

Complex Eigenvalue Analysis and Brake Squeal: Traps, Shortcomings and their Removal

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ABSTRACT

Among many NVH problems brake squeal continues to be a difficult topic for design engineers and scientists. Both the experimental and the simulation approaches so far have failed to provide robust and reliable guidelines for the design of squeal free brakes. On the experimental side the problem clearly lies in the wide range of operating conditions which the brake encounters in its lifetime, in which it should be squeal free. From lab experiments alone, it is hardly possible to judge how far the system is from squeal, which implies that an extremely wide range of conditions is mandatory. Brake squeal simulation presents different challenges. Once a model for the brake has been formulated, including the excitation mechanism(s), it should be possible to check the robustness of a given design and system parameters against squeal. Complex eigenvalue analysis has become a standard industrial tool for squeal prediction, and is routinely applied to the simulation models. Despite many years of research and development along these lines, the reliability of squeal prediction is still not adequate. This lack of reliability of the simulation results persists, even if great care is taken to properly model the friction and damping, gyroscopic terms, etc. and a more fundamental difficulty remains in the simulations. One of the basic assumptions of complex eigenvalue analysis is that the system is time-invariant and behaves like one with constant coefficients. Brake disks however typically have ventilation channels, and therefore are not axially symmetric. This leads to periodic coefficients in the equations of motion of the brake with a rotating brake disk, to which the standard complex eigenvalue analysis is not applicable. Using basic case studies, in this paper it is shown that the conclusions drawn from complex eigenvalue analysis can be meaningless in this case, at least concerning the real part of the eigenvalue. It is shown that in situations which should be squeal free according to the standard eigenvalue analysis, the system can actually squeal, and vice versa. In order to overcome this problem, a method is outlined, which is capable to take into account the lack of axial symmetry of the brake rotors and should yield much more accurate predictions than the standard complex eigenvalue analysis.

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INTRODUCTION

Complex eigenvalue analysis has become a standard tool for the analysis of brake squeal in industry. Unfortunately, despite of several years of development of the method the results are insufficient to predict squeal. One of the shortcomings is that too many squealing frequencies are predicted. The goal of this paper is to explain this phenomenon and to suggest an improvement of the method in that direction. In contrast to many other structural problems the eigenvalue analysis in brake squeal problems is more involved because it is not performed about a configuration at rest but around a moving configuration with contacts. In recent years analytic minimal models were developed which showed certain features that were considered as important for example gyroscopic effects, damping due to friction and the

role of mode reduction methods [2,3,5,7]. Benchmarks with FE calculations in industry showed that these aspects are in fact problematic, however, in some codes models have been set in which all of the effects can be observed. Nevertheless, the problem of inaccuracy of the results remains. A crucial point is that complex eigenvalue analysis in brake squeal tacitly assumes that the brake rotor is rotationally symmetric. In the procedure the model is perturbed about a certain position of the disc and the rotation is artificially introduced by adding gyroscopic terms. Therefore the results of the eigenvalue analysis depend on the position of the disk and furthermore the real part of the calculated eigenvalues is in fact not a suitable measure for stability. This will be shown and explained on a minimal model in the following sections. Afterwards a modification of the method will be suggested.

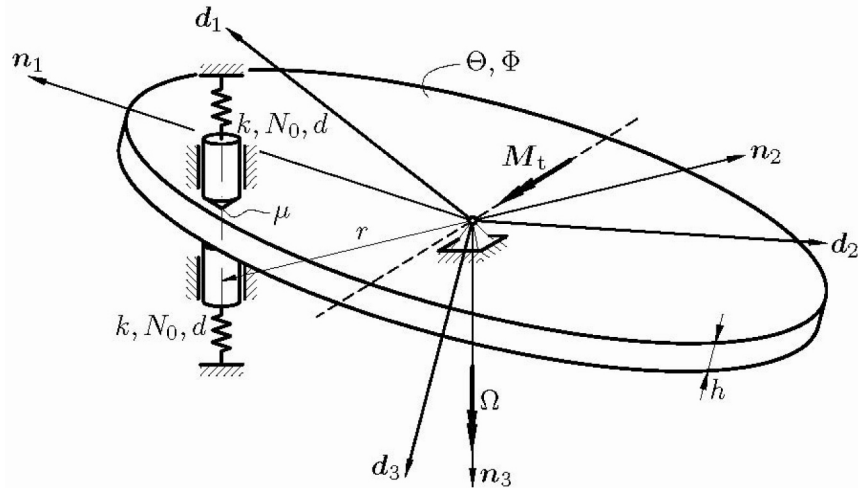


Figure 1. Disk brake model with wobbling disk.

COMPLEX EIGENVALUE ANALYSIS ON A MINIMAL MODEL

In [7] a minimal model for brake squeal was introduced involving a rotating wobbling disk in frictional contact with idealized brake pads. It was shown that the motion of the disk can be destabilized by friction forces. At the SAE meeting in Phoenix 2010 the model was used to show that asymmetries of the brake rotor have a stabilizing effect [6]. The detailed modeling assumptions are now stated.

The asymmetric model shown in figure 1 consists of a rigid disk rotating about the $\mathbf{n}_3$ axis at constant angular velocity Ω and free to tilt with respect to the $\mathbf{n}_1$ - $\mathbf{n}_2$ plane. The elastic restoring torque

$$M_{tk} = -k_{t1} \left(\arccos(\mathbf{n}_3 \cdot \mathbf{d}_2) - \frac{\pi}{2} \right) \dots$$

$$\mathbf{d}_1 - k_{t2} \left(\arccos(\mathbf{n}_3 \cdot \mathbf{d}_1) - \frac{\pi}{2} \right) \mathbf{d}_2 \quad (1)$$

is assumed proportional to the tilting angles between the axes $\mathbf{d}_1$ and $\mathbf{d}_2$ and the normal vector $\mathbf{n}_3$, the corresponding stiffnesses are k_{t1} , k_{t2} . In addition a linear viscous damping proportional to the angular velocity about $\mathbf{d}_1$ and $\mathbf{d}_2$ is assumed (damping constants d_{t1} , d_{t2}). The disk is in contact with massless pins which are pressed on the disk by linear springs (stiffness constant k , prestress N_0). Between the disk and the pads Coulomb friction is assumed with the constant μ . If the tilting stiffnesses about $\mathbf{d}_1$, $\mathbf{d}_2$, namely

k_{t1} , k_{t2} as well as d_{t1} , d_{t2} are chosen equal, the formulation agrees with [7] in the linear terms. The resulting equations set up with respect to an inertial frame read

$$\begin{bmatrix} \Theta & 0 \\ 0 & \Theta \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \frac{\mu N_0 h^2}{\Omega r} & \Omega \Phi \\ -\Omega \Phi & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \dots$$

$$+ \begin{bmatrix} k_t + N_0 h + 2kr^2 & \frac{1}{2} \mu N_0 \frac{h^2}{r} \\ -\mu(2N_0 r + khr) & k_t + (1 + \mu^2) N_0 h \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Clearly these equations can be studied using complex eigenvalue analysis and a positive real part of the eigenvalues can be interpreted as squeal. It is easily seen that the equations feature gyroscopic terms and a viscous term due to the frictional contact. If the restoring terms are not equal, i.e. $k_{t1} \neq k_{t2}$, $d_{t1} \neq d_{t2}$ the equations of motion are no longer time invariant. This is easily understood by imagining that if we statically apply a unit force at the points of the pads, the resulting tilting angle depends on the position of the disk. The corresponding equations of motion read

$$\begin{bmatrix} \Theta & 0 \\ 0 & \Theta \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} d_{t1} + \frac{1}{2} \mu N_0 \frac{h^2}{r \Omega} - \Delta d \sin^2 q_3 & \Phi \Omega + \Delta d \sin q_3 \cos q_3 \\ -\Phi \Omega - d_{t2} \mu r + \Delta d \sin q_3 \cos q_3 & d_{t2} + \Delta d \sin^2 q_3 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} +$$

$$\begin{bmatrix} k_{t1} + 2kr^2 + N_0 h - \Delta k \sin^2 q_3 & \frac{1}{2} \mu N_0 \frac{h^2}{r} + \Delta k \sin q_3 \cos q_3 \\ -\mu(khr + 2N_0 r) + \Delta k \sin q_3 \cos q_3 & k_{t2} + (1 + \mu^2) N_0 h + \Delta k \sin^2 q_3 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (2)$$

where $q_3 = \Omega t$. The equations are still linear but now feature periodic coefficients. The procedure of complex eigenvalue analysis now to perturb the system (2) around a fixed position q_3 , as if the equations had constant coefficients. The real part of the calculated eigenvalues is interpreted as a measure for squeal. We will now follow the usual steps of the