

Analysis of Gear Geometry and Durability with Asymmetric Pressure Angle

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Abstract

Gear design is one of the most critical components in the Mechanical Power Transmission industry. Among all the gear design parameters pressure angle is the most critical parameter, which mainly affects the load carrying capacity of the gear. Generally gears are designed with a symmetric pressure angle for drive and coast side. It means that both flank side of gear are able to have same load carrying capacity. In some applications, such as in wind turbines, the gears experience only uni-directional loading. In such cases, the geometry of the drive side need not be symmetric to the coast side. This allows for the design of gears with asymmetric teeth. Therefore new gear designs are needed because of the increasing performance requirements, such as high load capacity, high endurance, long life, and high speed. These gears provide flexibility to designers due to their non-standard design. If they are correctly designed, they can make important contributions to the improvement of designs in aerospace industry, automobile industry, and wind turbine industry.

In this paper we present a mathematical model of helical gear pair with an asymmetric pressure angle. We have increased the pressure angle of gear on the drive side to increase the load capacity and performance of the gear pair in terms of noise and mesh stiffness while transmitting power. Also we have analyzed the results of bending and contact stresses generated on gear pair with the asymmetric pressure angle.

CITATION: Sandooja, A. and Jadhav, S., "Analysis of Gear Geometry and Durability with Asymmetric Pressure Angle," *SAE Int. J. Commer. Veh.* 5(2):2012, doi:10.4271/2012-01-1995.

INTRODUCTION

In conventional involute spur and helical gears, both drive and coast flanks experience the same bending and contact strength. However, in most of the cases for automotive applications drive side and coast side are not equally loaded while transmitting power, the drive side of gear tooth is loaded for longer period of time as compared to coast side. Therefore an asymmetric gear tooth is preferable for such applications. In conventional symmetric gear design, bending and contact strength of a particular gear pair can be increased by an increase in pressure angle. But there is a limitation of top land thickness of tooth which is reduced significantly with increasing pressure angle. With an asymmetric gear design, keeping same pressure angle on the drive side and a higher pressure angle on coast side, the bending strength can be improved without improving contact strength for drive side. Hence gear mass, noise and vibration can be reduced significantly. Similarly, by keeping the same pressure angle on the coast side and a higher pressure angle on the drive

side, both bending and contact strength can be significantly increased for the drive side significantly. Additionally, the asymmetric gear with a high pressure angle on drive side will reduce the vibration level due to a reduction in sliding and an increase in mesh stiffness. A gear pair with asymmetric pressure angle is shown in Fig.1.

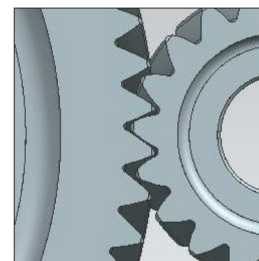


Fig 1. Asymmetric Pressure Angle Gear Pair

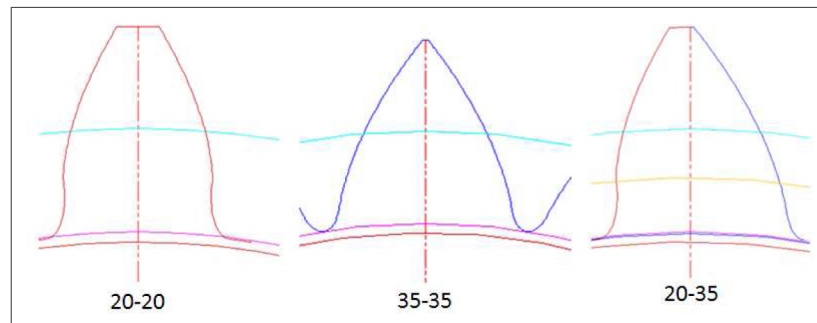


Fig. 2a. Change in Tooth Flank and Top land Thickness w.r.t Pressure Angle

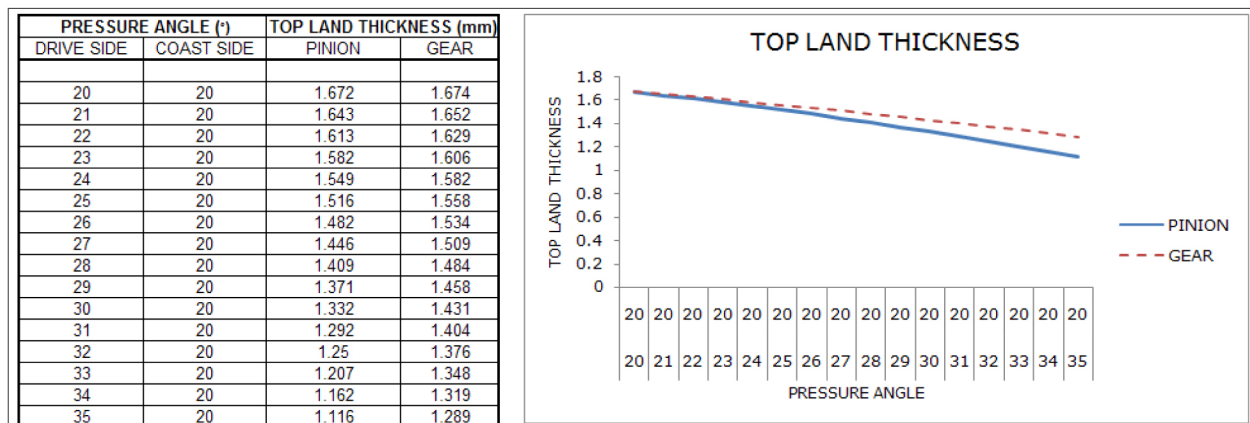


Fig. 2b. Top land thickness Vs pressure angle

INFLUENCE OF PRESSURE ANGLE

In the design of an asymmetric gear pair, proper selection of pressure angle on drive side and coast side is very important. Although the increase in pressure angle will increase the bending strength, there is a limit to the increase in pressure angle because it reduces the top land thickness of tooth which makes the tooth sharp at top which is known as ‘**Tooth Peaking**’. Also increasing the pressure angle decreases the contact and overlap ratio which affects the performance in terms of NVH. By increasing the pressure angle the sliding velocity of the gear reduces because of increase in flank curvature, this in turn reduces the noise and vibration and increases the stiffness of the gear tooth in loaded condition.

An asymmetric gear tooth design effects the following terms and factors related to gear geometry which will be discussed in detail.

1. Tooth Top land thickness
2. Contact Ratio
3. Sliding velocity
4. Bending strength
5. Contact Strength

TOOTH TOP LAND THICKNESS

A major issue or limitation with increasing the pressure angle is the reduction of gear tooth width at the addendum circle. Tooth shape becomes more and more pointed as the pressure angle increases and the top land becomes smaller which results in permanent deflection at the tooth tip. This phenomenon is termed as “**Tooth Peaking** “. The peaking limit sets the limit of the pressure angle. The effect on the tooth flank and top land thickness with the increase in pressure angle is shown in Fig. 2a and 2b.

The thickness of the gear tooth at the addendum circle decreases as the pressure angle increases, and the tooth tip becomes more pointed. Gear Standards such as DIN, ISO and AGMA recommend that the thickness at the addendum circle be a **minimum 0.4 times of module**. Tooth thickness on addendum circle is calculated by:

$$Sa = Do/2 * [(2 * Sna / d) + (inv\alpha_{nd} + inv\alpha_{nc}) \dots - (inv\alpha_{tcd} + inv\alpha_{tcc})] \geq 0.4 \text{ module} \quad \dots (1)$$

$$atcd = ACOS [(d / Do) * Cos (\alpha_{nd})] \quad \dots (2)$$

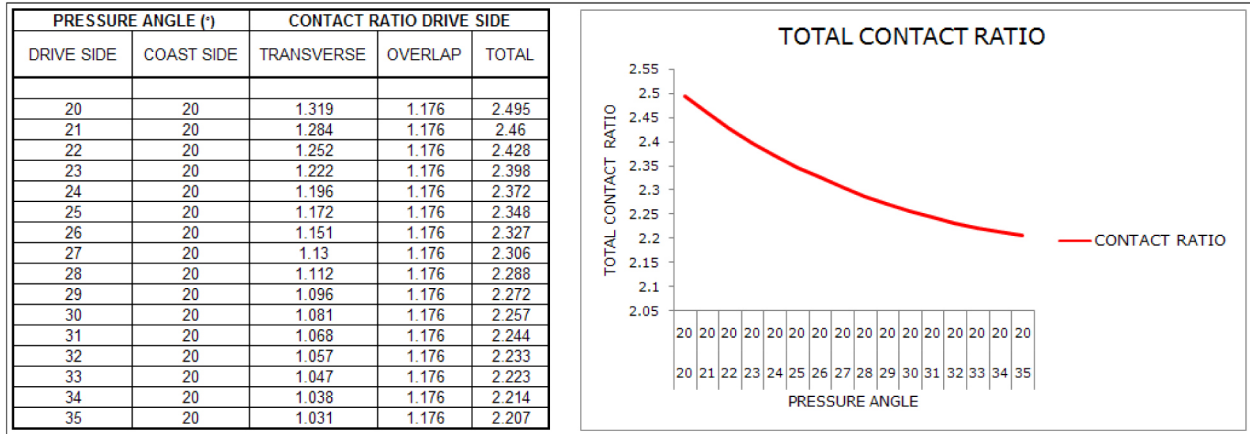


Fig. 3. Contact Ratio Vs Pressure Angle

$$\alpha_{tcc} = \text{ACOS}[(d/D_o) * \text{Cos}(\alpha_{nc})] \quad \dots\dots\dots(3)$$

CONTACT RATIO

Contact ratio is the measure of the average number of teeth in contact along the total path of contact. It means the contact period in which a tooth comes and goes in and out of contact with the mating gear. As the pressure angle increases the contact ratio reduces and load increases on gear tooth. In general the contact ratio of a gear pair should be a minimum **1.10**, below this limit the loading on gear tooth increases, which results in tooth breakage during cyclic loading. Also the highest point of single tooth contact (HPSTC) reaches near the tooth tip causes permanent deflection in gear teeth and affects the gear performance in terms of noise and vibration. Hence it is important to the keep maximum pressure angle on drive side so as to maintain a minimum contact ratio of 1.10 and keeping same pressure angle on coast side for better performance. The behavior of contact ratio with increase in pressure angle is shown in Fig. 3

Total contact ratio of gear pair is calculated by:

$$\epsilon_{ges} = \epsilon d + \epsilon \beta \quad \dots\dots\dots(4)$$

$$\epsilon d = (gadp + gadg) / Pbt \quad \dots\dots\dots(5)$$

$$\epsilon \beta = b * \text{Tan } \beta / \pi * mt \quad \dots\dots\dots(6)$$

SLIDING VELOCITY

Gear tooth sliding velocity is defined as a difference between rolling velocities of teeth in mesh. For two involute profile teeth in mesh, at a given point on the line of action, a product of radius of curvature and the rotational speed are not equal; therefore, the resultant rolling velocities are different. The pitch point is the only contact point for the meshing teeth where there is a pure rolling. At any other location during the interaction of tooth profile there is some sliding. Fig. 4a

shows the contact of gear and pinion at the tip of the tooth. The velocities at contact point of gear and pinion are denoted by V_g and V_p . The individual velocities of the gear and pinion can be resolved into two components normal and tangent to tooth surface as V_{gn} and V_{gt} for gears and V_{pn} and V_{pt} for pinion. If the teeth in meshing contact at the pitch point 'P', then V_{pn} and V_{gn} are same. But under loading conditions there is some misalignment and deflection on gear the tooth resulting in sliding. Therefore the tangential velocity components V_{pt} and V_{gt} are different and the difference between two is known as **sliding velocity**. The sliding velocity is directly proportional to the distance between the pitch point and point of contact. The maximum sliding velocity occurs with contact at tooth tip.

The sliding velocity at tip v_{ga} and at root v_{gf} of gear teeth is calculated by following equation:

$$v_{ga} = [\omega p * ga * (1 + \frac{1}{u})] \quad \dots\dots\dots(7)$$

$$v_{gf} = [\omega p * gf * (1 + \frac{1}{u})] \quad \dots\dots\dots(8)$$

$$ga = 1/2 * [(\sqrt{(dNa1)^2 - (db1)^2}) - (db1 * \text{Tan}(\alpha_{wtd}))] \quad \dots\dots\dots(9)$$

$$gf = 1/2 * [(\sqrt{(dNa2)^2 - (db2)^2}) - (db2 * \text{Tan}(\alpha_{wtd}))] \quad \dots\dots\dots(10)$$

As the pressure angle increases, the sliding velocity decreases due to, an increase in tooth thickness at the root and mesh stiffness, the deflection of gear teeth reduces. Hence performance of the gear pair increases in terms of reduction in noise and vibration. Fig. 4b shows the behavior of sliding velocity at the tooth tip as the pressure angle increases on the drive side.

GEAR TOOTH BENDING STRENGTH

Determination of the capacity of the gear to transfer the required torque for the desired operating life is completed by

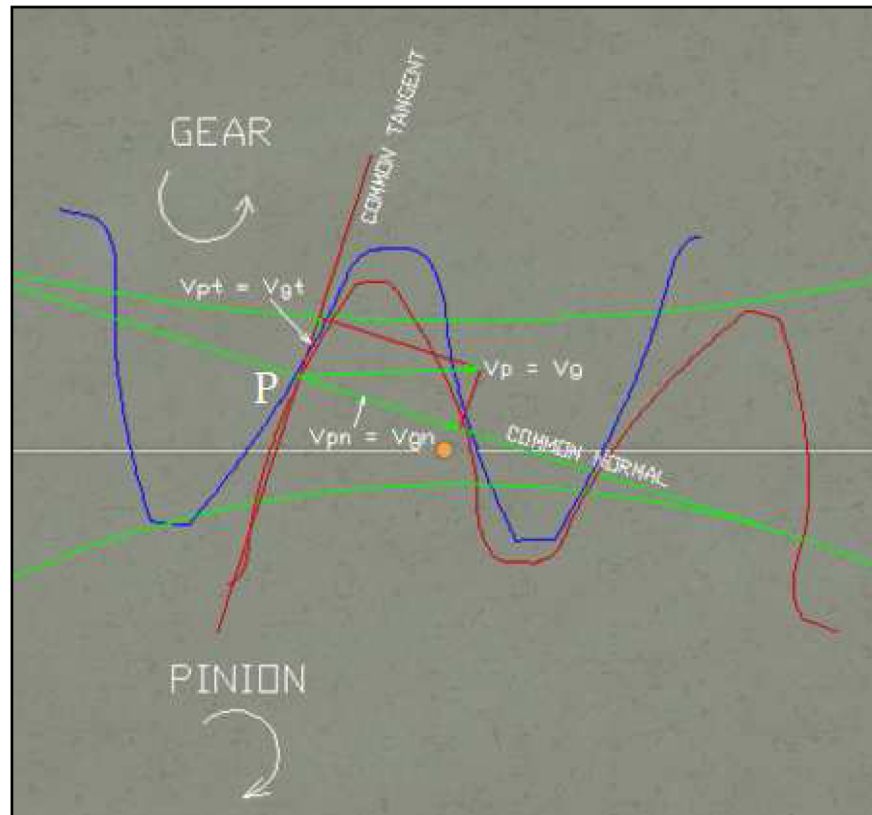


Fig. 4a. Single Tooth Contact

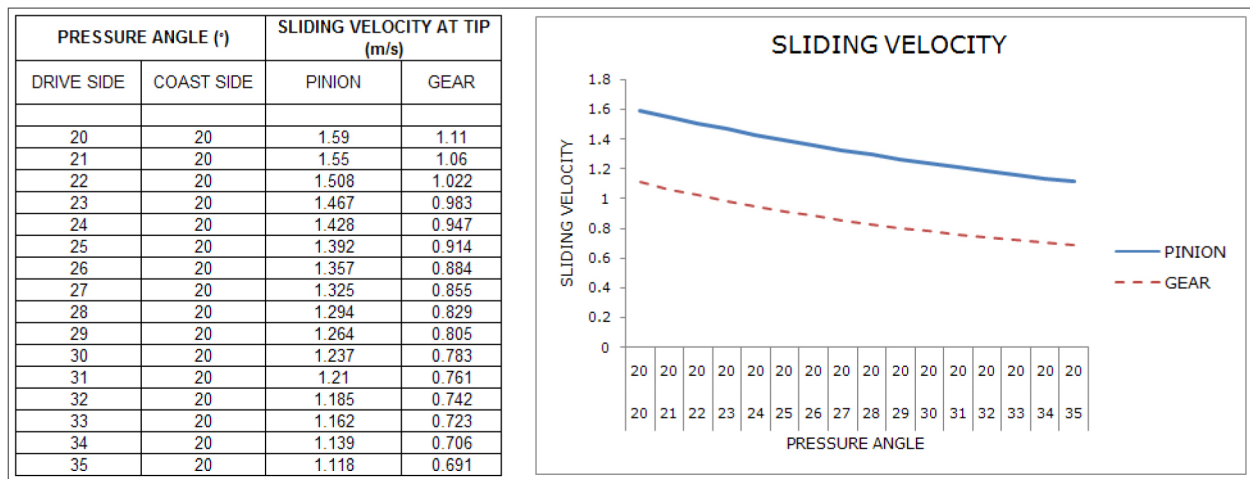


Fig. 4b. Sliding Velocity Vs Pressure angle

determining the strength of the gear teeth in bending and durability of teeth. Gear tooth bending strength increases as the stresses at the tooth root decreases. As the pressure angle increases the tooth root thickness at the critical section increases hence reducing stress at the tooth root. Fig. 5 shows, the direction of load applied on gear tooth while transferring torque.

According to ISO and DIN standards the maximum bending stress on a gear tooth is given by

$$\sigma_{Fb} = [F_t / (b \cdot mn)] \cdot Y_{Fa} \cdot Y_{sa} \cdot Y_{\epsilon} \cdot Y_{\beta} \cdot [K_A \cdot K_V \cdot K_{F\beta} \cdot K_{Fa}] \quad (11)$$