

Modeling Combined Braking and Cornering Forces Based on Pure Slip Measurements

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ABSTRACT

A novel predictable tire model has been proposed for combined braking and cornering forces, which is based on only a few pure braking and pure cornering tests. It avoids elaborate testing of all kinds of combinations of braking and side forces, which are always expensive and time consuming. It is especially important for truck or other large size tires due to the capability constraints of tire testing facilities for combined shear forces tests.

In this paper, the predictive model is based on the concept of slip circle and state stiffness method. The slip circle concept has been used in the COMBINATOR model to obtain the magnitude of the resultant force under combined slip conditions; however the direction assumption used in the COMBINATOR is not suitable for anisotropic tire slip stiffness. The state stiffness method, by which the variation of resultant force direction caused by anisotropy of tire slip stiffness can be described well and then the longitudinal and lateral forces under combined situations can be obtained separately. In addition, the friction ellipse is introduced in state stiffness method to express the anisotropic friction coefficients.

The UniTire pure slip model is introduced firstly in this paper, and then the slip circle concept and state stiffness method are discussed. Finally, the UniTire prediction tire model for combined braking and cornering forces is proposed and the effectiveness of the model is validated by the comparison with the experimental data.

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INTRODUCTION

Tires are the only components of a road vehicle to directly contact with the road surface, the forces generated in the contact patch under combined cornering/braking situations have significant effects on the vehicle directional control during driving and braking. There are already many tire models for use in calculating the shear forces at the tire-road interface. Since the theoretical models are generally based on simplifying assumptions which limit their practical use, the empirical and semi-empirical models based on experimental data and some theoretical boundary conditions are widely employed. For example, the Magic Formula model [1] is the typical empirical model which is widely used and has a high accuracy. However, the experimental data for combined slip is often difficult or expensive to obtain which is needed by the Magic Formula model for identifying model parameters.

Therefore, it is important to develop a concise model with less parameters and satisfactory accuracy. And more importantly, the model should be able to predict the tire

forces under combined slip conditions from pure cornering and braking tests, in order to avoid elaborate testing of all kinds of combinations of braking and side forces, which are always expensive and time consuming. Some scholars have been working on this study and have proposed some predictable tire models, such as the similarity method [1, 2], the COMBINATOR model [3] and so on. However, these models are not always accurate enough to describe tire forces under combined slip conditions, especially for anisotropic tire slip stiffness and friction coefficient.

Based on the concept of slip circle and state stiffness method, a novel predictable tire model is proposed in this paper, which can describe tire longitudinal and lateral forces under combined slip conditions based on only pure slip measurements. Firstly, the UniTire model for pure slip is introduced. It is the cornerstone for predicting tire forces under combined slip conditions. Then, some properties of tire forces with the slip circle concept and state stiffness method are discussed. Finally, the UniTire prediction model is proposed and the results are validated by the experimental

data, which shows that the accuracy is satisfactory even if the tire has a strong anisotropy.

THE UNITIRE MODEL FOR PURE SLIP

Tire Axis System and Slip Ratios

The tire axis system is shown in Fig.1. It is different from the SAE tire axis system. The positive direction of Xt-axis and Yt-axis are coincident with tire revolution direction (not wheel direction of travel); the Zt axis is perpendicular to road plane and upward. The wheel traveling speed is denoted as V , α is the slip angle and γ is inclination angle. The figure shows all the forces and moments associated with a wheel. The three moments shown in this figure are not discussed in this paper because the topic of interest is the shear force under combined braking and steering conditions.

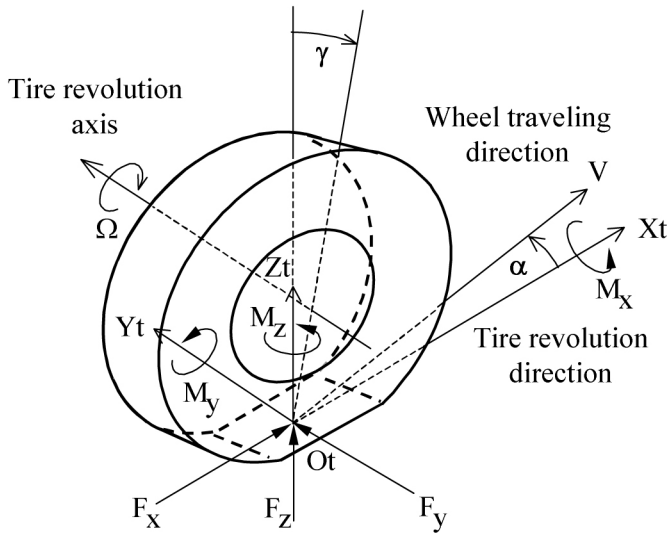


Figure 1. Tire axis system

The longitudinal slip ratio S_x and lateral slip ratio S_y used in the UniTire model are defined in the unified form as the sliding speed over the rolling speed of the tire [4, 5]:

$$\begin{cases} S_x = \frac{-V_{sx}}{\Omega R_e} = -\frac{V \cos \alpha - \Omega R_e}{\Omega R_e} & S_x \in (-\infty, +\infty) \\ S_y = \frac{-V_{sy}}{\Omega R_e} = -\frac{V \sin \alpha}{\Omega R_e} & S_y \in (-\infty, +\infty) \end{cases} \quad (1)$$

where, Ω is the angular speed, R_e is the effective rolling radius. V_{sx} and V_{sy} are the longitudinal and lateral sliding speeds of tire with respect to road surface.

Usually, the longitudinal slip ratio used in tire force test is defined as [1]:

$$\kappa = -V_{sx} / (V \cos \alpha) \quad (2)$$

The relationship between S_x and κ can be obtained easily:

$$S_x = \kappa / (1 + \kappa) \quad (3)$$

Tire Force Under Pure Slip Condition

The basic idea of the UniTire model is to employ non-dimensional expression and have a unified form for longitudinal and lateral forces. Under pure slip conditions, the longitudinal and lateral force model can be written as [4, 5]

$$\begin{cases} \bar{F}_x = 1 - \exp \left[-|\phi_x| - E_x \cdot |\phi_x|^2 - (E_x^2 + 1/12) \cdot |\phi_x|^3 \right] \\ \bar{F}_y = 1 - \exp \left[-|\phi_y| - E_y \cdot |\phi_y|^2 - (E_y^2 + 1/12) \cdot |\phi_y|^3 \right] \end{cases} \quad (4)$$

where

$$\begin{cases} \bar{F}_x \cdot \text{sgn}(\phi_x) = \frac{F_x}{\mu_x F_z} \\ \bar{F}_y \cdot \text{sgn}(\phi_y) = \frac{F_y}{\mu_y F_z} \end{cases}, \quad \begin{cases} \phi_x = \frac{K_x S_x}{\mu_x F_z} \\ \phi_y = \frac{K_y S_y}{\mu_y F_z} \end{cases} \quad (5)$$

$\bar{F}_x$ and $\bar{F}_y$ are the normalized longitudinal and lateral forces; ϕ_x and ϕ_y are the normalized longitudinal and lateral slip ratios; K_x , K_y represent the longitudinal slip stiffness and cornering stiffness; F_z is tire vertical load; μ_x , μ_y , E_x and E_y are the friction coefficients and curvature factors in the x and y directions.

The unified normalized longitudinal and lateral tire force curve, Eq. (4), can be illustrated in Fig.2. The unique parameter E_x (E_y) is used to control the curvature. The slope of tangent line at zero slip in Fig.2 is 1, which can make the slip stiffness of tire equals to K_x or K_y automatically in the dimensional space [6].

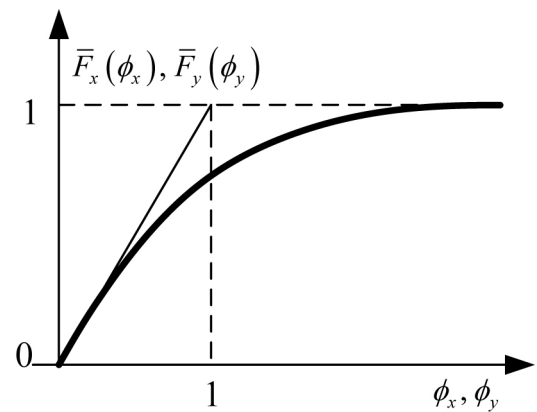


Figure 2. Unified normalized tire force

The friction coefficient μ_x and μ_y in the UniTire model are dynamic friction coefficients [7], which consider the

significant influence of slip speed. The expression is as follows

$$\begin{cases} \mu_x = \mu_{xs} + (\mu_{x0} - \mu_{xs}) \cdot \exp\left(-\mu_{xh}^2 \cdot \log^2\left(\left|\frac{V_{sx}}{V_{sxm}}\right| + N \cdot \exp\left(-\left|\frac{V_{sx}}{V_{sxm}}\right|\right)\right)\right) \\ \mu_y = \mu_{ys} + (\mu_{y0} - \mu_{ys}) \cdot \exp\left(-\mu_{yh}^2 \cdot \log^2\left(\left|\frac{V_{sy}}{V_{sym}}\right| + N \cdot \exp\left(-\left|\frac{V_{sy}}{V_{sym}}\right|\right)\right)\right) \end{cases} \quad (6)$$

where μ_{x0} , μ_{xs} , μ_{xh} and V_{sxm} are longitudinal friction characteristic parameters and μ_{y0} , μ_{ys} , μ_{yh} and V_{sym} are lateral friction characteristic parameters, N (usually $N=0.8$) is a factor to make the friction coefficient increase slightly around the origin [7]. From Eq.(1), V_{sx} and V_{sy} are the longitudinal and lateral slip speeds which can be expressed as:

$$V_{sx} = \frac{S_x}{1-S_x} V \cos \alpha, V_{sy} = \frac{S_y}{1-S_x} V \cos \alpha \quad (7)$$

With all the above equations, the longitudinal and lateral forces under pure slip conditions can be calculated. Fig.3 and Fig.4 show comparisons between measurements and calculations for F_x and F_y separately. It can be seen that the model can describe pure longitudinal and lateral forces accurately, which is the cornerstone for predicting tire forces under combined slip conditions.

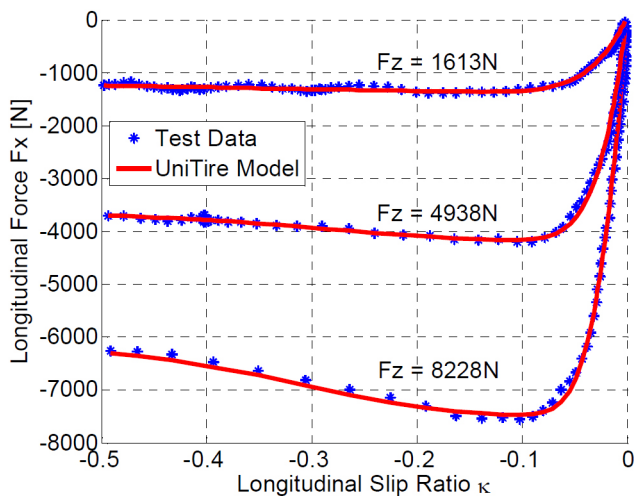


Figure 3. Longitudinal force at pure slip condition

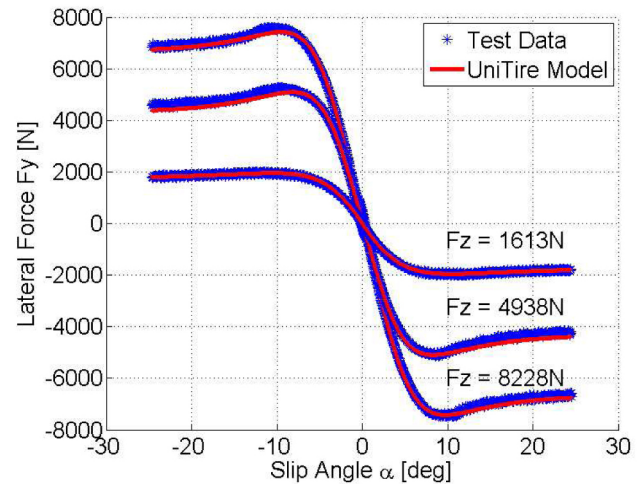


Figure 4. Lateral force at pure slip condition

THE STATE STIFFNESS METHOD

For modeling the combined braking and cornering forces, the state stiffness method will be discussed in this section. The shear forces acting on the tire and the velocity of the wheel are shown in Fig.5 under combined slip condition. F is the resultant force and F_x and F_y are the components in longitudinal and lateral direction; the wheel traveling speed V is composed of the slip speed vector V_s and the rolling speed vector V_r ; V_{sx} and V_{sy} are the components of V_s .

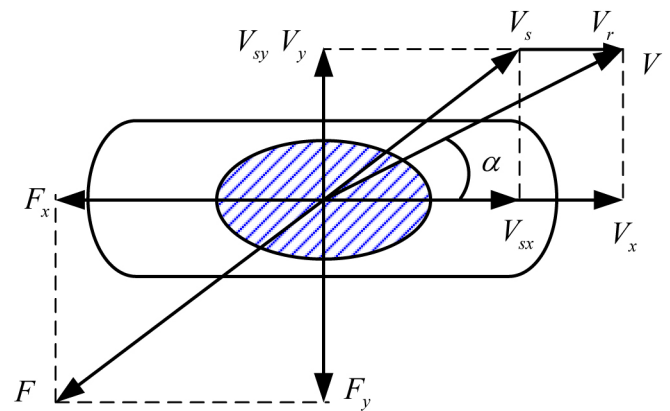


Figure 5. Top view of tire during combined cornering and braking

To model the resultant tire force F and its components, F_x and F_y , some properties of tire force under combined slip conditions should be discussed first.

The Concept of Slip Circle

The concept of slip circle was defined in [3], and was used in COMBINATOR model. From Eq.(1), under combined conditions, the two slip components S_x and S_y can be combined into a total slip ratio with a magnitude of

$$S = \sqrt{S_x^2 + S_y^2} \quad (8)$$

And a direction of

$$\tan \theta_s = S_y / S_x = V_{sy} / V_{sx} \quad (9)$$

For different combinations of S_x and S_y , the directions θ_s are located in different quadrants, as shown in Table 1.

Table 1. Sign Convention

Longitudinal Slip ratio κ	S_x	Slip angle α	S_y	$\theta_s = \arctan(S_y/S_x)$
+	+	+	-	Fourth Quadrant
+	+	-	+	First Quadrant
-	-	+	-	Third Quadrant
-	-	-	+	Second Quadrant

With a constant magnitude S , the different directions θ_s describe a full circle, which is called the *Slip Circle* [3]. Fig.6 gives an illustration of the slip circle. The concept of slip circle is very important, because the resultant slip ratio, S , reflects the totality of the rather complicated deformations and slip motions in the tire-road contact region. And it is notable that the tire shear force will reveal some important properties at a constant magnitude of resultant slip.

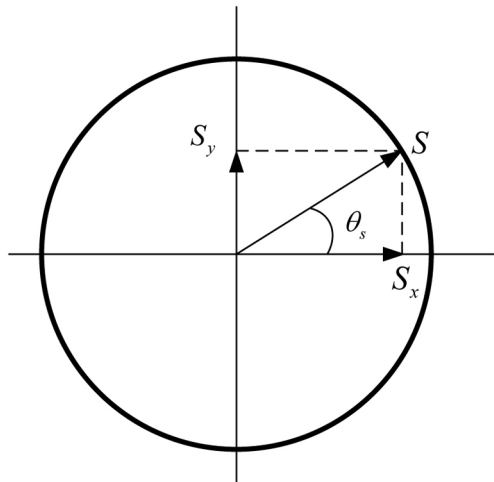


Figure 6. Slip circle

The Magnitude and Direction of Resultant Force

The magnitude of resultant force can be computed by

$$F = \sqrt{F_x^2 + F_y^2} \quad (10)$$

and its direction by

$$\tan \theta_F = F_y / F_x \quad (11)$$

From the tire theoretical model [8], we know that at small values of resultant slip ratio S , there is no sliding in the whole tire-road contact region, and the longitudinal and lateral forces can be written as

$$F_x = K_x S_x, F_y = K_y S_y \quad (12)$$

With the equations (8) and (9), equation (10) becomes:

$$F_x = K_x S \cdot \cos \theta_s, F_y = K_y S \cdot \sin \theta_s \quad (13)$$

Therefore,

$$\left(\frac{F_x}{K_x S} \right)^2 + \left(\frac{F_y}{K_y S} \right)^2 = 1 \quad (14)$$

With equations (10) and (11), the magnitude of resultant force can be expressed as

$$\left(\frac{F \cdot \cos \theta_F}{K_x S} \right)^2 + \left(\frac{F \cdot \sin \theta_F}{K_y S} \right)^2 = 1 \quad (15)$$

and the direction of resultant force

$$\tan \theta_F = \frac{F_y}{F_x} = \frac{K_y}{K_x} \cdot \tan \theta_s \quad (16)$$

It can be seen that the magnitude of resultant force satisfy the ellipse equation, when the resultant slip ratio S is constant and small. The direction θ_F is different from the slip direction θ_s , when the longitudinal slip stiffness K_x is different with the cornering stiffness K_y . At this time, θ_F represent the direction of the shear force in the adhesion region, and can be redefined as adhesion direction θ_{ad} in order to state conveniently. Fig.7 shows the tire-force ellipse, where it is assumed that $K_y=1$ and $K_x=2$.

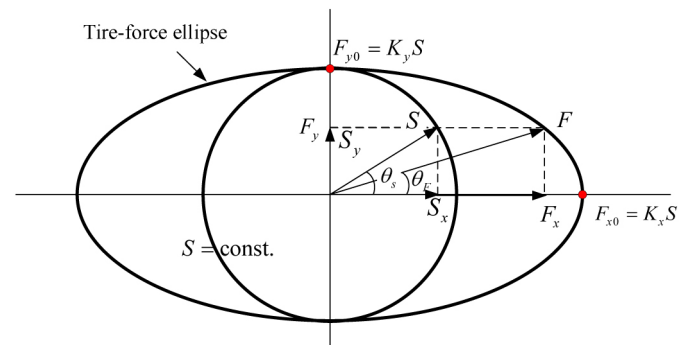


Figure 7. Tire-force ellipse at small value of slip condition

The magnitude of resultant force for large values of slip is usually assumed to satisfy the friction ellipse, and the