

Dynamics and Simulation of RWD Vehicles Drifting at Steady State using BNP-MNC Tire Model

Ronnapee Chaichaowarat and Witaya Wannasuphprasit
Chulalongkorn University

ABSTRACT

Drifting, a very popular challenge in the recent motorsport, is a cornering technique with large angle of sideslip when rear tires encounter with combined slip over the linear region of the slip-tire friction characteristic. Besides, this technique is frequently used by expert rally drivers in order to ensure that the vehicle will not understeer. In this work, steady state cornering of a RWD vehicle on plane, with constant speed, sideslip and radius of curvature, was simplified via the single track vehicle model. In addition, BNP magic formula, along with MNC model, was used to estimate tire frictions. The computer program was developed, based on EOMs derived via body fixed coordinate, in order to calculate a suitable cornering speed and corresponding open-loop driving control inputs, steering angle and rear slip ratio, for a given radius of curvature and vehicle sideslip. By comparing the obtainable cornering speeds relating to different sideslips at various radii of curvature, the low-sideslip driving provides maximum speed for almost all radii; however, drifting is faster in some special radius. Due to this advantageous, the development of automatic drifting assistant may be feasible.

KEYWORDS: Drifting, Vehicle dynamics, Automotive control, Single track vehicle model, Dynamics simulation

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INTRODUCTION

Nowadays, in order to increase the level of safety in transportation, various active stability control systems have been invented and installed into general passenger vehicles so that drivers still hold their controllability despite emergency maneuvers. However, the existent systems such as differential braking [1], active steering [2], or even independent wheel torque control [3], appeared in electric and hybrid vehicle, were developed based on a simplified vehicle model of steady state cornering with small sideslip [4]. Moreover, in all modern automobile, rudimentary active safety systems, such as the Anti-lock Braking System (ABS) and the Traction Control System (TCS), primarily aim to avoid slip phenomena, such that tires operate within the linear region of the wheel slip-tire friction characteristic where ordinary drivers still maintain their controllability. It is ponderable that rather than restricting tire slip limit in linear region, both full handling performance of vehicle and expert skills of driver in extreme slip condition may be imperative to avert accident in emergency case.

Drifting is a cornering technique with large angle of sideslip where tires incur large slip in both lateral and

longitudinal directions. This technique is a very popular challenge in recent motorsport; also, is frequently used by expert rally drivers when turning sharply on dirt track to ensure that the vehicle will not understeer. Although drifting is not a natural thing for normal drivers, also requires expert skills of driver to control suitable slip of each four tires to sustain equilibrium of forces and moment so that the vehicle travel along desired trajectory, it is still useful in some cornering condition as mentioned in [5, 6]. In those experiments, the test car had been modified by installing inertial measurement unit and GPS module, also other equipment to measure and record vehicle states during drift by an expert rally driver. The recorded value of state variables was used to analyze cornering behaviors in order that some popular drifting techniques, such as Trail-braking and Pendulum-turn, could be simulated. Simulation results show that drifting does not provide the minimum-time trajectory as does small-slip driving, but provides ability to return to a controllable straight line driving state, which allows the driver to react to emergencies and unexpected changes in the environment, faster than that of the conventional driving. Furthermore, there was an alternative technique, via neural network, to estimate sideslip and yaw

acceleration while drifting by using data measured from the array of linear accelerometers [7].

That the number of papers about drifting has been increasing over the last decade reflects the awareness of researchers on how important the understanding of vehicle dynamics at high slip maneuver is; however, limited progress on the design and development of controlling systems for automatic drifting. With this vision in mind, based on assumptions of the single track vehicle model, the optimal controller for steady state drifting of an AWD vehicle was initiated [8] and simulated computationally. In the study, tire frictions, the major factor affecting on motion behavior of vehicle, were estimated by the BNP magic formula [9], along with the friction circle model by assuming symmetrical tire characteristic in both lateral and longitudinal direction. The state space description was constructed by linearization of the vehicle and the tire models about operating condition at steady state in order that the quadratics optimal regulator could be designed. For the derivation of the open-loop control inputs corresponding to a given steady state condition, front steering angle was fixed so that front and rear wheel slip ratios could be calculated directly without requiring of computational iteration. However, most of the calculated results were difficult to be generated in real situations. For example, independent torque distribution among front and rear drive shaft was needed in case of positive slip ratio required in both front and rear wheels. Besides, the Left-foot-braking technique might be needful in case of negative slip ratio required at front wheels and positive slip ratio required at rear wheels.

Hence, it would be more appropriate to derive control inputs which could be controlled conveniently. Without fixing the steering angle to be constant, the algorithms to calculate the control inputs for steady state drifting of RWD vehicle were developed in [10, 11, 12] by neglecting rolling resistance and assuming no brake torque applied to both front wheels. The optimal controller for steady state drifting of RWD vehicle was initiated and simulated computationally in [10, 11]. In those study, the comprehensive Full-car vehicle model, along with the tire frictions estimating approach used in [8], was used to develop a computer program in order to iterate the driving control inputs, consisting of steering angle and rear drive torque; the details of the iteration, however, hadn't been revealed. Soon afterward, the single track vehicle model was embedded into the computer program [12] for the calculation of steady state control inputs, consisting of steering angle and rear slip ratio. On the other hand, the other tire friction model [13], which could provide more realistic slip-tire friction characteristic and more popular in simulation scheme, was used instead of the approach used in [8, 10, 11]. However, instead of focusing on steady state drifting controller, this project was emphasized on determining of maximum cornering speed among different sideslips in order to state whether drifting was faster than convenience driving for a given radius of curvature.

Due to the complexity of tire model [13] which tire frictions could not be explicitly calculated without computational iteration, it was not proper for control purpose. Furthermore, the estimation of tire forces by the approach used in [8, 10, 11] provided unsatisfactory results in highly degree of combined slip. Eventually, in this study the BNP magic formula [9], along with the MNC equations [14], was used so that the frictions at high slip condition could be precisely estimated. Building on the approach of [12] with application of the single track vehicle model, computer program was developed for calculation of the suitable cornering speed and corresponding open-loop driving control inputs, consisting of steering angle and rear slip ratio, for any given sideslip and radius of curvature. In the proposed of control scheme, stabilization is achieved by regulation of steering angle and rear slip ratio. LQR stabilizing controller had been designed and simulated via MATLAB.

In this paper, the single track vehicle model will be introduced first. After that, the application of BNP-MNC tire friction model will be briefly described. Consequently, the algorithm of computation for steady state driving control inputs will be explained via the program flowchart. Finally, the results of simulation will be discussed in terms of the comparison between cornering speeds corresponding to different sideslip. Furthermore, the equations of motion in terms of state space description will be proposed for control objective; however, the controller design procedure will not be included.

DYNAMIC MODEL

Single Track Vehicle Model

Dynamics of drifting vehicle along a constant radius curve on a horizontal plane, with constant cornering speed and angle of sideslip, was simplified via a two dimensional dynamics model, namely the single track or the bicycle model. Based on this simple model, the degrees of complexity on dynamics of an actual automobile can be reduced as the consideration of two wheels vehicle where dynamics of suspension and effects of lateral load transfer of a vehicle are neglected.

To describe the definition and sign convention of symbols used in the single track vehicle model, the translation of vehicle drifting along the arc from position A to B is illustrated in Figure 1.

In this figure, $x - y$ coordinate is the moving frame fixed to the body of a vehicle where $x - axis$ points to the vehicle heading in longitudinal direction and $y - axis$ positions in lateral direction. The other moving frame located at CG is $n - t$ coordinate, the direction of which always changes depending on direction of vehicle velocity. a and b are the longitudinal distances from the CG to the front axle and the rear axle respectively. The velocity of vehicle $\vec{V}$ measured at CG is perpendicular to the direction of curved radius ρ measured from the instantaneous center zero velocity ICZV

to the CG. The angle between vehicle velocity and the heading direction is given by the sideslip β . Likewise, the direction of local velocities at the front axle $\vec{V}_f$ and the rear axle $\vec{V}_r$ are perpendicular to the directions of curved radii measured from ICZV to the front and the rear axles respectively. The sideslip β , the steering angle δ , the front wheel slip angle α_f and the rear wheel slip angle α_r are positive in clockwise direction whereas the yaw angle ψ , yaw rate $\dot{\psi}$ and yaw acceleration $\ddot{\psi}$ are measured in counterclockwise direction. Finally, the positive directions and the point of applications of the tire forces are also shown in the same figure. The vehicle parameters used in this study is show in [Table 1](#).

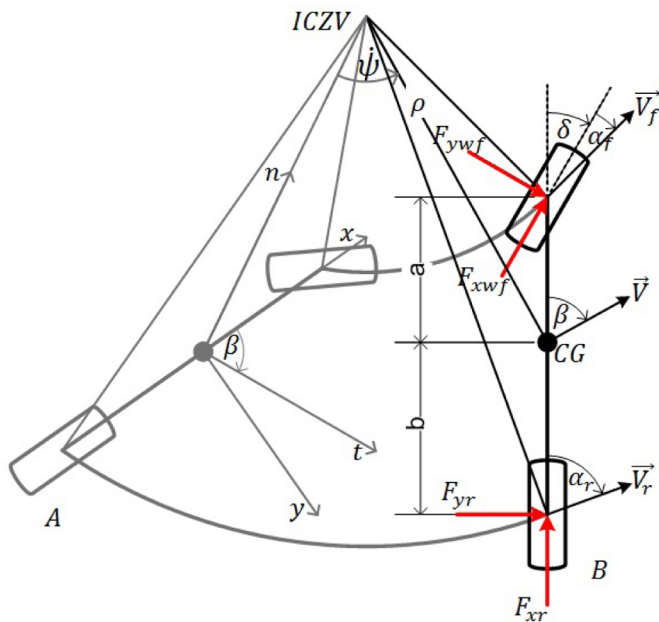


Figure 1. Single track vehicle model

Table 1. Vehicle parameters

Parameters	Value	Parameters	Value
m (kg)	1250	L (m)	2.52
I_{CG}	2500	h (m)	0.28
a (m)	1.13	r_f (m)	0.3
b (m)	1.39	r_r (m)	0.3

Equations of Motion

According to the body-fixed $x - y$ coordinate, the set of equations of motion is obtained by applying the Newton's second law of motion. The equilibriums of forces in both longitudinal and lateral directions are shown in [equations \(1\)](#) and [\(2\)](#) respectively. Also, the equilibrium of moment about the vertical axis is shown in [equation \(3\)](#)

$$F_{xwf} \cos \delta - F_{ywf} \sin \delta + F_{xr} = ma_x \quad (1)$$

$$F_{xwf} \sin \delta + F_{ywf} \cos \delta + F_{yr} = ma_y \quad (2)$$

$$-(F_{xwf} \sin \delta + F_{ywf} \cos \delta) a + F_{yr} b = I_{CG} \ddot{\psi} \quad (3)$$

where lateral and longitudinal accelerations can be derived from certain normal and tangential accelerations via the coordinate transformation between $x - y$ and $n - t$ coordinates, as shown in [equations \(4\)](#) and [\(5\)](#)

$$a_x = a_n \sin \beta + a_t \cos \beta \quad (4)$$

$$a_y = -a_n \cos \beta + a_t \sin \beta \quad (5)$$

In the special case of steady state drifting, tangential acceleration is always zero as [equation \(6\)](#). Besides, normal acceleration can be simply calculated from [equation \(7\)](#)

$$a_t = \dot{\rho} \dot{\psi} + \rho \ddot{\psi} = 0 \quad (6)$$

$$a_n = \rho \dot{\psi}^2 = V_{ss}^2 / \rho_{ss} \quad (7)$$

According to the assumptions that no rolling resistance, no braking and driving torques are applied to the front wheels, the equations of motion describing steady state drifting of RWD vehicle can be modified as shown in [equations \(8\), \(9\), \(10\)](#)

$$-F_{ywf} \sin \delta + F_{xr} = \frac{m V_{ss}^2}{\rho_{ss}} \sin \beta_{ss} \quad (8)$$

$$F_{ywf} \cos \delta + F_{yr} = -\frac{m V_{ss}^2}{\rho_{ss}} \cos \beta_{ss} \quad (9)$$

$$-F_{ywf} \cos \delta a + F_{yr} b = 0 \quad (10)$$

The obtained EOMs were rearranged to the usable form as the following equations

$$F_{ywf} = \frac{ma_y b}{L \cos \delta} \quad (11)$$

$$F_{xr} = ma_x + F_{ywf} \sin \delta \quad (12)$$

$$F_{yr} = ma_y - F_{ywf} \cos \delta \quad (13)$$

This set of EOMs was used to construct the computer program for calculation of the suitable cornering speed and

corresponding open-loop driving control inputs for any desired steady state.

Vertical Loads

As previously introduced that lateral load transfer can be neglected by using the single track vehicle model. However, longitudinal load transfer is still significant. The calculations of vertical loads at front and rear axles are shown in equations (14) and (15) respectively

$$F_{zf} = \frac{m}{L}(gb - a_x h) \quad (14)$$

$$F_{zr} = \frac{m}{L}(ga + a_x h) \quad (15)$$

where h is the vertical distance of CG from ground.

Velocities and Tire Slips

The local velocities at any position on the vehicle must be derived via kinematic analysis so that the tire slips at front and rear axles can be calculated. For a given steady state triplet (ρ, β, V), the rate of changing in direction or yaw rate $\dot{\psi}$ is shown in equation (16). Also, the components of local velocity at CG are shown in equations (17) and (18)

$$\dot{\psi} = V/\rho \quad (16)$$

$$V_x = V \cos \beta \quad (17)$$

$$V_y = V \sin \beta \quad (18)$$

The components of the front wheel velocity along longitudinal and lateral directions of the vehicle are shown as equations (19) and (20). However, to calculate actual tire slip, the components of velocity parallel and perpendicular to the heading direction of the front tire, as shown in equations (21) and (22), are imperative.

$$V_{xf} = V_x \quad (19)$$

$$V_{yf} = V_y - \dot{\psi} a \quad (20)$$

$$V_{xwf} = V_{xf} \cos \delta + V_{yf} \sin \delta \quad (21)$$

$$V_{ywf} = -V_{xf} \sin \delta + V_{yf} \cos \delta \quad (22)$$

The front tire slip angle can be calculated as equation (23.1) or (23.2); also, slip ratio as equation (24)

$$\alpha_f = \tan^{-1} \left(\frac{V_{ywf}}{V_{xwf}} \right) \quad (23.1)$$

$$\alpha_f = \tan^{-1} \left(\frac{V_{yf}}{V_{xf}} \right) - \delta \quad (23.2)$$

$$k_f = \frac{\omega_f r_f - V_{xwf}}{\omega_f r_f} \quad (24)$$

where ω_f is rotational speed and r_f is radius of the front tire.

Likewise, the components of local velocity at the rear wheel can be calculated by the following equations

$$V_{xr} = V_x \quad (25)$$

$$V_{yr} = V_y + \dot{\psi} b \quad (26)$$

The rear tire slip angle and slip ratio can be calculated as equations (27) and (28) respectively

$$\alpha_r = \tan^{-1} \left(\frac{V_{yr}}{V_{xr}} \right) \quad (27)$$

$$k_r = \frac{\omega_r r_r - V_{xr}}{\omega_r r_r} \quad (28)$$

where ω_r is rotational speed and r_r is radius of the rear tire.

Slip angle and slip ratio obtained in this section are the crucial quantities directly affecting on magnitude of tire frictions in both lateral and longitudinal directions, as will be described in the following section.

BNP-MNC Tire Friction Model

As previously mentioned in the beginning that extremely combined slip of rear tire is inevitable during drift; therefore, appropriate tire friction model must be deliberately chosen.

In this study, the Modified Nicolas-Comstock (MNC) tire model for combined slip [14] was used to estimate tire frictions in both longitudinal and lateral directions as shown in equations (29) and (30) respectively

$$F_x(\alpha, k) = \frac{F_x(k)F_y(\alpha)k}{\sqrt{k^2 F_y^2(\alpha) + F_x^2(k) \tan^2 \alpha}} \times \frac{\sqrt{k^2 C_{\alpha}^2 + (1-|k|)^2 \cos^2 \alpha F_x^2(k)}}{k C_{\alpha}} \quad (29)$$