

Estimation of Lateral Tire Force from Objective Measurement Data for Handling Analysis

Jungsik Kim

Hankook Tire Co., Ltd. R&D Center

Seongho Kim

Hankook Tire Co., Ltd. R&D Center

ABSTRACT

A methodology is presented to estimate lateral tire force dynamics. The methodology is based on EKF (extended Kalman filter) and makes it possible to determine lateral tire force dynamics on the basis of the results obtained from various on-road handling manoeuvres such as lane change, slalom, and circuit driving. A vehicle model having four degrees of freedom is used to estimate the lateral tire dynamics where the lateral tire forces are described by the Magic formula with transient behavior. Several handling tests including subjective and objective are performed. The objective data are used to extract lateral tire dynamics during handling test manoeuvres. The estimated lateral tire forces are compared with the experimental one measured from indoor Flat-trac machine. Also, the subjective ratings are compared with the estimated lateral tire characteristics such as cornering stiffness, lateral peak friction and load sensitivity. The results show good agreement between subjective rating and lateral tire dynamics, which confirm the availability of the methodology in vehicle and tire design.

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INTRODUCTION

In the vehicle and tire design, designer must consider many performances such as handling, ride, fuel efficiency, endurance and so forth. In these various kinds of performances, handling is very important because it is directly connected to safety of passengers during driving. The computer simulation allows designers today to predict handling performance of vehicle before making a prototype. Thus, much experimental activity can be avoided or at least reduced. If there is a correspondence between the simulation and the real system, the worst ones can be dropped in the variations to the original plan. Fewer prototypes have to be made, and the design activity becomes quicker and less expensive. That correspondence is based on the choice of an adequate model and above all the accuracy of the parameters of the model concerning the tires and suspension. But, the main source of uncertainty in the model is certainly due to the interaction between the tires and road surface because of the non-linear dependence of the lateral tire forces on several variables such as longitudinal slip, sideslip angle, normal

load, camber angle, tire pressure, temperature, wear, and road surface characteristics.

Notable attempts have been made to work out both analytical tire force models based on physical principles and empirical models. Analytical models are usually difficult to handle and require an accurate knowledge of tire component parameters, which are hardly measurable or identifiable in practice. Conversely, in the empirical models, tire forces are described by a family of curves, whose coefficients are identified from experimental laboratory tests [1]. But, these methods have limitations to represent real tire force characteristics due to different conditions from road tests. Therefore, the need for a correction arises, which can be made only using tests performed in real conditions, which is on the road, and a suitable elaboration of the measurements is required. To solve this problem, some identification methodologies have been suggested. Kiencke and Daiß [2] and Stephant and *et al.* [3] present linear and non-linear observation methods using a bicycle model. Sierra and *et al.* [4] suggested both time-domain and frequency-domain based methods, aiming to estimate the cornering stiffness. The estimation methods are based on the simple bicycle model

and common lateral and yaw measurement. Here, it is found that the under-determinateness is a major issue and should be avoided if possible. As other methods, Kim [5, 6], Bolzern *et al.* [7] and Mancosu *et al.* [8] suggest EKF method to determine the tire lateral force on the basis of a pre-established theoretical model and the results of experimental tests carried out on the corresponding real model. It is therefore necessary to define a mathematical model and have access to the real system and its experimental results; the estimations can be determined using an identification technique after making the results of the theoretical model reproduce those of the real system. Therefore, the more complete the mathematical model is, the closer the obtained values will be to the real ones: although this will require a great deal of computing load it will produce a model that will be able to use the identified ones to make a comprehensive analysis of the vehicle's dynamics. On the other hand, if the model is simplified, the identified ones will be less close to the real ones and this will entail a lighter computing work load but the model will enable analysis of a limited field of vehicle's operating conditions, namely those used for its identification. It is therefore necessary to first define the objective so as to choose the most fitting model and, consequently, the identification technique.

The goal of this article is to offer a methodology to obtain reliable lateral tire forces between real roads and tires off-line. The main idea is that the lateral tire forces are identified by the EKF, whose design depends on estimation model of vehicle and statistical information of power and measurement noises. As is widely known, EKF has high reliability to observe state and parameter at the same time [5, 6]. Therefore, there is no need to measure them with expensive instrument. At first, in order to simplify EKF design, the estimation model is simplified as the four degrees of freedom vehicle including two axle suspension-tire forces that are only modeled as a function of the slip angle. Moreover, the introduction of the suspension-tire relaxation length in the estimation model allows one to better evaluate the transient dynamic behaviour, too. The EKF data are then applied to off-line in order to construct lateral tire forces as functions of slip angle and normal load. This approach can provide more reliable data between real roads and tires compared to other estimation techniques. Therefore, designers can easily obtain the simulation models that behave like the real vehicle in actual vehicle tests. Such a model can be used to design advanced control systems and to analyze vehicle handling characteristics. The model of vehicle used is briefly described and some of the results obtained from the identification procedure are presented along with a description of the identification.

CONSTRUCTION OF VEHICLE MODELS

The mathematical vehicle-road system model used for the identification procedure is four degrees of freedom model

accounting for longitudinal, lateral displacements, yaw, and roll rotations as shown in Figure 1.

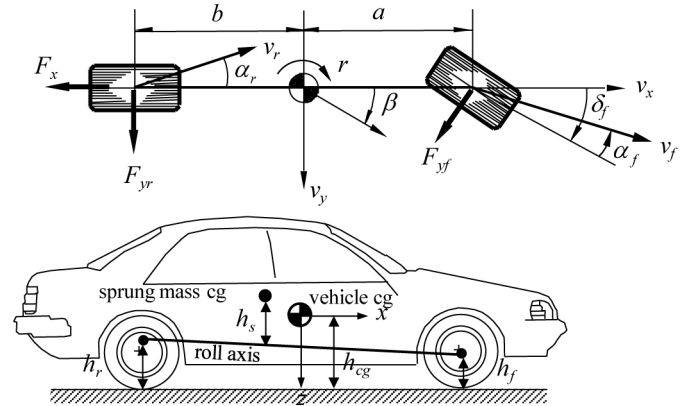


Figure 1. Mathematical vehicle-road system model

Assuming a moving reference system attached to the vehicle and taking the longitudinal velocity v_x , lateral velocity v_y , yaw rate r , roll angle ϕ , and roll rate p as state variables, the equation of motion becomes:

$$\begin{aligned} m\dot{v}_x &= mv_y r - F_{yf} \sin \delta_f - F_x \\ m\dot{v}_y + m_s h_s \dot{p} &= -mv_x r + F_{yf} \cos \delta_f + F_{yr} \\ I_{zz} \dot{r} - I_{xzs} \dot{p} &= aF_{yf} \cos \delta_f - bF_{yr} \\ \dot{\phi} &= p \\ I_{xxs} \dot{p} + m_s h_s \dot{v}_y - I_{xzs} \dot{r} &= -m_s h_s v_x r \\ &\quad + m_s g h_s \sin(\phi) - L_\phi \phi - L_p p \\ \dot{F}_{yf} &= \frac{v_x}{\sigma_f} (-F_{yf} + \bar{F}_{yf}) \\ \dot{F}_{yr} &= \frac{v_x}{\sigma_r} (-F_{yr} + \bar{F}_{yr}) \end{aligned} \quad (1)$$

where m is the total vehicle mass, m_s is the sprung mass that is the mass supported by the suspension, and I_{zz} is the moment of inertia of the entire vehicle around the vertical axes, I_{xxs} is the moment of inertia of the sprung mass around longitudinal axis, I_{xzs} is the inertia product of the sprung mass, L_ϕ and L_p are total roll stiffness and damping. Moreover, a and b are the distances from vehicle centre of gravity to front and rear axles, and h_s is the distance from roll axis to sprung mass centre of gravity. F_x is equivalent longitudinal force due to driving torque, aero dynamic, rolling resistance and so forth. Also, F_{yf} and F_{yr} are the transient lateral tire force on the front and rear wheels respectively. In general, these forces are function mainly of

the related slip angle α_i , dynamic normal loads Z_i , camber angles γ_i , and longitudinal slip s_i ($i=f, r$).

The lateral tire force values F_{yi} can be evaluated from the steady-state lateral tire force $\bar{F}_{yi}$ introducing the relaxation lengths σ_i . The load transfer, which is responsible for fluctuations of the normal loads, can be approximated by a static function of the lateral forces themselves [8]. With these hypotheses, it is possible to view the lateral tire forces only as a function of the slip angle:

$$\bar{F}_{yi} = \bar{F}_{yi}(\alpha_i, Z_i(\bar{F}_{yi})) \quad (2)$$

Equation (1) only includes steering input and ignores the effect of other inputs such as road-bank angle. Therefore, the estimation will be obtained by observing the variation of state output due to steering input perturbations. In this article, the road-bank angle is assumed to be zero. In the model of Figure 1, it is easy to recognize that the front and rear slip angles α_f and α_r are obtained as a function of the independent variables as:

$$\begin{aligned} \alpha_f &= -\delta_f + \varepsilon_f \phi + \tan^{-1} \left(\frac{v_y + ar}{v_x} \right) \\ \alpha_r &= \varepsilon_r \phi + \tan^{-1} \left(\frac{v_y - br}{v_x} \right) \end{aligned} \quad (3)$$

where δ_f is the steer angle, which can be obtained by dividing the steering wheel angle by an overall steering gear ratio of steering gear box. Also, ε_f and ε_r are front and rear roll steer coefficients, which include roll steer effect in the vehicle model. From Equations (3) slip angle can be estimated during vehicle manoeuvring.

In the true vehicle model, the normal load depends on suspension dynamics. The normal load to produce a lateral tire force can be made as [10]

$$\begin{aligned} [F_{zfl} F_{zfr}] &= \frac{mg}{2} \left(\frac{b}{l} - \frac{a_x h_{cg}}{gl} \right) \pm \frac{L_{\phi f} R_{\phi} a_y + m_f h_f a_y}{t_{wf}} \\ [F_{zrl} F_{zrr}] &= \frac{mg}{2} \left(\frac{a}{l} + \frac{a_x h_{cg}}{gl} \right) \pm \frac{L_{\phi r} R_{\phi} a_y + m_r h_r a_y}{t_{wr}} \end{aligned} \quad (4)$$

where, $l=a+b$ is wheelbase and $R_{\phi}=mgh_s/(L_{\phi f}+L_{\phi r}-mgh_s)$, which represents the roll angle change according to lateral acceleration. From Equations (4) normal load can be estimated during vehicle manoeuvring.

Magic formulas [1] are used to represent when the vehicle approaches the physical limit of adhesion in tire model. In order to reproduce the non-linear relationship between slip angles and steady-state tire lateral forces, the following formula is adopted:

$$\bar{F}_{yi}(\alpha_i) = D_i \sin \left[C_i \tan^{-1} \left\{ B_i \alpha_i - E_i (B_i \alpha_i - \tan^{-1}(B_i \alpha_i)) \right\} \right] \quad (5)$$

where $i=f$ for front axle and $i=r$ for rear axle. As is well known, the parameters in Equation (5) have a direct interpretation. Precisely, C is a shape factor, usually fixed to 1.3, D indicates the peak value of the force, E determines the curvature at the peak value. Note that the product BCD represents the initial slope and corresponds to the linearized cornering stiffness. Equations (1), (3) and (5) are the vehicle-road system models used in the estimation of vehicle handling dynamics.

THE EKF ESTIMATION DESIGN

As the road test gives the time history of a few system states, it is necessary to deduce all other ones in order to identify the vehicle parameters. So a technique based on the Kalman filter has been adopted because of its high reliability as state and parameter observer at the same time. The identification technique adopted here, namely the continuous extended nonlinear Kalman filter, calls for the model and the measurements to be available in the form of continuous functions of time. It is thus necessary to have measurements sampled at a sufficiently high frequency so that the measurements can be made continuous during algorithm execution through linear interpolation. Letting

$$\mathbf{x}(t) = [v_x(t) \quad v_y(t) \quad r(t) \quad \phi(t) \quad p(t) \quad F_{yf}(t) \quad F_{yr}(t)]^T \quad (6)$$

be the state vector and $u(t) = \delta_f(t)$ be the measured input variable, the model of the vehicle described by equation of motion of Equation (1), the kinematics relations of Equation (3) and the expression of the equivalent suspension-tire axle lateral forces of Equations (5) can be written in a compact form as:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), u(t), t) \quad (7)$$

Assumed known the geometric and inertial parameters, the uncertainty are restricted to the parameters appearing in the tire model of Equations (5) and to the relaxation lengths. Equivalent longitudinal force, F_x can be determined using a sensor set that does not require direct measurement. To reduce model complexity, longitudinal slip ratio isn't taken into account for the estimation of the tire lateral force. So, for the estimation problem such as severe braking and accelerating, both lateral slip and longitudinal slip ratio should be taken into account simultaneously. A second-order random walk model is appended to model longitudinal drag force to be determined:

$$\begin{bmatrix} \dot{y}_0 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \end{bmatrix} + \mathbf{w}_y$$

where y_0 represents the force to be determined, y_1 is its first time derivative, and $\mathbf{w}_y$ is random white noise. Thus, in the problem of estimating the lateral tire force F_{yi} of Equation (1), the steady-state lateral tire force $\bar{F}_{yi}$ of Equation (2) and relaxation length σ_i of Equation (1) should be identified. Therefore, in case that non-linear tire model of Equation (5) is used, the parameters vector $\mathbf{p}$ to be identified is given by:

$$\mathbf{p} = [B_f \ D_f \ E_f \ B_r \ D_r \ E_r \ \sigma_f \ \sigma_r \ F_x \ \dot{F}_x]^T \quad (9)$$

In order to use an EKF for the identification of the unknown parameters, the augmented state vector is defined as:

$$\underline{\mathbf{x}}(t) = \begin{Bmatrix} \mathbf{x}(t) \\ \mathbf{p}(t) \end{Bmatrix} \quad (10)$$

and suppose that the parameters appearing Equation (5) and the relaxation lengths are constant with time. Then, Equation (7) can be written as:

$$\begin{aligned} \dot{\underline{\mathbf{x}}}(t) &= \underline{\mathbf{f}}(\underline{\mathbf{x}}(t), t) + \mathbf{w}(t) \\ \underline{\mathbf{z}}(t) &= \underline{\mathbf{h}}(\underline{\mathbf{x}}(t), t) + \mathbf{v}(t) \end{aligned} \quad (11)$$

where $\mathbf{w}(t)$ is a random zero-mean process in Gelb [9]. The process-noise matrix describing the random process $\mathbf{w}(t)$ for the describing model is given by $\mathbf{Q}(t) = E[\mathbf{w}(t)\mathbf{w}(t)^T]$. Also, $\mathbf{v}(t)$ is a zero-mean random process described by the measurement noise matrix $\mathbf{R}(t)$, which is defined as $\mathbf{R}(t) = E[\mathbf{v}(t)\mathbf{v}(t)^T]$.

The vector $\underline{\mathbf{x}}$ and $\underline{\mathbf{f}}$ has the following structure:

$$\underline{\mathbf{x}}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_7(t) \\ p_1 \\ \vdots \\ p_8 \\ p_9(t) \\ p_{10}(t) \end{bmatrix}, \quad \underline{\mathbf{f}}(t) = \begin{bmatrix} f_1(\underline{\mathbf{x}}(t), t) \\ \vdots \\ f_7(\underline{\mathbf{x}}(t), t) \\ 0 \\ \vdots \\ 0 \\ x_{17}(t) \\ 0 \end{bmatrix} \quad (12)$$

In which $x_i(t)$ indicates the 7 states in Equation (6) and p_j indicates the parameters that are to be identified. Consequently, the first 7 terms in the vector $\underline{\mathbf{f}}(t)$ are deduced from Equation (7), while the medium part of the vector contains some 0's to impose the fact that the parameters do not vary over time. The third part is constructed from random

walk model of Equation (8). The overall size n of the two vectors is thus given by the sum of the 7 system states and the 10 parameters to be identified. The vector $\mathbf{z}(t)$ of the measurements has a size m equal to the number of quantities recorded during a test and is, in the most general case, dependent on the system state through the non-linear relation $\mathbf{h}(\mathbf{x}(t), t)$. Having posed the problem in the form described above, the value of the 10 parameters p_i together with the estimate of 7 states x_i , is provided by the steady state solution of the following system of first order differential equations:

$$\begin{aligned} \dot{\underline{\mathbf{x}}}(t) &= \underline{\mathbf{f}}(\underline{\mathbf{x}}(t), t) + \mathbf{K}(t)[\underline{\mathbf{z}}(t) - \underline{\mathbf{h}}(\underline{\mathbf{x}}(t), t)] \\ \dot{\mathbf{P}}(t) &= \mathbf{F}(\underline{\mathbf{x}}(t), t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T(\underline{\mathbf{x}}(t), t) \\ &\quad + \mathbf{Q}(t) - \mathbf{P}(t)\mathbf{H}^T(\underline{\mathbf{x}}(t), t)\mathbf{R}^{-1}(t)\mathbf{H}(\underline{\mathbf{x}}(t), t)\mathbf{P}(t) \\ \mathbf{K}(t) &= \mathbf{P}(t)\mathbf{H}^T(\underline{\mathbf{x}}(t), t)\mathbf{R}^{-1}(t) \end{aligned} \quad (13)$$

where the matrices $\mathbf{F}$ and $\mathbf{H}$ are given by:

$$\begin{aligned} \mathbf{F}(\underline{\mathbf{x}}(t), t) &= \left[\frac{\partial \underline{\mathbf{f}}(\underline{\mathbf{x}}(t), t)}{\partial \underline{\mathbf{x}}(t)} \right]_{\underline{\mathbf{x}}(t) = \underline{\hat{\mathbf{x}}}(t)} \\ \mathbf{H}(\underline{\mathbf{x}}(t), t) &= \left[\frac{\partial \underline{\mathbf{h}}(\underline{\mathbf{x}}(t), t)}{\partial \underline{\mathbf{x}}(t)} \right]_{\underline{\mathbf{x}}(t) = \underline{\hat{\mathbf{x}}}(t)} \end{aligned} \quad (14)$$

Considering that the estimate error covariance matrix $\mathbf{P}$ is symmetrical and that, therefore, only the terms of the main diagonal and one of the two triangles need to be calculated, the number of differential equations in the system (13) is given by: $N = n + (n \cdot n - n)/2 + n = (n^2 + 3n)/2$. In the present paper, the solution of the system of N differential equations (13) was obtained by adopting the Runge-Kutta method, starting from known initial conditions for the state and approximated conditions for the parameters and the terms of the matrix $\mathbf{P}$. At each step of the integration, the terms of the $\mathbf{F}$ and $\mathbf{H}$ matrices need to be updated by numerically executing the derivatives in Equation (14).

For each road test carried out, a file is generated containing the time history of the steering wheel input and measurement outputs of longitudinal speed, lateral acceleration, and yaw rate, $\mathbf{z}(t) = [v_x \ a_y \ r]$. These make possible to start up an integration cycle for the system of Equation (13) starting from initial time $t=0$. The integration proceeds by interpolating the data up to the final time corresponding to the end of the manoeuvre. During integration, the values assumed by the unknown parameters are stored and are then averaged at the end. The values thus obtained are used as input values for a further integration cycle and the described procedure is iterated until the estimated parameter values are stabilized. For the estimation problem of model of Figure 1, as there are 7 states and 3 measurements and as 10 parameters have to be identified, the number N of simultaneous differential equations of the