

Composing Tradeoff Studies under Uncertainty based on Parameterized Efficient Sets and Stochastic Dominance Principles

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ABSTRACT

Tradeoff studies are a common part of engineering practice. Designers conduct tradeoff studies in order to improve their understanding of how various design considerations relate to one another and to make decisions. Generally a tradeoff study involves a systematic multi-criteria evaluation of various alternatives for a particular system or subsystem. After evaluating these alternatives, designers eliminate those that perform poorly under the given criteria and explore more carefully those that remain.

One limitation of current practice is that designers cannot combine the results of preexisting tradeoff studies under uncertainty. For deterministic problems, designers can use the Pareto dominance criterion to eliminate inferior designs. Prior work also exists on composing tradeoff studies performed under certainty using an extension of this criterion, called parameterized Pareto dominance. The capability to compose preexisting tradeoff studies is advantageous to the designers of complex systems, such as aircraft, military equipment, and automobiles. For example, automotive systems engineers could combine tradeoff studies from the engine and transmission subsystems quickly to produce a comprehensive tradeoff study for the power train. This level of knowledge reuse is in keeping with good systems engineering practices. However, existing procedures for generating tradeoff studies involve assumptions that preclude the valid composition of tradeoff studies under uncertainty.

In this paper we describe a new approach that permits engineers to compose preexisting subsystem-level tradeoff studies under uncertainty into mathematically valid system-level tradeoff studies. The approach is based on two key ideas: the use of stochastic dominance methods to enable the tradeoff evaluation when the values of the design criteria are not known with certainty (i.e., tradeoff studies under uncertainty) and the use of parameterized efficient sets to enable reuse and composition of subsystem-level tradeoff studies. The key ideas and their mathematical validity are described. The overall approach is demonstrated in the context of a tradeoff study for a motor vehicle.

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INTRODUCTION

Designers often must compose unique system designs out of common types of components or subsystems. In composing the system, designers must determine which combination of components maximizes the overall performance of the design. Frequently components are employed in previous designs where the components' concept tradeoffs were already considered. Enabling designers to leverage this previous knowledge about component concepts is advantageous in design and especially applicable in composing system-level concepts from reusable tradeoff-space models of common functional components. This is in keeping with good engineering practice and enables a

designer to incorporate detailed tradeoff considerations into their decision making without requiring specialized domain knowledge about every component concept.

For example, consider an automobile company who decides to develop a new gas efficient vehicle to attract budget and environmentally conscious consumers. System level requirements are generated for the vehicle drivetrain and the design is separated into multiple teams, each tasked with designing a specific component. Each team considers multiple concepts and must decide which will maximize system performance, as illustrated in [Figure 1](#). If the teams individually consider the component tradeoffs in choosing a concept, they may not select the best system design. They must consider the effects of each component concept on the

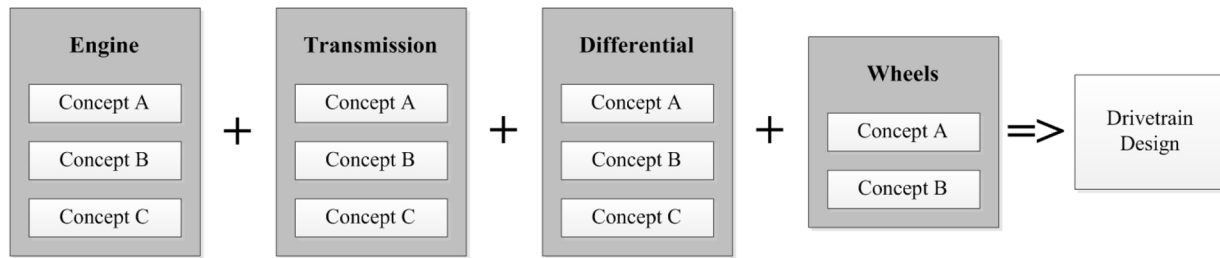


Figure 1. Selecting component concepts and composing vehicle drivetrain.

system performance. Consequently a means to compose the component tradeoffs to form system-level tradeoffs is necessary to determine which set of concepts results in the best design for the drive train.

Researches have demonstrated the composition of tradeoff studies under deterministic conditions [1] and techniques for the non-compositional tradeoff studies under uncertainty [2]. In this paper, we combine these two techniques in a novel way to enable compositional tradeoff modeling under uncertainty.

We demonstrate our approach on the conceptual design of an automobile drivetrain, composed of an engine, transmission, differential, and wheel components. The engine and wheel components are predetermined. Single physical configuration concepts are considered for the transmission and differential. For each concept, we identify the parameterized efficient set using stochastic dominance principles and fit a predictive tradeoff model to it for reusability. We compose the models into a system-level tradeoff analysis. We support our results by comparing them to a baseline method of exploring the design space from the system level.

BACKGROUND

TRADEOFF STUDIES UNDER CERTAINTY WITH PARETO DOMINANCE

The term tradeoff refers to a decision-making situation in which an improvement in one property of a solution is impossible without a sacrifice in some other property. For example, in designing an automobile, an engineer may be unable to improve vehicle acceleration without sacrificing fuel efficiency. In this paper, we refer to the performance metrics with which designers evaluate designs as *attributes*. Mapping component concepts to the tradeoff space, where under deterministic conditions each dimension represents a design attribute, allows one to visualize various alternative design approaches. Designers then select the final design or may decide to develop additional concepts after viewing a variety of feasible alternatives. Some authors refer to this approach as “design by shopping” [3].

When visualizing or generating a mathematical model of the tradeoff space, it is beneficial to eliminate alternatives a rational designer would never select, known as dominated alternatives. Non-dominated alternatives are known as

efficient alternatives, and the set of all non-dominated alternatives is known as the *efficient set*. Considering only the efficient set of alternatives eliminates clutter and allows designers to focus on what is important [4]. In deterministic problems, the efficient set of alternatives is known as the Pareto set [5]. Pareto dominance criteria eliminates all points where an alternative is at least better in one attribute without making any other attribute worse off.

Mathematically expressing classical Pareto dominance requires defining some notation. Suppose a designer's preferences are monotonically increasing in each decision attribute, x_i for $i=1, \dots, N$, associated with a particular component. If $x=[x_1, x_2, \dots, x_N]$ denotes an attribute vector and X is the set of all feasible attribute values, then classical Pareto dominance can be expressed as

Classical Pareto dominance: An alternative with attribute vector $x'' \in X$ is said to be Pareto dominated by one with attribute vector $x' \in X$ if and only if $x'_i \geq x''_i \forall i = 1, \dots, N$ and $x'_i > x''_i \exists i = 1, \dots, N$.

Figure 2 illustrates the deterministic mapping of two unique design spaces for a component into the tradeoff space. Each design space represents a different concept with a unique design configuration. Since the tradeoff criteria are related to the designer's objectives, each concept for a component has the same tradeoff space. In deterministic decisions, each design implementation maps to a single point in the attribute space. Thus in this case we refer to the attribute space as the tradeoff space because attributes are all we need to test dominance.

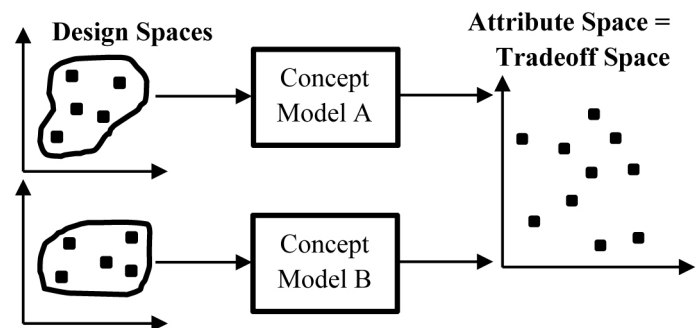


Figure 2. Deterministic mapping of design spaces to tradeoff space.

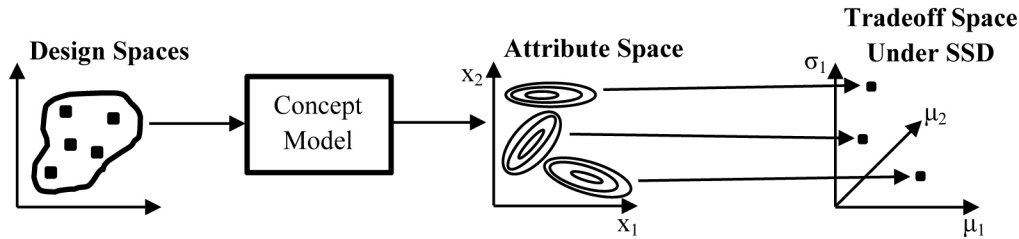


Figure 3. Mapping of design space to attribute space to tradeoff space under uncertainty. The tradeoff space is a 3d visualization of the tradeoff space that is higher dimensional.

TRADEOFF STUDIES UNDER UNCERTAINTY WITH STOCHASTIC DOMINANCE

In decisions under uncertainty, the design space maps into probability distributions in the attribute space, illustrated in first part of [Figure 3](#). Thus to identify the efficient set of alternatives, designers cannot use Pareto dominance, which is inherently tied to deterministic decisions. Other authors have discussed how to handle uncertainty in tradeoff studies. Mattson and Messac approach uncertainty by identifying the Pareto frontier using the attribute means then worsen each based on their corresponding standard deviations, creating a conservative shifted frontier [6]. However, this approach may be over-conservative and lacks the ability to allow the one to consider tradeoffs between the mean and variance of an attribute, which is often important to a designer. We employ the theory of stochastic dominance for this task.

Stochastic dominance has been primarily used in the areas of economics, finance, and statistics [7]. Multiple stochastic dominance rules exist which depend on the mathematical structure of the utility function employed, where utility is a measure of overall satisfaction. With the exception of our prior work, stochastic dominance has not been applied to engineering design problems [2].

Using stochastic dominance, designers compare two cumulative distribution functions and establish an order of preference between the two. In literature there are three common single-attribute classes of stochastic dominance: first-degree stochastic dominance (FSD), second-degree stochastic dominance (SSD), and third-degree stochastic dominance (TSD). [Table 1](#) is a summary of the three classes with their assumption and domination criteria. Each successive class become more restrictive; that is each class is a subset of the previous. For example, an efficient set in FSD will have solutions which are considered dominated and eliminated under SSD. It is best to evaluate solutions with the most restrictive class justifiable by the intended use of the solutions in order to reduce problems with identifying the efficient set with the less restrictive assumptions.

The stochastic dominance classes described in [Table 1](#) are for single-attribute decisions. This is rarely the case in engineering design where designers often must make decisions based on multiple attributes. Research has expanded the single-attribute stochastic dominance to multi-attribute problems, but this process can be difficult to apply [8, 9]. Multi-attribute stochastic dominance involves comparing multivariate distribution functions. However, if one assumes the distributions are statistically independent, one can test the multi-attribute dominance condition by testing the corresponding single-attribute dominance condition on the marginal distributions [9]. This is not a perfect assumption for all engineering problems, but is a good approximation in many cases. We use this assumption in this paper and leave a more robust investigation for future work.

If one also assumes the distributions are normal, the second-degree stochastic domination (SSD) criterion simplifies to comparing the corresponding means and variances. This assumption largely simplifies the mathematics in applying SSD. When using this assumption, it is important to collect enough data to produce a reliable estimator of the true mean and variance. This is an imperfect assumption like the independence assumption; however it is a good approximation for many engineering problems. We use this assumption in this paper and leave a more robust investigation for future work.

One can define several common distributions with a few parameters (e.g. mean and variance for normal distributions, interval boundaries for uniform distributions). If these common probability distributions can accurately represent the uncertainty, designers can map the attribute space to a space of distribution parameters. If the distributions are normal, we consider this the tradeoff space because we can do dominance reasoning using SSD in this space. The second part of [Figure 3](#) shows the mapping of the attribute space to the tradeoff space in terms of the distribution parameters.

Consider a performance attribute of a design implementation under uncertainty. The attribute is known to within a distribution function. If the distribution can be described as normal, it can be represented by a mean and

Table 1. Summary of three common single-attribute stochastic dominance classes.

Class	Utility Function Assumptions	Interpretation	Dominance Criterion
U_0	None	All utility functions.	None
U_1	$U_1 = \left\{ u \in U_0 \left \frac{du(x)}{dx} \geq 0 \right. \right\}$	Monotonic utility functions.	Option a with CDF F_a dominates option b with CDF F_b if and only if for all x : $F_a(x) \leq F_b(x)$
U_2	$U_2 = \left\{ u \in U_1 \left \frac{d^2u(x)}{dx^2} \geq 0 \right. \right\}$	Monotonic and non-risk taking utility functions.	Option a with CDF F_a dominates option b with CDF F_b if and only if for all x : $\int_{-\infty}^x [F_b(t) - F_a(t)] dt \geq 0$
U_3	$U_3 = \left\{ u \in U_2 \left \frac{d^3u(x)}{dx^3} \geq 0 \right. \right\}$	Monotonic, non-risk taking, and decreasing absolute risk aversion utility functions.	Option a with CDF F_a dominates option b with CDF F_b if and only if for all x : $\int_{-\infty}^x \int_{-\infty}^v [F_b(t) - F_a(t)] dt dv \geq 0$

variance. Often a designer is willing to tradeoff the mean performance in order to reduce variability in the attribute. Thus representing alternatives in the tradeoff space by distribution parameters is a good representation of the criteria which designers base decisions. This type of tradeoff is common in robust design [10, 11].

Mathematically expressing the multivariate second-degree stochastic dominance criterion with the independence and normality assumptions requires defining some additional notation. Suppose a designer's preferences are monotonically increasing in each decision attribute. Let a and b represent two decision alternatives. Let x represent an N -dimensional attribute vector where the distribution of attributes is represented by $F_a(x)$ and $F_b(x)$. The mean of the i th attribute for each of the two alternatives is $\mu_{a,i}$ and $\mu_{b,i}$. The variance of the i th attribute for each of the two alternatives is $\sigma_{a,i}^2$ and $\sigma_{b,i}^2$. The multivariate stochastic dominance criterion can now be expressed as

Multivariate (multi-attribute) second-degree stochastic dominance: If $F_a(x)$ and $F_b(x)$ are independent and normally distributed, then $F_a(x)$ dominates $F_b(x)$ by multivariate SSD if and only if $\mu_{a,i} \geq \mu_{b,i}$ and $\sigma_{a,i}^2 \leq \sigma_{b,i}^2$ for all $i=1 \dots N$ where at least one of the inequalities is strict.

Figure 4 illustrates the differences in data flow when generating the efficient set of data for deterministic and uncertain decision making, where the tradeoff exploration method under uncertainty utilizes second-order stochastic dominance with the statistical independence and normality assumptions.

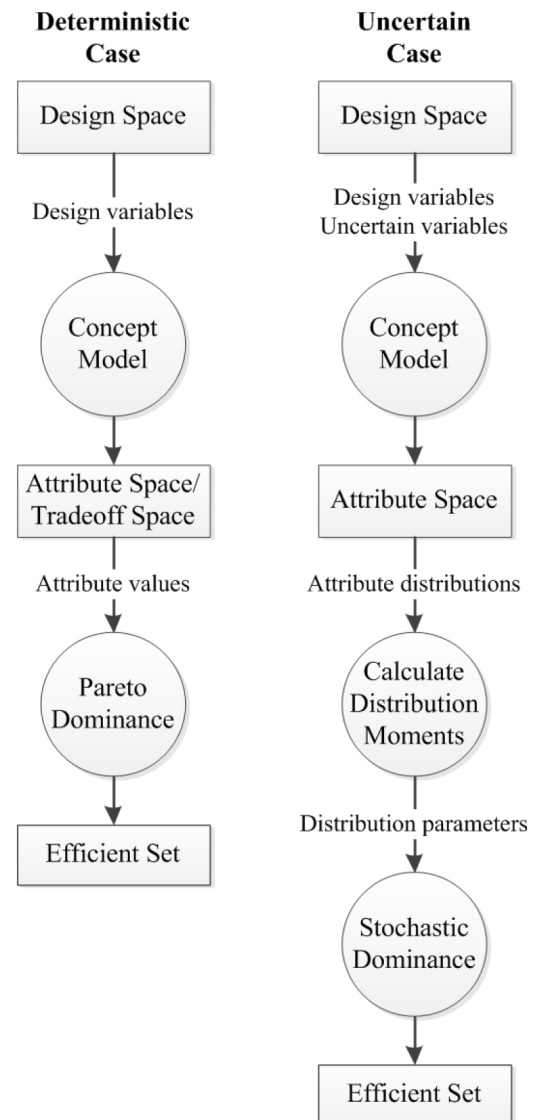


Figure 4. Efficient data set generation flow chart for deterministic and uncertain decision making.