

A Multi-Objective LMI-Based Antiroll Control System

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ABSTRACT

A long standing problem with heavy vehicle stability has been rollover. With the higher center of gravity, heavier loads, and narrower tracks (as compared to passenger vehicles), they have a lower rollover stability threshold. In this paper, a rollover stability control algorithm based on a two degrees of freedom (DOF) and a three DOF vehicle model for a two-axle truck was developed. First, the 3DOF model was used to predict the future Lateral load Transfer Rate (LTR). Using this LTR value, the dynamic rollover propensity was estimated. Then, a robust output feedback gain control rollover stability control algorithm based on the combination of active yaw control and active front steering control was developed. A H_2/H_∞ /poles placement multi-objective control strategy was developed based on the 2DOF reference model. After the control objectives were determined, the Linear Matrix Inequality (LMI) technique was utilized to solve the restrictions and to carry out the output feedback gain for the multi input multi output (MIMO) system. Finally, utilizing the commercial software TruckSim, the rollover stability control was evaluated using fishhook and step steering maneuvers. The simulation results show that the LMI robust controller could effectively improve the vehicle rollover stability.

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INTRODUCTION

Since rollover is one of the most serious accidents for heavy trucks, more research has been focused on active roll control (ARC) systems in recent years. Both reduction of roll angle and improvement of occupant comfort can be achieved by an ARC system. Rollover stability could be improved significantly by active front or four wheel steering, direct yaw control, active stabilizer bar or active suspension system. Although an integrated system comprising of all of what was mentioned would improve the roll stability significantly, integrating a subset of the systems mentioned will be a significant step forward. In this research, an integrated control system has been developed for active front steering and direct yaw control.

Recently, there have been several research programs on the integration of active steering control and active yaw moment control. Various kinds of linear 2 degree of freedom (DOF) model-based feed-forward or feed-back algorithms are adopted to achieve desired steering angle and yaw moment. A model matching controller was studied in 1998 [1]. In [2,3,4], feedback linearization technique, solution of state-dependent Riccati equation as a nonlinear optimal regulator, and fuzzy/

optimal LQR control strategies were used separately. The integrated control was also proved to be effective in ARC system. In [5], the integration of electronic stability control (ESC) and continuous damping control (CDC) was adopted to improve rollover and lateral stability. Active steering control and active yaw moment control were integrated for yaw stability and emergency roll control in [6].

As is well known, output feedback control is an effective control methodology. However, it is difficult to find the best parameter set for a complex multi input multi output (MIMO) system, and it is also difficult to combine complex vehicle handling dynamics and control subsystems using a single set of parameters. Therefore, a robust control technique is used to adjust the MIMO output feedback control parameters based on different H_2/H_∞ restriction functions. For example, 1) a H_∞ restriction for the robust stability of the system; 2) a H_2 restrictions for transient performance of the system, 3) restrictions for placing the poles of closed-loop system into a reasonable field on the complex plane to ensure the desired transient response. In addition, Linear Matrix Inequality (LMI) technique was used since it provides a powerful design tool for solving these mixed objectives [7][8].

In this paper, a multi-objective robust output feedback controller is developed for anti-roll control, as shown in [figure 1](#). The active steering control and active yaw moment control are combined based on a 2 DOF reference model. While the further research on the distribution from active yaw moment to individual brake control is not considered in this work. An LMI-based multi-objective algorithm is developed for mixed H_2/H_∞ /poles region restrictions. By setting the desired yaw rate and side slip angle, both of the roll stability and yaw stability can be improved.

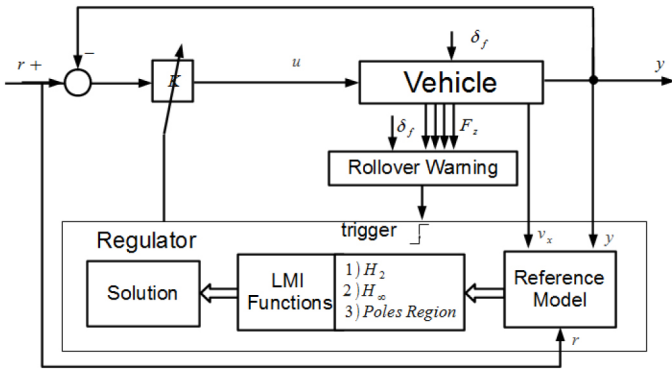


Figure 1. Schematic Frame for Multi-Objective-based Output Feedback Controller

REFERENCE MODEL

The 3 DOF reference vehicle model is shown in [figure 2](#) and the following force and moment equilibrium equations are listed as follows,

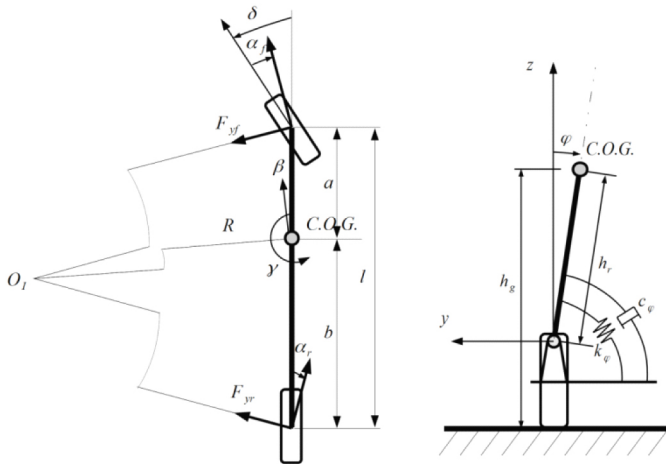


Figure 2. A 3 DOF reference vehicle model

$$\begin{cases} m(\dot{v}_y + v_x \gamma) - m_s h_r \ddot{\phi} &= F_{yf} + F_{yr} \\ I_z \dot{\gamma} &= a F_{yf} - b F_{yr} + M_z \\ I_x \ddot{\phi} - m_s h_r (\dot{v}_y + v_x \gamma) \cos \phi &= m_s g h \sin \phi - c_\phi \dot{\phi} - k_\phi \phi \end{cases} \quad (1)$$

Where

$$\begin{cases} F_{yf} = 2k_f \alpha_f = 2k_f \left(\frac{v_y + \gamma a}{v_x} - \delta_f \right) \\ F_{yr} = 2k_r \alpha_r = 2k_r \left(\frac{v_y - \gamma b}{v_x} \right) \end{cases} \quad (2)$$

It should be noted that under coordinate system adopted in this paper, $\alpha_f < 0$, $\alpha_r < 0$, $k_f < 0$, and $k_r < 0$. Let v_x be constant and ϕ be small, then the model can be linearized as $\cos \phi = 1$ and $\sin \phi = \phi$. Furthermore, it is noticed that $\dot{v}_y = \beta \dot{v}_x$ as v_x is constant.

Let $\mathbf{x} = (\gamma \quad \beta \quad \phi \quad \dot{\phi})^T$, $\mathbf{u} = (\Delta \delta_f \quad \Delta M_z)$, $w = \delta_f$, [equation \(1\)](#) could be written as

$$\begin{cases} \mathbf{A}_0 \dot{\mathbf{x}} = \mathbf{A}_1 \mathbf{x} + \mathbf{B}_1 \mathbf{u} + \mathbf{F}_1 w \\ \mathbf{y} = \mathbf{C} \mathbf{x} \end{cases} \quad (3)$$

where

$$\mathbf{A}_0 = \begin{pmatrix} 0 & m v_x & 0 & -m_s h \\ I_z & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -m_s h v_x & 0 & I_x \end{pmatrix}$$

$\mathbf{A}_1 =$

$$\begin{pmatrix} \frac{-m v_x^2 + 2k_f a - 2k_r b}{v_x} & 2k_f + 2k_r & 0 & 0 \\ \frac{2k_f a^2 + 2k_r b^2}{v_x} & 2a k_f - 2b k_r & 0 & 0 \\ 0 & 0 & 0 & 1 \\ m_s h v_x & 0 & m_s g h - k_\phi & -c_\phi \end{pmatrix}$$

$$\mathbf{B}_1 = \begin{pmatrix} -2k_f & 0 \\ -2a k_f & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{F}_1 = \begin{pmatrix} -2k_f \\ -2a k_f \\ 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Thus, the 3DOF model yields

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{x} + \mathbf{A}_0^{-1} \mathbf{B}_1 \mathbf{u} + \mathbf{A}_0^{-1} \mathbf{F}_1 w \\ \mathbf{y} = \mathbf{C} \mathbf{x} \end{cases} \quad (4)$$

If the roll degree of freedom is not considered, the 2DOF model could also be calculated by equation (2), where

$$\mathbf{A}_0 = \begin{pmatrix} 0 & mv_x \\ I_z & 0 \end{pmatrix}, \mathbf{A}_1 = \begin{pmatrix} \frac{-mv_x^2 + 2k_f a - 2k_r b}{v_x} & 2k_f + 2k_r \\ \frac{2k_f a^2 + 2k_r b^2}{v_x} & 2ak_f - 2bk_r \end{pmatrix}$$

$$\mathbf{B}_1 = \begin{pmatrix} -2k_f & 0 \\ -2ak_f & 1 \end{pmatrix}, \mathbf{F}_1 = \begin{pmatrix} -2k_f \\ -2ak_f \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In this paper, the 3DOF model is used for predicting future LTR and the 2DOF model is used for controller design.

ROLLOVER WARNING

There are several factors which could be chosen as rollover warning index. Lateral load Transfer Rate (LTR) is an effective one [9],

$$LTR = \frac{|F_{zl} - F_{zr}|}{F_{zl} + F_{zr}} \quad (5)$$

In [10], active roll control is triggered if LTR is larger than 0.8. However, this may result in late prediction for sudden steering inputs at high speeds. Considering the lateral load transfer as shown in figure 3 and using the moment equilibrium equations on right and left wheels with the assumption that ϕ is small, we can have

$$\frac{F_{zl} - F_{zr}}{F_{zl} + F_{zr}} = \frac{2(m_s g h_r \phi - (m(\dot{v}_y + v_x \gamma) - m_s h_r \ddot{\phi}) h_g)}{d_{tr} m g} \quad (6)$$

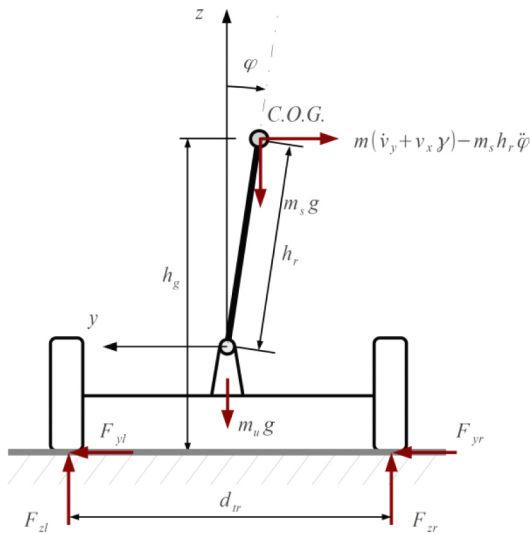


Figure 3. Rear View of a Cornering Vehicle

Thus, the following state space model can be written using equation (3)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{x} + \mathbf{A}_0^{-1} \mathbf{F}_1 w \\ LTR = \mathbf{C}_{LTR} \mathbf{x} + \mathbf{F}_{LTR} w \end{cases} \quad (7)$$

Where

$$\mathbf{C}_{LTR} = \frac{4}{d_{tr} m g} \begin{pmatrix} -\frac{h_g (k_f a - k_r b)}{v_x} & -h_g (k_f + k_r) & \frac{m_s g h_r}{2} & 0 \end{pmatrix}$$

$$\mathbf{F}_{LTR} = \frac{4k_f h_g}{d_{tr} m g}$$

Based on the above analysis, the rollover control will be triggered if $|LTR| > 0.8$. In addition, if $|LTR| > 0.5$ and the LTR predicted using the 3-DOF model is greater than one, $|LTR| > 1$, within the 1 second following these conditions being true, the rollover control will also be triggered.

CONTROLLER DESIGN

Output Feedback Controller

For output feedback control, the control input is

$$\mathbf{u} = \mathbf{K} \mathbf{e} = \mathbf{K}(\mathbf{r} - \mathbf{y}) = \mathbf{K}(\mathbf{r} - \mathbf{C} \mathbf{x}) \quad (8)$$

In which $\mathbf{r}$ is the reference input. For normal yaw rate control, desired yaw rate could be set as

$$\gamma_1 = \frac{v_x / L}{1 + K_u v_x^2} \delta_f = a_{\gamma 1} \delta_f \quad (9)$$

Where

$$K_u = \frac{m}{L^2} \left(\frac{a}{k_r} - \frac{b}{k_f} \right) \quad (10)$$

However, for a truck with higher C.O.G., the corresponding lateral acceleration to γ_1 might be too large to maintain the roll stability. A maximum lateral acceleration $a_{y \max}$ should be set up and the corresponding yaw rate is

$$\gamma_2 = \frac{a_{y \max} g}{v_x} \text{sgn}(\delta_f) = \frac{a_{y \max} g}{v_x \delta_f} \delta_f = a_{\gamma 2} \delta_f \quad (11)$$

Thus, the desired yaw rate could be set as

$$\gamma_{des} = \min(|a_{\gamma 1}|, |a_{\gamma 2}|) \cdot \delta_f = a_{\gamma} \delta_f \quad (12)$$