

Improving the Convergence of the Fast Multipole BEM for the Exterior Sound Radiation of a Truck Muffler

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ABSTRACT

In the last few years, some of the computational limitations imposed by the classical Boundary Element Methods (BEM) have been overtaken thanks to the Fast Multipole BEM (FMBEM) which allows solving large acoustic models much faster.

Nevertheless, as the BEM, the FMBEM suffers from non-uniqueness of the solution and the exterior prediction is polluted by fictitious resonances. Since the FMBEM is based on an iterative solver, not only the accuracy is degraded. In fact, the number of iterations to the convergence, in correspondence of a fictitious resonance, is larger and the total computational time increases. Applying the impedance condition over the whole inner boundary allows to completely damp fictitious resonances with a drastic increase of computational cost. Nevertheless, when the condition is applied on a well-chosen percentage of elements, the accurate solution is obtained even faster.

The efficiency of this approach and its applicability to industrial-size problems is proved in this paper. Numerical predictions of the exterior noise emitted by a truck muffler are compared with experimental data. After a comparison between classical BEM and FMBEM solutions, the improvement obtained by properly applying the internal absorption is illustrated both in terms of solution accuracy and computational efforts.

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INTRODUCTION

Nowadays, one of the main sources of environmental noise exposure is the traffic noise. Even though the European regulations were introduced in the 1970s [1], noise levels are not decreased in the last forty years. In the early stages, the directive was focused on defining noise limits for vehicle and the objective was mainly market oriented. Later the attention was extended to higher level of protection from noise pollution.

The overall vehicle noise is mainly emitted by three sources: the propulsion (engine, powertrain, exhaust and intake systems), tyre/road contact and aerodynamic turbulence. At low speed, the engine is the main source of noise, while tyre noise is dominant for higher speed; the aerodynamic noise is variable and dependent on the speed and the shape of the vehicle.

To regulate the noise emission and the technical approval of new vehicle, European standards are laid down in the directive 2007/34/EC (amending the directive [1]). The

directive 2007/34/EC concerns the permissible sound level and the exhaust system of motor vehicles. The noise limit defined within the directive includes all the sources of noise from the vehicle. Currently, the limitation for passenger cars (< 9 persons) is set to 74dB (A) while for trucks (≥ 150 kW) it is 80dB (A).

Meeting the regulation and reducing the noise pollution generated by vehicles are two challenging tasks for acoustic engineers. The exterior noise radiation of trucks is an excellent example since they have always been one of the major contributors to traffic noise [2, 3]. In this contest, CAE tools are necessary to simulate and improve the performance of products.

When dealing with exterior acoustic problems, the Boundary Element Method (BEM) [4] represents a reliable and accurate technique. If compared to the Finite Element Method (FEM) [5] the dimensionality of the problem is decreased by one. For instance, three-dimensional models are described by two-dimensional integral equations and only

their surfaces are required to be meshed. Consequently, the BEM formulation is suitable for handling exterior unbounded problems which are more difficult to solve with the FEM. The BEM matrices are complex and fully populated. Moreover, since reaching an accurate solution requires at least six elements per wavelength, moving to higher frequencies or solving large models may be computationally very demanding.

In the last few years, the Fast Multipole BEM (FMBEM) [6] allowed tackling this limitation. Firstly, Greengard and Rokhlin [7, 8] introduced the multipole expansion of the Helmholtz equation and later these concepts have been applied to large acoustic problems as aircraft, ships, trucks and complete models as engines, turbines, i.e. [9]. Instead of solving the model in one go, the FMBEM splits it up in subdomains which are successively split in smaller ones and treated as classic BEM models. Building such a hierarchical structure allows a matrix reorganization and a reduction of solving time and memory usage.

The main disadvantage related to the BE techniques is the non-uniqueness problem. When solving exterior problems, the BEM fails in providing the correct solution at characteristic values which are the eigenfrequencies of the related internal problem [10]. At those frequencies, the BEM matrices suffer from ill-conditioning [11] and the solution can be seriously compromised by the presence of fictitious resonances. Due to the iterative nature of the solver, the non-uniqueness problem may become even more critical for the FMBEM. In fact, in correspondence of a fictitious resonance, a larger number of iterations are needed to reach the convergence and the simulation becomes more expensive computationally. Furthermore, since the modal density increases with frequency, the solution will be more polluted at small wavelengths, where the FMBEM shows the major benefits.

In order to overcome this problem several methods are proposed in the literature. For the direct BEM (DBEM), the Combined Helmholtz Integral Equation Formulation (CHIEF) method [12] suggests to add to the system of equations integral relations referred to points placed inside the cavity. This approach is implemented in many acoustic softwares, and applications to acoustic cases are present in the literature [13, 14]. Another widely used approach is the Burton and Miller [15] method which is based on an integral equation that is a combination of the original surface integral and its normal derivative.

Besides some enhanced formulations, practical approaches are common for the indirect BEM (IBEM). Dummy elements with impedance can be placed inside the cavity of the model to damp internal resonances and reduce the pollution on the exterior [16].

Wu [17] proposes to simultaneously apply the velocity and unequal impedance boundary condition on each side of the domain.

The approach used in this work is the so-called internal impedance method [18]. It consists in to apply an impedance boundary condition on the interior of the model cavity. To

reduce the high computational load required, the boundary condition is applied only over a limited number of elements [19]. This allows reaching a good balance between accuracy of solution and computational effort.

This paper investigates how the application of the internal impedance method improves the FMBEM convergence and speeds up the calculation. Moreover, the enhanced technique is compared with the classic BEM both in terms of solution accuracy and computational performances. The exterior acoustic radiation of a Volvo truck muffler is used as test case. Numerical results are compared with measurements to show the accuracy of the predictions.

The paper is organised as follows. In the first part, the BE techniques and the non-uniqueness of the solution are introduced. The second part deals with the application case: numerical models and measurements are described and compared. Finally, the computational advantages of the FMBEM techniques are highlighted and discussed.

THE BOUNDARY ELEMENT METHOD AND THE FAST MULTIPOLE BOUNDARY ELEMENT METHOD

THE HELMHOLTZ EQUATION AND BOUNDARY CONDITIONS

Many problems related to steady-state oscillations lead to the Helmholtz equation,

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0 \quad \text{on } D \quad (1)$$

where ∇^2 is the Laplacian operator, p is the acoustic pressure at point $\mathbf{x}$, interior to the domain D and k is the wavenumber. D is supposed to be partitioned into an exterior D^+ and an interior D^- domain, of boundaries ∂D^+ and ∂D^- , respectively. Solving the acoustic problem (1) requires the application of prescribed boundary conditions. The most general type of boundary condition is the so-called Robin or impedance boundary condition,

$$\frac{\partial p}{\partial n}(\mathbf{x}) + i \frac{\rho \omega}{Z} p(\mathbf{x}) = g \quad \text{on } \partial D_Z \quad (2)$$

By means of the impedance Z , the pressure at $\mathbf{x}$ is directly connected to its normal derivative on the boundary partition ∂D_Z . The function g is zero for acoustic scattering problems and non-zero for radiation problems.

The rate of increase of the pressure in $\mathbf{n}$ direction at a point $\mathbf{x}$ corresponds to the Neumann boundary condition on the boundary partition ∂D_N , and is expressed as function of the normal velocity v_n ,

$$\frac{\partial p}{\partial n}(\mathbf{x}) = -i\rho\omega v_n \quad \text{on } \partial D_N \quad (3)$$

Finally, the Dirichlet boundary condition defines the pressure over the boundary partition ∂D_p ,

$$p(\mathbf{x}) = \hat{p} \quad \text{on } \partial D_p \quad (4)$$

THE BOUNDARY ELEMENT METHOD

Integrating eq.(1) twice via Green's theorem over one side of the domain (i.e. ∂D^+), using the free-space Green's functions

$$g(\mathbf{x}, \mathbf{y}) = \frac{e^{-ikr}}{4\pi r} \quad (5)$$

where $r = |\mathbf{x} - \mathbf{y}|$, leads to the direct Helmholtz Integral Equation (dHIE),

$$c_s(\mathbf{x})p(\mathbf{x}) = \int_{\partial D} p(\mathbf{y}) \frac{\partial g(\mathbf{x}, \mathbf{y})}{\partial n(\mathbf{y})} ds - \int_{\partial D} g(\mathbf{x}, \mathbf{y}) \frac{\partial p(\mathbf{y})}{\partial n(\mathbf{y})} ds \quad (6)$$

where $c_s(\mathbf{x})$ is a coefficient dependent on the position of the point $\mathbf{x}$ and $\mathbf{y}$ is the integration variable. If $\mathbf{x}$ is inside the domain, c_s is equal to 1; if it is outside the domain, c_s is equal to 0; if it is placed on the boundary, c_s is equal to $\frac{1}{2}$ for smooth surfaces. The direct Normal Derivative Helmholtz Integral Equation (dNDHIE) can be obtained computing the derivative of eq.(6) with respect to the normal direction at point $\mathbf{x}$. The dHIE and dNDHIE allow to solve the problem either on the exterior D^+ or on the interior D^- domain. To extend their validity to the whole D , the solutions on both the subdomains can be superposed obtaining an equivalent formulation [20]. Taking into account the normal to the surface leads to the so-called indirect Helmholtz Integral Equation (iHIE)

$$c_s(\mathbf{x})p(\mathbf{x}) = \int_{\partial D} \mu(\mathbf{y}) \frac{\partial g(\mathbf{y}, \mathbf{x})}{\partial n} ds(\mathbf{y}) - \int_{\partial D} g(\mathbf{y}, \mathbf{x}) \sigma(\mathbf{y}) ds(\mathbf{y}) \quad (7)$$

where $\mathbf{x} \in D^+ \cup D^-$, and to the indirect Normal Derivative Helmholtz Integral Equation (iNDHIE)

$$c_s(\mathbf{x}) \frac{\partial p(\mathbf{x})}{\partial n(\mathbf{x})} = \int_{\partial D} \mu(\mathbf{y}) \frac{\partial g^2(\mathbf{x}, \mathbf{y})}{\partial n(\mathbf{y}) \partial n(\mathbf{x})} ds - \int_{\partial D} \frac{\partial g(\mathbf{x}, \mathbf{y})}{\partial n(\mathbf{x})} \sigma(\mathbf{y}) ds \quad (8)$$

Eq.(7) expresses the acoustic pressure as the field in free-space due to monopoles and dipoles distributed over the boundary, known as double layer potential, with density μ , and single layer potential, with density σ ,

$$\mu(\mathbf{y}) = p^+(\mathbf{y}) - p^-(\mathbf{y}) \quad (9)$$

$$\sigma(\mathbf{y}) = \frac{\partial p^+(\mathbf{y})}{\partial n} - \frac{\partial p^-(\mathbf{y})}{\partial n} \quad (10)$$

with $p^+(\mathbf{y})(p^-(\mathbf{y}))$ and $\frac{\partial p^+(\mathbf{y})}{\partial n(\mathbf{y})} (\frac{\partial p^-(\mathbf{y})}{\partial n(\mathbf{y})})$ are the pressure and its derivative on the positive (negative) side of the boundary ∂D^+ (∂D^-). Computing the closed-form solution to problem (1) is impossible for complex cases, thus the previous integral equations are solved numerically on discretized domains [17].

Usually, the direct formulation is solved by a collocational approach and the resulting matrices are complex and non-symmetric. On the other hand, the variational scheme is used for the indirect formulation [21], this allows to reduce hypersingular integrals to a singular form. Moreover, minimizing the functional leads to a symmetric and complex system of equations

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_\sigma \\ \mathbf{f}_\mu \end{bmatrix} \quad (11)$$

where A , B , C and D are $n \times n$ matrices, n is the number of dof, $\mathbf{f}_\sigma$ and $\mathbf{f}_\mu$ represent the terms containing the boundary conditions of the problem. To solve the system (11), n conditions are required. Their expression can be written as function of the densities using eq.(9) and (10).

If the same type of condition is applied on both the exterior and the interior boundaries, the system of equations can be reduced. Supposing that the pressure (the velocity) is known over ∂D^+ and ∂D^- both, the density of the double layer potential μ (the single layer potential σ) is consequently known from eq.(9) (eq.(10)); the only unknown is the single layer density (the double layer potential) on the boundary and the system dimension can be reduced to n equations.

If discontinuous boundary conditions are applied, these kind of simplifications are not allowed and the number of equations to solve is $2n$.

The methodology used in the following requires the application of discontinuous Robin boundary conditions across the boundary [22]. The system dimension will be increased proportionally to the number of elements on which the Robin boundary condition is applied.

THE FAST MULTIPOLE BEM

Even if the BEM is a reliable and accurate technique for acoustic simulation, solving large models can potentially lead to prohibitive computational times and memory requirements. Moreover, the direct connection between the frequency of analysis and the required number of elements limits the applicability of the standard BEM to the low frequency range.

The FMBEM represents an enhancement of the standard BEM: the computational limitations are overcome and the solving process is accelerated. The algorithm divides the domain in near and far field. The former is solved by using the classical BEM, whereas for the latter, a clustering

operation is applied to the model and the solution is evaluated by a multipole expansion.

The interaction between nodes in the standard BEM is described by the Green's kernel eq.(5), whose intensity decreases with the distance r . The FMBEM exploits this characteristic to approximately calculate the far interactions between x and y , Figure 1. To get separate integrals in the formulation, the vector connecting the source x and target y points of the Green's function is split in three parts: $\overrightarrow{x_i M_1}$ (Moment-to-Moment translation), $\overrightarrow{M_1 M_2}$ (Moment-to-Local translation), and $\overrightarrow{M_2 y_i}$ (Local-to-Local translation), as illustrated in Figure 1 and carefully explained in [6].

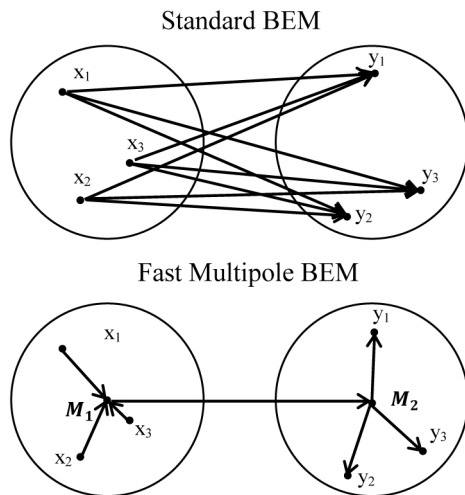


Figure 1. Comparison between the standard BEM and the FMBEM algorithms.

According with [23], the FMBEM is based on a six-step algorithm,

1. Discretization of the boundary. The model is discretized as in the classical BEM.

2. Tree structure. In practice, the model is divided in cells, defining the so-called *octree* and building a *multi-level* hierarchical structure. Each *parent* cell, at level l , is divided into 2^d *child* sub-cells (where d is the dimensionality of the problem) of equal dimension. To benefit of the multipole expansion by treating distant cells, the octree must be divided at least twice, or equivalently be a “level 2” octree. The size of the smallest cells is set equal to a quarter of the wavelength of analysis. The higher the frequency, the more distant cells will be present in the octree and consequently, the more accelerated will be the computation. This explains why the FMBEM solver is very efficient at high frequencies where the wavelength is small if compared to the size of the structure. A detailed explanation of the FMBEM terminology is provided in Figure 2.

3. Upward pass. Expansions on all cells at all levels $l \geq 2$ are computed: the contribution from every source x is

expanded at point $M1$, Figure 1. For a parent cell, the expansion is evaluated by computing the sum of the expansions from all its child cells. This operation is called *Moment-to-Moment translation* and allows expressing the expansion computed at x as an expansion computed at the cell centroid $M1$.

4. Downward pass. Contributions from all the far cells are computed by means of the *Moment-to-Local translation*. This operation calculates the expansion at point $M1$ as an expansion calculated in $M2$, Figure 1. Contributions from the *interaction list* (Figure 2) to the child cell, in which the target point is situated, are computed by the *Local-to-Local translation*. Using these operators, the expansion is computed in $M2$ at observation point y (Figure 1).

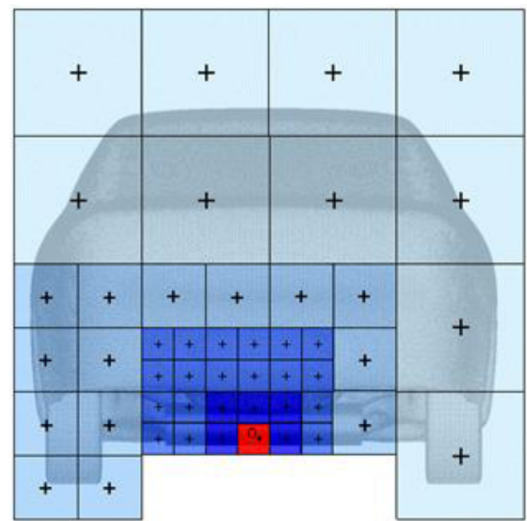


Figure 2. FMBEM definitions. Every parent cell is divided into child cells; a cell without child cells is called *leaf*. Two cells at the same level that have at least one common vertex are called *adjacent cells at level l*. If two cells are not adjacent at level l but their parents are adjacent at level $l-1$, they are called *well-separated at level l*. The list of the well-separated cells from a cell C is called *interaction list of C*. Cells are called to be *far cells of C* if their parent cells are not adjacent to the parent cell of C . (Courtesy of LMS International)

5. Final summation and integration. The contribution of the cell C and its *adjacent cells* are evaluated by means of the standard BEM. Computing the local expansion it is possible to calculate all the contributions from the other cells of the octree. The expansion point is shifted from the centroid of the cell C to the point where the integral is evaluated. This allows to reduce the number of transfer operators and thus to accelerate the computation.

6. Iteration of the solution. The unknown vector of the linear system is updated and the systems is solved iteratively until the solution converges within a given tolerance.