

Decoupled 3D Moment Control for Vehicle Motion Using In-Wheel Motors

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ABSTRACT

Vehicles equipped with in-wheel motors are being studied and developed as a type of electric vehicle. Since these motors are attached to the suspension, a large vertical suspension reaction force is generated during driving. Based on this mechanism, this paper describes the development of a method for independently controlling roll and pitch as well as yaw using driving force distribution control at each wheel. It also details the theoretical calculation of a method for decoupling the dynamic motions. Finally, it describes the application of these 3D dynamic motion control methods to a test vehicle and the confirmation of the performance improvement.

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INTRODUCTION

Compared with vehicles powered by an internal combustion engine, the layout of the power source of electric vehicles (i.e. the motor) can be designed with a relatively high degree of freedom. It is also feasible to install power sources inside the wheels as in-wheel motors [1,2,3]. Although this type of layout has the potential to help innovate vehicle packaging, another extremely interesting aspect of in-wheel motors is related to vehicle dynamics. Installing motors in each wheel allows the driving force to be distributed freely, thereby making it easier to apply vehicle dynamics control. The direct yaw moment control method of distributing driving force has been investigated in a number of previous studies. This method is known to improve yaw motion. Since conventional driving force distribution control has limitations in terms of the response and durability of the control devices used, the main form of control that has been adopted is yaw control close to the limits of the tires. In contrast, vehicles powered by in-wheel motors can be controlled in the normal range of operation, and the controllability of motors means that these vehicles have a strong potential for improved performance. In addition, considering that a suspension reaction force is generated independently at each wheel, there is the possibility that roll, pitch, and heave motions, as well as yaw motion can be controlled [4]. This paper focuses on vehicle dynamics in the linear range. The following sections describe the construction of a 3D moment control method and the verification of its effect.

3D MOMENT CONTROL BY IN-WHEEL MOTOR

In-wheel motors are structured such that the reaction force from the road due to driving acts on the tire contact point. For this reason, as the vehicle is driven, a large reaction force is also generated in the vertical direction of the vehicle through the suspension, thereby affecting the behavior of the sprung mass. As shown in [Figure 1](#), the front and rear vertical forces generated by motor driving can be calculated by [Equation \(1\)](#), in which the front and rear angles formed between the lines connecting the suspension rotational centers and the ground at the tire contact points are shown as θ_f and θ_r , respectively [5]. Accordingly, these vertical forces can be used proactively to construct a method that controls the sprung mass motion simultaneously as well as simply yaw.

$$\begin{bmatrix} F_{zf} \\ F_{zr} \end{bmatrix} = \begin{bmatrix} -\tan \theta_f & 0 \\ 0 & \tan \theta_r \end{bmatrix} \begin{bmatrix} F_{xf} \\ F_{xr} \end{bmatrix} \quad (1)$$

If in-wheel motors are installed in all four wheels, then four-degree of freedom control becomes possible in principle. The control targets can be selected from the six degrees of freedom of the vehicle. However, it must be noted that the elements not selected for control may be affected by the other controls. Consequently, longitudinal motion and yaw cannot be excluded from the control targets. In addition, it is considered sensible to eliminate the lateral control because the layout of the motors makes this an inefficient control target and because it is less susceptible to the effects of other

controls. The remaining control elements are roll, pitch, and heave. Since vehicles generally have left/right symmetry, roll can be controlled independently. However, independent control is not possible for pitch and heave because both are cancelled out by controlling torque at the front and rear wheels as a result of the restrictions in the vehicle longitudinal motion. In a vehicle with front/rear symmetry in terms of suspension and weight distribution, it would be possible to select heave without the controlling pitch. As the degree of symmetry becomes less, pitch becomes easier to control than heave. Therefore, the control targets are selected based on the required dynamic performance and the control efficiency determined by the vehicle specifications. Here, the relationships between the driving force at each wheel and the targeted control forces or moments can be expressed as shown in Equations (2),(3),(4),(5),(6). The front, left, and upward directions of the vehicle are, respectively, expressed as the x , y , and z axes in a coordinate system. The meanings of symbols used with no explanation are described in the Appendix at the end of this paper.

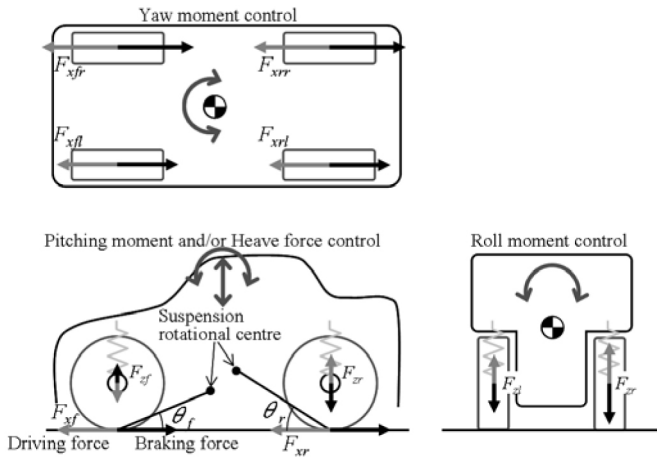


Figure 1. Mechanism of 3D moment control due to suspension reaction force on driving.

Longitudinal force:

$$F_x = F_{xfl} + F_{xfr} + F_{xrl} + F_{xrr} \quad (2)$$

Heave force:

$$F_z = -(F_{xfl} + F_{xfr})\tan\theta_f + (F_{xrl} + F_{xrr})\tan\theta_r \quad (3)$$

Roll moment:

$$M_x = \frac{t_f}{2}(-F_{xfl} + F_{xfr})\tan\theta_f + \frac{t_r}{2}(F_{xrl} - F_{xrr})\tan\theta_r \quad (4)$$

Pitch moment:

$$M_y = l_f(F_{xfl} + F_{xfr})\tan\theta_f + l_r(F_{xrl} + F_{xrr})\tan\theta_r \quad (5)$$

Yaw moment:

$$M_z = \frac{t_f}{2}(-F_{xfl} + F_{xfr}) + \frac{t_r}{2}(-F_{xrl} + F_{xrr}) \quad (6)$$

If the longitudinal force and the roll, pitch, and yaw moments are selected as the control targets, the control command value for these motions is represented as $\mathbf{u}$, the driving force of each wheel to achieve this command value is represented as $\mathbf{F}$, and the coefficient matrix obtained from Equations (2),(3),(4),(5),(6) is represented as $\mathbf{C}$, then the following equations can be formulated.

$$\mathbf{F} = \mathbf{C}_1^{-1} \mathbf{u}_1 \quad (7)$$

$$\mathbf{u}_1 = [F_x \quad M_x \quad M_y \quad M_z]^T \quad (8)$$

$$\mathbf{F} = [F_{xfl} \quad F_{xfr} \quad F_{xrl} \quad F_{xrr}]^T \quad (9)$$

$$\mathbf{C}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\frac{t_f}{2}\tan\theta_f & \frac{t_f}{2}\tan\theta_f & \frac{t_r}{2}\tan\theta_r & -\frac{t_r}{2}\tan\theta_r \\ l_f\tan\theta_f & l_f\tan\theta_f & l_r\tan\theta_r & l_r\tan\theta_r \\ -\frac{t_f}{2} & \frac{t_f}{2} & -\frac{t_r}{2} & \frac{t_r}{2} \end{bmatrix} \quad (10)$$

If heave is selected instead of pitch as a control target, Equations (7), (8) and (10) are replaced as follows:

$$\mathbf{F} = \mathbf{C}_2^{-1} \mathbf{u}_2 \quad (11)$$

$$\mathbf{u}_2 = [F_x \quad F_z \quad M_x \quad M_z]^T \quad (12)$$

$$\mathbf{C}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\tan\theta_f & -\tan\theta_f & \tan\theta_r & \tan\theta_r \\ -\frac{t_f}{2}\tan\theta_f & \frac{t_f}{2}\tan\theta_f & \frac{t_r}{2}\tan\theta_r & -\frac{t_r}{2}\tan\theta_r \\ -\frac{t_f}{2} & \frac{t_f}{2} & -\frac{t_r}{2} & \frac{t_r}{2} \end{bmatrix} \quad (13)$$

These equations can be used to control the sprung mass behavior as an active suspension. A previous report [6] described simulation and test results when damping control of road surface disturbances was applied based on skyhook theory, using this principle. In this example, Equations (7) and (11) were substituted as the control command values of damping forces for roll, pitch, and heave using the skyhook theory, and the average $\mathbf{F}$ value of each was applied to the vehicle. As a result, although the control effect in the event of the composite pitch and heave inputs was reduced by half, an

effective vehicle control combination was verified in the event of a single input.

Applying the force or moment corresponding to that command value to the vehicle in each direction of motion enables each motion to be controlled independently. This has the merit of enabling the construction of a control theory using models defined for each plane of motion. However, since vehicle behavior is coupled dynamically, particularly between yaw and the other motions, this control cannot be considered to be truly independent in [Equations \(7\)](#) and [\(11\)](#). This is because, for example, controlling yaw has an effect on transient roll. The following section describes the dynamic coupling of the control that affects each form of motion.

DECOUPLING OF EACH MOTION

The equations for vehicle longitudinal, lateral, heave, yaw, roll, and pitch motions are defined below for investigating the effects of the coupled terms [7]. The final term on the right side of each equation is the command value. It should also be noticed that changes in alignment caused by the suspension stroke are ignored.

Longitudinal:

$$m\ddot{x} = F_d + F_x \quad (14)$$

Lateral:

$$mV(\dot{\beta} + r) = m_s h_s \ddot{\phi} + K_f \left(\delta - \frac{l_f r}{V} - \beta \right) + K_r \left(\frac{l_r r}{V} - \beta \right) + F_y \quad (15)$$

Heave:

$$m\ddot{z} = F_d \left[-p \tan \theta_f + (1-p) \tan \theta_r \right] - K_z z - C_z \dot{z} + F_z \quad (16)$$

Yaw:

$$I_z \dot{r} = I_{zx} \ddot{\phi} + I_f K_f \left(\delta - \frac{l_f r}{V} - \beta \right) - I_r K_r \left(\frac{l_r r}{V} - \beta \right) + M_z \quad (17)$$

Roll:

$$I_x \ddot{\phi} = I_{zx} \dot{r} + m_s h_s V(\dot{\beta} + r) - K_\phi \phi - C_\phi \dot{\phi} + M_x \quad (18)$$

Pitch:

$$I_y \ddot{\theta} = F_d \left[p l_f \tan \theta_f + (1-p) l_r \tan \theta_r - h \right] - K_\theta \theta - C_\theta \dot{\theta} + M_y \quad (19)$$

The explanation below assumes that the longitudinal, yaw, roll, and pitch motions defined in [Equation \(7\)](#) have been selected as the control targets. The complex equations for yaw, roll, and pitch are expressed as [Equations \(20\),\(21\), \(22\)](#), which have been simplified by substituting $F_y=0$, $F_z=0$ and F_x is also 0 to avoid unintended longitudinal motion into

[Equations \(14\),\(15\),\(16\),\(17\),\(18\),\(19\)](#) and arranging the results obtained by Laplace transformation into the control terms. Note that driving force F_d is included in all the transfer functions, except for [Equation \(22\)](#).

Yaw:

$$r = \frac{N_r^r(s)}{D(s)} \delta + \frac{N_{M_z}^r(s)}{D(s)} M_z + \frac{N_{M_x}^r(s)}{D(s)} M_x \quad (20)$$

Roll:

$$\phi = \frac{N_\phi^r(s)}{D(s)} \delta + \frac{N_{M_x}^\phi(s)}{D(s)} M_x + \frac{N_{M_z}^\phi(s)}{D(s)} M_z \quad (21)$$

Pitch:

$$\theta = \frac{N_{F_x}^\theta}{D^\theta(s)} F_d + \frac{N_{M_y}^\theta}{D^\theta(s)} M_y \quad (22)$$

[Equations \(20\)](#) and [\(21\)](#) show that when control moments M_x and M_z are applied to control yaw and roll, they affect each other. Consequently, these motions can be understood to be coupled. Therefore, it is important to decouple these dynamic motions to apply transient motion control derived from the respective single planar motions to the yaw, roll, and pitch. This research considered methods of decoupling yaw and roll. Actually, roll and pitch may also become coupled due to the asymmetric damping coefficient between the compression side and rebound side, however, this is not considered here. The respective decoupling equations for roll and pitch can be defined as follows.

$$r = \frac{N_r^r(s)}{D(s)} \Big|_{F_d=0} \cdot \delta + \frac{N_{M_z}^r(s)}{D(s)} \Big|_{F_d=0} \cdot M_z' \quad (23)$$

$$\phi = \frac{N_\phi^r(s)}{D(s)} \Big|_{F_d=0} \cdot \delta + \frac{N_{M_x}^\phi(s)}{D(s)} \Big|_{F_d=0} \cdot M_x' \quad (24)$$

Each can be calculated as follows by obtaining M_x' and M_z' from [Equations \(20\), \(21\), \(23\), and \(24\)](#).

$$M_x' = \frac{D(s)}{N_{M_x}^\phi(s)} \Big|_{F_d=0} \cdot \frac{1}{D(s)} \left[N_{M_x}^\phi(s) \cdot M_x + N_{M_z}^\phi(s) \cdot M_z + \dots \right. \\ \left. + \left(N_\phi^r(s) - \frac{N_\phi^r(s)}{D(s)} \Big|_{F_d=0} \cdot D(s) \right) \delta \right] \quad (25)$$