure-outlet, reformulated Euler equations are solved in cartesian coordinates on the boundary of the domain in an algorithm similar to the flow equations applied to the interior of the domain:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0 \\ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot [\mathbf{u}(\rho \mathbf{u})] + \nabla p = \nabla \cdot \mathbf{T} \\ \frac{\partial \rho E}{\partial t} + \nabla \cdot [\mathbf{u}(\rho E)] + \nabla \cdot (\mathbf{u}p) = -\nabla \cdot \mathbf{T} \cdot \mathbf{u} + \nabla \cdot \mathbf{q} \end{cases} \quad (2)$$

where p is the thermodynamic pressure, m_i is the momentum density in the x_i direction, ρE is the total energy density (kinetic + thermal):

$$\rho E = \frac{1}{2} \rho u_k \cdot u_k + \frac{p}{\gamma - 1} \quad (3)$$

$\mathbf{T}$ in [Eq. \(2\)](#) is the deviatoric stress tensor, whose elements are defined as follows:

$$\tau_{i,j} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \quad (4)$$

The heat flux along the direction x_i , namely q_i , is given by

$$q_i = -\lambda \frac{\partial T}{\partial x_i} \quad (5)$$

where the thermal conductivity λ is expressed as follows:

$$\lambda = \frac{\mu c_p}{Pr} \quad (6)$$

In the NSCBC approach, wave propagation is assumed to be associated only with the hyperbolic part of the Navier-Stokes equations, by neglecting waves associated with the diffusion process. While at high Reynolds numbers this approximation is probably well justified, at low Reynolds numbers the mathematical justification may be questionable. Despite of this, simulation results at low Reynolds numbers look satisfying.

DEVELOPMENT OF NSCBC IN LOCAL COORDINATES

In this work, the formulation of the characteristic boundary condition treatment for Navier-Stokes equations (NSCBC) originally suggested in Cartesian coordinates by Poinso and Lele [1] has been extended to local coordinates and it has been solved by a multistage time stepping scheme to improve the numerical performance. Results will confirm that the novel boundary condition contributes to achieve reliable predictions of TL when complex silencers are simulated, in particular in the high frequency range.

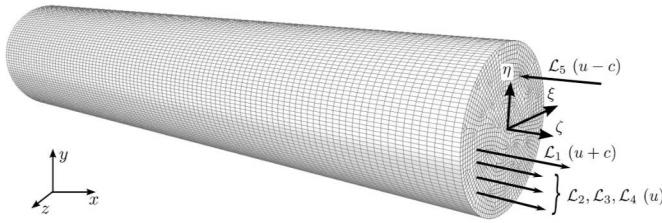


Figure 1. For each cell face at the boundary end, a local reference frame (ξ, η, ζ) has been defined. Each reference frame has its origin in the cell face center and the vector ζ is set as perpendicular to the cell face.

To transform the conservative system (2) from a physical domain (ξ, η, ζ) to a computational domain (x, y, z), a change of variables is required because, as it will be discussed later, the characteristic theory to include in the compressible subsonic Navier Stokes equations refers to a wave element that is perpendicular to the outlet domain. Hence, the following general transformation of the independent variables has been applied:

$$\begin{aligned} x &= x(\xi, \eta, \zeta) \\ y &= y(\xi, \eta, \zeta) \\ z &= z(\xi, \eta, \zeta) \end{aligned} \quad (7)$$

Being u_i the flow velocity in the local reference frame:

$$\begin{aligned} u_1 &= u_\xi \\ u_2 &= u_\eta \\ u_3 &= u_\zeta \end{aligned} \quad (8)$$

and the momentum density in the i -th direction (still written for the local reference frame):

$$m_i = \rho \cdot u_i \quad (9)$$

For a generic boundary end, under the assumption of inviscid flow, the system of governing equations (2) in local coordinates takes the form:

Mass conservation

$$\frac{\partial \rho}{\partial t} + d_1 + \frac{\partial m_2}{\partial \eta} + \frac{\partial m_3}{\partial \zeta} = 0 \quad (10)$$

Energy equation

$$\begin{aligned} \frac{\partial \rho E}{\partial t} + \frac{1}{2}(u_k \cdot u_k)d_1 + \frac{d_2}{\gamma - 1} + m_1 d_3 + m_2 d_4 + m_3 d_5 \\ + \frac{\partial[(\rho E + p)u_2]}{\partial \eta} + \frac{\partial[(\rho E + p)u_3]}{\partial \zeta} = -\nabla \cdot q + \frac{\partial \mu_j \tau_{ij}}{\partial x_i} \end{aligned} \quad (11)$$

Momentum equations

$$\begin{aligned} \frac{\partial m_1}{\partial t} + u_1 d_1 + \rho d_3 + \frac{\partial m_1 u_2}{\partial \eta} + \frac{\partial m_1 u_3}{\partial \zeta} &= \frac{\partial \tau_{1j}}{\partial x_j} \\ \frac{\partial m_2}{\partial t} + u_2 d_1 + \rho d_4 + \frac{\partial m_2 u_2}{\partial \eta} + \frac{\partial m_2 u_3}{\partial \zeta} &= -\frac{\partial p}{\partial \eta} + \frac{\partial \tau_{2j}}{\partial x_j} \\ \frac{\partial m_3}{\partial t} + u_3 d_1 + \rho d_5 + \frac{\partial m_3 u_2}{\partial \eta} + \frac{\partial m_3 u_3}{\partial \zeta} &= -\frac{\partial p}{\partial \zeta} + \frac{\partial \tau_{3j}}{\partial x_j} \end{aligned} \quad (12)$$

The different terms of the system of Eqs. (12) contain derivatives normal to the boundary (d_1 to d_5) and derivatives parallel to the boundary like $\frac{\partial}{\partial \eta}(m_2 u_\eta)$.

According to the characteristic analysis of Thompson [13], the vector $\mathbf{d}$ can be written as:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{c^2} [L_2 + \frac{1}{2}(L_5 + L_1)] \\ \frac{1}{2}(L_5 + L_1) \\ \frac{1}{\rho c}(L_5 - L_1) \\ L_3 \\ L_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial m_1}{\partial \xi} \\ \frac{\partial c^2 m_1}{\partial \xi} + u_1 \frac{\partial p}{\partial \xi} \\ u_1 \frac{\partial u_1}{\partial \xi} + \frac{1}{\rho} \frac{\partial p}{\partial \xi} \\ u_1 \frac{\partial u_2}{\partial \xi} \\ u_1 \frac{\partial u_3}{\partial \xi} \end{bmatrix} \quad (13)$$

where L_i represents the amplitude variation of the i_{th} characteristic wave crossing the boundary [14]:

$$\mathbf{L} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \end{bmatrix} = \begin{bmatrix} \lambda_1 \left(\frac{\partial p}{\partial \xi} - \rho c \frac{\partial u_1}{\partial \xi} \right) \\ \lambda_2 \left(c^2 \frac{\partial \rho}{\partial \xi} - \frac{\partial p}{\partial \xi} \right) \\ \lambda_3 \frac{\partial u_2}{\partial \xi} \\ \lambda_4 \frac{\partial u_3}{\partial \xi} \\ \lambda_5 \left(\frac{\partial p}{\partial \xi} + \rho c \frac{\partial u_1}{\partial \xi} \right) \end{bmatrix} \quad (14)$$

L_i are the amplitudes of characteristic waves associated with each characteristic velocity λ_i :

$$\begin{cases} \lambda_1 = u_1 - c \\ \lambda_2 = \lambda_3 = \lambda_4 = u_1 \\ \lambda_5 = u_1 + c \end{cases} \quad (15)$$

In particular, in (15), c is the local speed of sound, λ_1 and λ_5 are the velocity of sound waves moving along the positive and negative stream-wise direction, λ_2 is the velocity for

entropy advection, while λ_3 and λ_4 are the velocity at which u_2 and u_3 are advected in the ζ direction.

The complete system of governing equations (10),(11), (12) for compressible and viscous flows can therefore be written at the boundary as:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{1}{c^2} [L_2 + \frac{1}{2}(L_5 + L_1)] + \frac{\partial m_2}{\partial \eta} + \frac{\partial m_3}{\partial \zeta} = 0 \\ \frac{\partial p}{\partial t} + \frac{1}{2} (L_5 + L_1) + \frac{\partial[(\rho E + p)u_2]}{\partial \eta} + \frac{\partial[(\rho E + p)u_3]}{\partial \zeta} = -\nabla \cdot q \\ \frac{\partial u_1}{\partial t} + \frac{1}{2\rho c} (L_5 - L_1) + \frac{\partial m_1 u_2}{\partial \eta} + \frac{\partial m_1 u_3}{\partial \zeta} = 0 \\ \frac{\partial u_2}{\partial t} + L_3 + \frac{\partial m_2 u_2}{\partial \eta} + \frac{\partial m_2 u_3}{\partial \zeta} = -\frac{\partial p}{\partial \eta} \\ \frac{\partial u_3}{\partial t} + L_4 + \frac{\partial m_3 u_2}{\partial \eta} + \frac{\partial m_3 u_3}{\partial \zeta} = -\frac{\partial p}{\partial \zeta} \end{cases} \quad (16)$$

The chain rule for changing variables in equation (2) requires the operators:

$$\begin{aligned} \frac{\partial(\cdot)}{\partial \xi} &= \frac{\partial(\cdot)}{\partial x} x_\xi + \frac{\partial(\cdot)}{\partial y} y_\xi + \frac{\partial(\cdot)}{\partial z} z_\xi \\ \frac{\partial(\cdot)}{\partial \eta} &= \frac{\partial(\cdot)}{\partial x} x_\eta + \frac{\partial(\cdot)}{\partial y} y_\eta + \frac{\partial(\cdot)}{\partial z} z_\eta \\ \frac{\partial(\cdot)}{\partial \zeta} &= \frac{\partial(\cdot)}{\partial x} x_\zeta + \frac{\partial(\cdot)}{\partial y} y_\zeta + \frac{\partial(\cdot)}{\partial z} z_\zeta \end{aligned} \quad (17)$$

The partial derivatives in these equations occur in the differential expressions:

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{bmatrix} \begin{bmatrix} d\xi \\ d\eta \\ d\zeta \end{bmatrix} \quad (18)$$

In a similar manner, from the inverse transformation equations, the differentials can be derived:

$$\begin{bmatrix} d\xi \\ d\eta \\ d\zeta \end{bmatrix} = \begin{bmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \quad (19)$$

The transformations (18) and (19) are inverse of each other and so we can write:

$$\begin{aligned} \begin{bmatrix} x_\xi & y_\xi & z_\xi \\ x_\eta & y_\eta & z_\eta \\ x_\zeta & y_\zeta & z_\zeta \end{bmatrix} &= \begin{bmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{bmatrix}^{-1} = \\ &= \vec{J} \cdot \begin{bmatrix} \eta_y \zeta_z - \eta_z \zeta_y & -(\xi_y \zeta_z - \xi_z \zeta_y) & \eta_x \zeta_y - \xi_x \eta_y \\ -(\xi_y \zeta_z - \xi_z \zeta_y) & \xi_x \zeta_z - \xi_z \xi_x & -(\xi_x \zeta_y - \xi_y \zeta_x) \\ \xi_y \eta_z - \eta_y \xi_z & \xi_x \eta_z - \eta_x \xi_z & \xi_x \eta_y - \eta_x \xi_y \end{bmatrix} \end{aligned} \quad (20)$$

where $\vec{J}$ is the Jacobian matrix, not singular:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} & \frac{\partial \xi}{\partial z} \\ \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} & \frac{\partial \eta}{\partial z} \\ \frac{\partial \zeta}{\partial x} & \frac{\partial \zeta}{\partial y} & \frac{\partial \zeta}{\partial z} \end{bmatrix} \quad (21)$$

By comparing like elements in equation (20) it follows:

$$\begin{aligned} x_\xi &= \mathbf{J}(\eta_y \zeta_z - \eta_z \zeta_y) \\ x_\eta &= -\mathbf{J}(\xi_y \zeta_z - \xi_z \zeta_y) \\ x_\zeta &= \mathbf{J}(\xi_y \eta_z - \xi_z \eta_y) \\ y_\xi &= -\mathbf{J}(\eta_x \zeta_z - \eta_z \zeta_x) \\ y_\eta &= \mathbf{J}(\xi_x \zeta_z - \xi_z \zeta_x) \\ y_\zeta &= -\mathbf{J}(\xi_z \eta_z - \xi_z \eta_x) \\ z_\xi &= \mathbf{J}(\eta_x \zeta_y - \eta_y \zeta_x) \\ z_\eta &= -\mathbf{J}(\xi_x \zeta_y - \xi_y \zeta_x) \\ z_\zeta &= \mathbf{J}(\xi_x \eta_y - \xi_y \eta_x) \end{aligned} \quad (22)$$

The governing equations may be therefore written as:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_1}{\partial x} + \frac{\partial \mathbf{F}_2}{\partial y} + \frac{\partial \mathbf{F}_3}{\partial z} = -\nabla p \quad (23)$$

where

$$\begin{aligned} \mathbf{U} &= \frac{\hat{\mathbf{U}}}{J} \\ \frac{\partial \mathbf{F}_1}{\partial x} &= \frac{\hat{\mathbf{F}}_1 x_\xi + \hat{\mathbf{F}}_2 x_\eta + \hat{\mathbf{F}}_3 x_\zeta}{J} \\ \frac{\partial \mathbf{F}_2}{\partial y} &= \frac{\hat{\mathbf{F}}_1 y_\xi + \hat{\mathbf{F}}_2 y_\eta + \hat{\mathbf{F}}_3 y_\zeta}{J} \\ \frac{\partial \mathbf{F}_3}{\partial z} &= \frac{\hat{\mathbf{F}}_1 z_\xi + \hat{\mathbf{F}}_2 z_\eta + \hat{\mathbf{F}}_3 z_\zeta}{J} \end{aligned} \quad (24)$$

If we compare the system Eq. (10),(11),(12) with Eq. (23)-(24), it follows:

$$\frac{\partial \hat{\mathbf{F}}_1}{\partial \xi} = \begin{bmatrix} \rho d_1 \\ \rho u_1 d_1 + \rho d_3 \\ \rho u_2 d_1 + \rho d_4 \\ \rho u_3 d_1 + \rho d_5 \\ \frac{1}{2}(u_k \cdot u_k) d_1 + \frac{d_2}{\gamma-1} + m_1 d_3 + m_2 d_4 + m_3 d_5 \end{bmatrix} \quad (25)$$