

Experimental Two-Port Characterization of the Aeroacoustic Transmission Properties of a Truck's Exhaust System

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ABSTRACT

Maximizing the acoustic attenuation is one of the important design criteria of automotive exhaust systems. Although both analytical and numerical approaches exist to evaluate the acoustic transmission properties of exhaust systems, they are, at present, insufficient to model the full geometrical complexity and to accurately assess the influence of thermal and aerodynamic phenomena onto the acoustic attenuation characteristics. For this reason, an experimental test campaign is often still indispensable to evaluate the aeroacoustic performance of exhaust systems. One of the most commonly used experimental characterization techniques for flow duct systems is the two-port characterization. Although this measurement approach is limited to low frequencies, below the traversal cut-on frequency of the inlet and outlet ducts, it allows making an in-depth analysis of the acoustic transmission properties of duct systems, including flow and thermal effects with an arbitrary geometrical complexity. Furthermore, using a two-port characterization allows combining various duct elements, which are measured and/or modeled independently from each other, in a network approach, enabling a straightforward acoustic evaluation of various flow-duct system design alternatives. In this paper, an experimental two-port determination technique is used to assess the influence of aerodynamic effects on the acoustic transmission properties of a part of a truck's exhaust system, containing four different components: an inlet bent duct; a flexible, corrugated, connection pipe; a complex muffler configuration; and a tailpipe. The experimental methodology is based on a multiple microphone calibration technique to determine the various environmental parameters. In this way, the final accuracy is increased and the need of accurate instrumental assessment of these parameters is avoided. The aeroacoustic performance of the various flow duct elements is determined and analyzed for each component and the use of a network approach, combining the various subcomponents is evaluated. It is shown that an experimental two-port characterization technique provides useful insight in the noise attenuation principles of flow duct systems and is ideally suited to be adopted in a network approach.

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INTRODUCTION

Linear acoustic network models are commonly used to describe the propagation of acoustic waves in confined flows, such as exhaust systems, HVAC-ducts, compressed air supplies,... The major advantage of using a network representation is the fact it describes each element of a duct system 'proper', i.e. independent of the inlet and outlet acoustic impedance, which is imposed by the acoustic elements lying before and after the system under

consideration. As a result, the different network models of separate sub-elements can be easily combined and the acoustic attenuation characteristics in the whole or parts of the flow duct system can be evaluated with only some simple matrix multiplications. Thus, acoustic network modeling offers a strong design tool for the determination of the acoustic transmission properties for confined flow applications.

Acoustic systems, located between two straight ducts in which acoustic plane wave propagation is occurring (i.e. below the cut-on frequency of the traversal duct modes), are often referred to as acoustic two-ports [1] (Figure 1). Every acoustic two-port is uniquely characterized by an acoustic transfer matrix T also known as the two-port or four-pole parameter representation, coupling the acoustic variables (e.g. acoustic pressure and velocity fluctuations) at the inlet (1) and outlet (2) of the acoustic element.



Figure 1. Graphical representation of an acoustic two-port

One of the most commonly used representations of the transfer matrix is using the pressure of the right- and left-running acoustic waves (p^+ ; p^-) (instead of the pressure and velocity fluctuations) at both ends of the two-port. In this way, the four matrix elements T^+ ; T^- ; R^+ and R^- can be physically interpreted as the transmission and reflection coefficients in, respectively, the downstream and upstream direction. If non-linear active noise generation and/or dissipation processes, caused by e.g. wall vibrations or aerodynamic noise generating mechanisms occur inside the acoustic element, an active two-port representation can be adopted by adding two additional source terms [2,3]. Since this paper focuses solely on the aeroacoustic attenuation characteristics of exhaust systems, the latter terms are not taken into account. As such, the system is considered to be passive which leads to following matrix formulation:

$$\begin{Bmatrix} p_2^+ \\ p_2^- \end{Bmatrix} = \begin{bmatrix} T^+ & R^- \\ R^+ & T^- \end{bmatrix} \begin{Bmatrix} p_1^+ \\ p_1^- \end{Bmatrix} \quad (1)$$

The passive two-port components T^+ ; T^- ; R^+ and R^- are independent of the upstream and downstream impedance. As such, they describe the 'filter proper'. Although the acoustic element is treated as a 'black-box' and the formulation is only valid below the traversal cut-on frequency of the inlet and outlet ducts, the active two-port formulation (provides a useful design framework for the aeroacoustics analysis of flow duct systems with potential applications of a.o.: the determination of the noise attenuation properties [4]; the characterization of acoustic absorbent materials[5]; the aeroacoustic design of duct networks [1]; the identification of flow-acoustic feedback coupling [6]; and the quantification of aerodynamic noise generation mechanisms [7,8].

Although both analytical and numerical approaches [9,10] exist to evaluate the passive two-port coefficients of flow ducts systems, they are, at present, insufficient to model the

full geometrical complexity (perforates, SCR elements,...) and to accurately assess the influence of thermal and aerodynamic phenomena onto the acoustic attenuation characteristics. For this reason, an experimental two-port characterization approach is used in this paper to evaluate the attenuation characteristics of a multi-component truck's exhaust system. The experimental methodology is based on a two-step approach:

1. The determination of the amplitudes of the upstream and downstream propagating waves at both ends of the acoustic two-port: p_1^+ , p_1^- , p_2^+ and p_2^- using a multiple flush-mounted microphone array. The adopted approach is based on an analytical expression for the (linear) plane wave propagation in a rigid straight duct which imposes some limitations to the applicability range of two-port characterization techniques: the sound field has to be linear in the measurement ducts, which is generally considered to be the case for sound pressure levels below 140 dB [11]; the temperature and flow velocity should be constant, both in time and in the longitudinal direction over the whole length of the microphone array; the acoustic damping, introduced by visco-thermal or aerodynamic effects should obey the analytical reference model; and the maximum frequency is limited to the cut-on frequency of the traversal modes in the measurement duct sections.

2. Characterization of the scattering matrix coefficients (passive two-port components): T^+ , T^- , R^+ and R^- using a multiple source/multiple load methodology. Besides the assumption that the acoustic system is linear, this assumes that the attenuation mechanisms are independent on the acoustic 'load' and/or excitation imposed at the upstream and/or downstream end of the acoustic two-port and that the system characteristics, such as e.g. the mean flow field, do not vary with the applied 'load' cases.

In this paper, this approach is adopted for the aeroacoustic configuration of a truck's exhaust system configuration containing four different components: an inlet bent, a flexible connection pipe, a muffler and a tailpipe. The analysis involves an in-depth study of the aeroacoustics attenuation characteristics of all sub-components separately as well as of the whole exhaust system configuration. The influence of non-uniform mean flow effects on the transmission behavior is investigated by performing two different experiments: a first one under quiescent conditions and ambient temperature and the second experiment is carried out with a non-uniform mean flow (Mach number $M \approx 0.05$) and ambient temperature.

The outline of this paper is the following: In the next section, the theory and the adopted approach for the experimental determination of the scattering matrix coefficients is discussed in further detail. Subsequently, the experimental test rig, measurement parameters and test configurations are described. The next section discusses at first the results for the various sub-components of the exhaust system and afterwards the attenuation characteristics of the

full exhaust system configuration. Finally, the major conclusions, drawn from this analysis are summarized.

EXPERIMENTAL TWO-PORT CHARACTERIZATION

The major challenge that arises, when using an experimental aeroacoustic two-port characterization technique, is the fact that the signal-to-noise ratio can become relatively small. The low value of the S/N is primarily originating from the presence of aerodynamic pressure fluctuations, also known as *pseudo-sound*; the acoustic attenuation caused by the acoustic filter; and uncertainties in the experimental and environmental conditions, such as the flow velocity or Mach number M , temperature, acoustic damping of the test sections, microphone locations,... For this reason, additional measures are needed in the various steps of the active two-port determination methodology, to improve S/N or to reduce the influence of environmental uncertainties.

DETERMINATION OF UPSTREAM AND DOWNSTREAM PROPAGATING WAVES

For the determination of the upstream and downstream propagating waves (p^+ , p^-) at each end of the two-port, the dynamic pressure signal p' at a minimum of two longitudinal positions z_i are needed in the measurement duct. The left- and right running amplitudes are directly linked to the acoustic pressure fluctuations, since at any longitudinal location z in the duct the following analytical relation holds for plane acoustic wave propagation:

$$p'(z, f) = p^+(z, f) + p^-(z, f) = p^+(0, f)e^{-ik^+z} + p^-(0, f)e^{ik^-z} \quad (2)$$

with $z=0$ the position of the reference plane and $k^+ \approx k_0/(1+M)$ and $k^- \approx k_0/(1-M)$ the acoustic wave numbers of, respectively, the downstream and upstream propagating waves where $k_0=\omega/c_0$ and $M=U_0/c_0$ the mean flow Mach number. For non space-uniform mean flows, or when the (visco-thermal) acoustic damping inside the measurement duct should be taken into account, a more accurate formulation for k^+ and k^- can be obtained using a complex wave number [12,13]. In this case, the wavenumber of the upstream and downstream propagating wave can be written as:

$$k^+ = k_0 \frac{\vartheta}{1+\vartheta M} \quad k^- = k_0 \frac{\vartheta}{1-\vartheta M} \quad (3)$$

with ϑ , a complex frequency-dependent number which takes into account visco-thermal losses and the influence of mean flow inhomogeneities.

Equation (2) can also be written when using multiple microphone positions, which leads to an overdetermined

system of n equations, n being the total number of microphones, with two unknowns:

$$\begin{bmatrix} p^+ \\ p^- \end{bmatrix} = \begin{bmatrix} e^{-ik^+z_1} & e^{ik^-z_1} \\ \vdots & \vdots \\ e^{-ik^+z_j} & e^{ik^-z_j} \\ \vdots & \vdots \\ e^{-ik^+z_n} & e^{ik^-z_n} \end{bmatrix}^{\otimes} \begin{bmatrix} p'_1 \\ \vdots \\ p'_j \\ \vdots \\ p'_n \end{bmatrix} \quad (4)$$

where subscript $\otimes$ indicates the Moore-Penrose pseudo matrix inverse.

Using more than two microphones at each end of the acoustic two-port, and thus solving the overdetermined system of equations (4) with a least-squares approach, improves the S/N and minimizes the influence of aerodynamic pressure fluctuations. Since the aerodynamic pressure fluctuations, present in the microphone signals, do not satisfy equation (2) they are partially filtered out by the solution procedure. Furthermore, the fact that the system of equations becomes overdetermined allows discarding microphone signals which have a low coherence with respect to the excitation signal [14]. In this paper the threshold level of the coherence between the microphone signals and the loudspeaker excitation signal is set equal to 0.9.

A further improvement of the S/N is achieved by using a transfer function relation in equation (4) instead of directly using the measured pressure fluctuations. These transfer functions are taken between the microphone signals and a reference signal which is correlated with the acoustic field inside the duct. In this sense the electric signal e , which drives the external loudspeaker is a convenient choice. Also using an appropriate choice of excitation signal (i.e. a stepped sine excitation), a large number of averages and optimized microphone placeholders leads to an increased measurement accuracy [15].

However, the main parameters, that influence the accuracy of the determination of the upstream and downstream propagating waves [16], are the environmental parameters, i.e. microphone distances, Mach number, temperature (and as a consequence also speed of sound) and the acoustic damping. For this reason, an iterative procedure, an extended version as the one proposed in reference [17], is adopted to obtain the different variables, mentioned above.

Following initial conditions are chosen for the iterative procedure:

- The microphone distances are determined using an acoustic calibration methodology with a rigid termination of the upstream and downstream measurements ducts [18]. However, a sensitivity analysis of the iterative procedure with respect to the accuracy of the initial conditions [19] showed no difference in the finally obtained microphone distance in comparison with an initial guess obtained using an estimation based on ruler measurements
- The flow rate and the incoming temperature are obtained using a vortex-flowmeter, installed upstream of the first

measurement duct. The temperature is used to obtain an initial value for the speed of sound and, combined with the cross section of the upstream measurement duct, the according Mach number. According to the sensitivity analysis [19], an over/under-estimation of the temperature and Mach number with 100% or higher does not yield a different estimation of the final values.

- The acoustic damping, represented by the complex term ϑ in [equation \(3\)](#) is obtained by following expression [20]:

$$\vartheta = 1 + \left(\frac{1-i}{\sqrt{2}s}\right) \left(1 + \frac{\gamma-1}{\sqrt{Pr}}\right) \quad (5)$$

with $s = r\sqrt{\omega/\nu}$, the shear wave number, r the duct radius and ν the kinematic viscosity; γ the specific heat ratio; and Pr the Prandtl number.

In total, this iterative system, based on [equation \(4\)](#) leads to a system of $n \cdot n_f$ equations, n_f being the number of frequencies, with $3n_f + n + 2$ unknowns. As a result, when $n_f > n \geq 4$, the system remains overdetermined. The different steps in the iterative procedure to obtain the upstream and downstream propagation plane wave coefficients (p^+ , p^-) for both the upstream and downstream measurement ducts can be summarized as:

1. Determination of the coherence of the various microphone signals and discarding microphone signals with a coherence level lower than 0.9. If only one (or zero) microphones are maintained, the measurement is discarded and no upstream- and downstream propagating waves are determined.

2. Determination of the initial guess for the microphone distances; Mach number; temperature and the according speed of sound; and acoustic damping using the values described above.

3. Determination of the upstream and downstream propagating waves (p^+ , p^-) based on the least square solution of [equation \(4\)](#).

4. Determination of the microphone distances z_i at every frequency using [equation \(4\)](#). Since the microphone distances are independent on the frequency, the final distance values are obtained using the averaged values over all frequencies. Since the system of [equations \(4\)](#) has an infinite number of solutions for z_i (with $i=1, \dots, n$), which accord to a shift of the reference plane $z=0$, the position of the first microphone z_1 at both the upstream and downstream end, is kept constant.

5. Determination of the flow velocity, U_0 , at every frequency using [equation \(4\)](#), where the Mach number, M , is replaced by U_0/c_0 . Similar as for the microphone distances, the final value for U_0 is obtained using the averaged value over all frequencies. The choice of solving for the flow

velocity, U_0 , rather than for the Mach number, M , is motivated by the fact that the Mach number is dependent on both the flow velocity and the speed of sound. As a result, a change in Mach number can be caused by either a variation of U_0 or c_0 , the latter resulting also in a change of the wavenumber k_0 .

6. Determination of the speed of sound c_0 or temperature using [equation \(4\)](#), where the Mach number, M , is replaced by U_0/c_0 . and averaging over all frequencies.

7. Determination of the acoustic damping factor ϑ using [equation \(4\)](#). Since it can be assumed that this parameter is frequency dependent, no frequency averaging is carried out.

8. Verification if convergence of all parameters (p^+ , p^- , z_i , U_0 , c_0 and ϑ) is obtained and if the total residual of [equation \(4\)](#) is sufficiently small. If not, all variables are updated with the result of the current iterative step and the procedure is repeated, starting from step 3.

To illustrate the accuracy of the iterative procedure and to make a comparison with the traditional two-microphone technique and the multiple microphone technique without parameter estimation, an analytical investigation of the errors, introduced by an inappropriate estimation of the microphone distances, the Mach number and the speed of sound, is carried out. For this purpose the microphone signals are calculated using [equation \(2\)](#) with $p^+ = p^- = 1 \text{ Pa}$; $M=0.3$; $c_0=344 \text{ m/s}$; and $\vartheta=1+0i$ at positions $z_1=0.0\text{m}$ and $z_2=0.12\text{m}$ for the two-microphone technique and at $z_1=0.0\text{m}$, $z_2=0.12\text{m}$, $z_3=0.28\text{m}$ and $z_4=0.52\text{m}$ for the multiple microphone technique for a frequency range between 20Hz and 2020Hz. Based on these microphone signals, the upstream and downstream propagating waves are calculated where the microphone locations are alternately shifted to the real value with a distance $\Delta z=0.01\text{m}$ ($z_1=0.0\text{m}$ and $z_2=0.13\text{m}$, for the two-microphone technique, and $z_1=0.0\text{m}$, $z_2=0.13\text{m}$, $z_3=0.27\text{m}$ and $z_4=0.53\text{m}$ for the multiple microphone technique); the Mach number is estimated as $M=0.275$ and the speed of sound as $c_0=347 \text{ m/s}$. As such, realistic errors are introduced in the estimation of the environmental parameters.

The left of [Figure 2](#) shows the relative amplitude error of the upstream and downstream propagating waves with an estimation error of only the microphone location for the different approaches. The two-microphone methodology shows a singularity at approximately 1200Hz due to a singularity in the matrix formulation of [equation \(4\)](#). In general the two-microphone method has the worst performance for the current configuration. A different combination of errors in the microphone positions of the multiple microphone techniques can, however, yield different results. It is clear that the iterative multiple microphone approach (with only 20 iterations) significantly improves the accuracy of the determination the upstream and downstream propagating waves. Similar results are obtained when an error