

A Comparative Study of Car-Trailer Dynamics Models

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ABSTRACT

The paper examines typical vehicle dynamics models used for the design of car-trailer active safety systems, including active trailer braking and steering. A linear 3 degree-of-freedom (DOF), a nonlinear 4 DOF and a nonlinear 6 DOF car-trailer model are generated. Then, these models are compared with a car-trailer model developed with the commercial software package, CarSim. The benchmark investigation of the car-trailer models is carried out through examining numerical simulation results obtained in two emulated tests, i.e., a single lane-change and a Fishhook maneuver. In the vehicle modeling, a mathematical model of a tire with flexible sidewalls is included to account for transient tire forces. Steady-state aerodynamic forces are included in these models. The deviation of the model dynamic responses, e.g., the variation of the articulation angle between the car and trailer, is discussed. With the benchmark investigation, the car-trailer models in terms of fidelity, complexity, and applicability for active safety system design are addressed.

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INTRODUCTION

A car-trailer system consists of a towing unit, e.g., a car, and a towed unit, i.e., a trailer. The car and trailer are connected at an articulated point by a hitch [1]. With respect to single-unit cars, car-trailer systems may exhibit poor lateral stability at high speeds because of their double-unit structure. For these vehicles, three unique unstable motion modes have been identified, including the trailer swing (oscillatory instability), jack-knifing, and rollover. These unstable motion modes are the common causes for fatal accidents occurring on highways [2]. For a car-trailer system, it is difficult for the driver to sense the trailer motions, as the driver's perception is based mainly on the car response, rather than the trailer. The articulation joint and the car suspension isolate the driver from trailer motions. The unstable motion modes of trailer swing, jack-knifing, and/or rollover are usually directly related to the trailer response. Moreover, most drivers of these articulated vehicles are non-professional driver, whose inexperience may make them susceptible to experiencing instabilities [2]. To date, the majority of car-trailer systems use passive mechanisms to enhance the lateral stability at high speeds. Unfortunately, the high-speed lateral stability of a car-trailer system cannot be guaranteed with a passive mechanism, because the operation conditions vary significantly, e.g., the variation of payload, location of payload, and center of gravity (CG) of the trailer [3].

To address the safety problem of car-trailer systems, various active safety systems have been proposed, which include active trailer steering and active trailer braking [2]. To develop an effective car-trailer active safety system, it is critical to design a robust control algorithm that prevents the unstable motion modes. In the initial development phase of road vehicle active safety systems, various vehicle models have been used to derive the corresponding control algorithms. In the controller design for articulated vehicle active safety systems, different vehicle models with various degrees of freedom (DOF) have been used [2, 4, 5]. However, the applicability of various articulated vehicle models for active safety systems design has not been addressed.

This paper will tackle this problem. Through the benchmark investigation, four car-trailer models are examined in terms of fidelity, complexity, and applicability for active safety system design. The comparison study of the car-trailer models is implemented through investigating numerical simulation results obtained in two emulated tests, i.e., a single lane-change and a Fishhook maneuver. The deviations of the model dynamic responses are discussed.

The rest of the paper is organized as follows: Section 2 introduces the four car-trailer models; the vehicle models are compared and analyzed in Section 4; finally, conclusions are drawn in Section 5.

VEHICLE SYSTEM MODELS

This section introduces the following four car-trailer models: 1) a linear yaw/plane model with 3 DOF; 2) a nonlinear yaw/plane model with 4 DOF; 3) a nonlinear yaw/roll model with 6 DOF; and 4) a nonlinear CarSim model with 21 DOF [6].

Linear Yaw/Plane Model with 3 DOF

Figure 1 shows a yaw/plane model with 4 DOF. For this model, the motions considered include: 1) the longitudinal motion of the car at velocity U at the car's CG; 2) the lateral motion of the car at velocity V ; 3) the yawing motion of the car at yaw rate $\dot{\psi}_1$; and 4) the articulation angle ψ_2 between the car and trailer. As shown in Figure 1, the system is telescoped laterally and each axle set is represented by one wheel. In this model, aerodynamic forces, rolling and pitching motions are ignored. From Newton's law of dynamics, the equations of motion for the car are [7]:

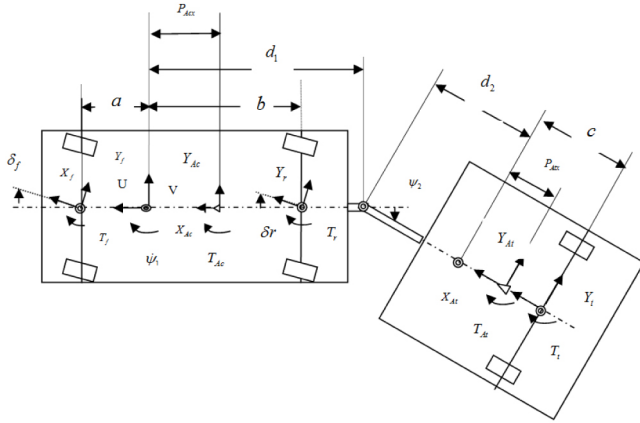


Figure 1. Schematic representation of the yaw plane model

$$m_c \cdot (\dot{U} - V \cdot \dot{\psi}_1) = -X_f \cdot \cos \delta_f - X_r + X_{\psi_2} \quad (1)$$

$$m_c \cdot (\dot{V} + U \cdot \dot{\psi}_1) = Y_f + Y_r + X_f \cdot \sin \delta_f - Y_{\psi_2} \quad (2)$$

$$I_c \cdot \ddot{\psi}_1 = a \cdot Y_f - b \cdot Y_r + a \cdot X_f \cdot \sin \delta_f + d_1 \cdot Y_{\psi_2} \quad (3)$$

and the equations of motion for the trailer are [7]:

$$m_t \cdot (\dot{U}_t - V_t \cdot \dot{\psi}_t) = -X_t - Y_{\psi_2} \cdot \sin \psi_2 - X_{\psi_2} \cdot \cos \psi_2 \quad (4)$$

$$m_t \cdot (\dot{V}_t + U_t \cdot \dot{\psi}_t) = Y_t + Y_{\psi_2} \cdot \cos \psi_2 - X_{\psi_2} \cdot \sin \psi_2 \quad (5)$$

$$I_t \cdot \ddot{\psi}_t = c \cdot Y_f - d_2 \cdot (X_{\psi_2} \cdot \sin \psi_2 - Y_{\psi_2} \cdot \cos \psi_2) \quad (6)$$

where Y_f , Y_r and Y_t are the lateral force on the car front tire, the lateral force on the car rear tire, and the lateral force on the trailer tire, respectively. The velocities at the articulation joint described in either the car-body fixed coordinate system or the trailer-body fixed coordinate system should be compatible. Expressing all the variables in car-body fixed coordinate system will reduce the above six equations to a set of four equations. The reader is referred to [7] for the transition of equations. The resulting four equations describe the general motion of a car-trailer system that may be accelerating or braking.

A linear tire model is used and it may be expressed by the following equations [8].

$$Y_f = C_f \cdot \alpha_f \quad (7)$$

$$Y_r = C_r \cdot \alpha_r \quad (8)$$

$$Y_t = C_t \cdot \alpha_t \quad (9)$$

where C_f , C_r and C_t are the tire cornering stiffness, α_f , α_r and α_t are the side-slip angles of the tires. The side-slip angles are given by [7]:

$$\alpha_f = \frac{V + a \cdot \dot{\psi}_1}{U} - \delta_f \quad (10)$$

$$\alpha_r = \frac{V - b \cdot \dot{\psi}_1}{U} \quad (11)$$

$$\alpha_t = \psi_2 - \frac{V - (d_1 + d_2 + c) \cdot \dot{\psi}_1 + (d_2 + c) \cdot \dot{\psi}_2}{U} \quad (12)$$

To derive the linear governing equations of motion of the car-trailer model, it is assumed that: the car forward speed U is constant; lateral tire forces, Y_f , Y_r and Y_t , are the only external forces; articulated angle ψ_2 is small, so that $\cos \psi_2 = 1$, $\sin \psi_2 = 0$; products of V , $\dot{\psi}_1$ and ψ_2 are small enough to ignore. With the above assumptions, only three motions are considered in the resulting model: 1) the lateral motion of the car at velocity V ; 2) the yawing motion of the car at yaw rate $\dot{\psi}_1$; and 3) the articulation angle ψ_2 between the car and trailer. The resulting model is a linear yaw/plane model with 3 DOF (hereafter called 3DOF-L). The linear yaw/plane model can be expressed in the state space as

$$M \{\dot{X}\} + D \{X\} + F \delta_f = 0 \quad (13)$$

where the state variable vector is defined as

$$\{X\} = \{\nu \quad \psi_1 \quad \psi_2 \quad \psi_2\} \quad (14)$$

The matrices M, D and F are listed in [Appendix A](#). [Table 1](#) lists the notations and the primary parameters of the car-trailer system.

Table 1. The primary parameters of the car-trailer system

Car mass	m_c	2000 kg
Car yaw inertia	I_c	3000 kgm ²
Car dimension	a	1.5 m
Car dimension	b	1.7 m
Car dimension	d_1	2.8m
Trailer mass	m_t	2300 kg
Trailer yaw inertia	I_t	3900 kgm ²
Trailer dimension	c	5 m
Trailer dimension	d_2	0 m
Front tire cornering stiffness (combined)	C_f	-80000 Nm/rad
Rear tire cornering stiffness (combined)	C_r	-80000 Nm/rad
Trailer tire cornering stiffness (combined)	C_t	-60000 Nm/rad

Nonlinear Yaw/Plane Model with 4 DOF

For the nonlinear yaw/plane model with 4 DOF (hereafter called 4DOF-NL), the motions considered include the longitudinal motion of the car and the other three motions that are considered in the 3DOF-L model. The 4DOF-NL model incorporates some effects that are not taken into account in the 3DOF-L model. The tire model used in the 4DOF-NL model is based on the Magic Formula by Pacejka [9]. Moreover, aerodynamic lift and drag on the car and the trailer bodies are included in the nonlinear yaw/plane model. However, the 4DOF-NL model does not consider the lateral load transfer due to lateral accelerations.

The derivation of the governing equations of motion for the 4DOF-NL model is conducted using Lagrange's equations. The governing equations of motion for this vehicle model is offered in [Appendix B](#). The reader is referred to [10, 11] for the detailed procedure for generating the governing equations of the nonlinear vehicle model.

Nonlinear Yaw/Roll Model with 6 DOF

The nonlinear yaw/roll model with 6 DOF (hereafter called 6 DOF-NL) is generated using the procedure proposed in [10, 11]. For the 6DOF-NL model, the motions considered are the same as those used for the 4DOF-NL with the addition of roll motions, ϕ_1 for the car body and ϕ_2 for the trailer body. Similar to the 4DOF-NL model, aerodynamic forces are taken into account in the 6DOF-NL model; the Magic Formula is introduced to model the tire/road forces. Not like the 4DOF-NL model, the 6DOF-NL model considers the lateral load transfer due to lateral accelerations.

In order to account for the roll motions of car-body and trailer-body, the roll model, shown in [Figure 2](#), is introduced in the 6DOF-NL model. [Figure 2](#) shows the body, either the car-body or trailer-body, rolling around a longitudinal axis with an angle ϕ . In [Figure 2](#), m_1 denotes the car sprung mass, m_2 the car unsprung mass, T the track between the left and

right wheels, S the distance between the right and left suspensions, h_R the vertical distance from the roll center to the road surface, and $h \cos \phi$ the vertical distance between the body CG to the roll center. With no other lateral motion of the CG, the rotation around an axis passing through the body CG results in a axle lateral motion of $h \sin \phi$. It is assumed that each axle moves laterally respect to the body CG and the lateral motion is calculated using the roll center shown in [Figure 2](#). The actual axle motion is determined by the motion of the whole vehicle that is governed by the forces applied on the system.

Similar to the 4DOF-NL model, the governing equations of motion for the 6DOF-NL model is generated using Lagrange's equations. The formulation of the governing equations is implemented using the method proposed by Anderson [10, 11].

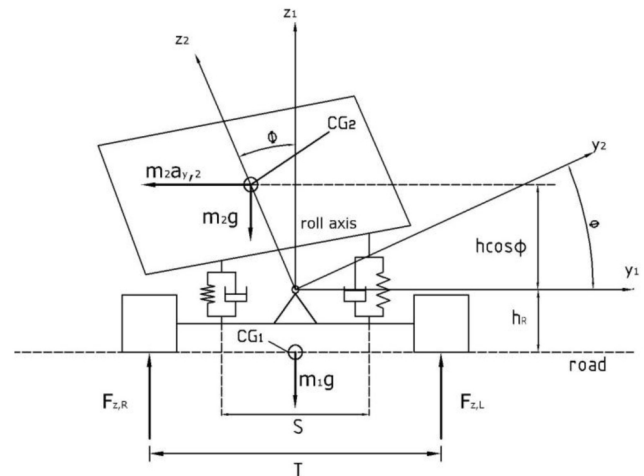


Figure 2. Roll model used in the 6DOF-NL vehicle model

Nonlinear CarSim Model with 21 DOF

In the research, a 21 DOF model based on the commercial software CarSim is generated to represent the car-trailer system. In this paper this model is called CarSim model [6]. For the towing unit, i.e., the car, the following motions are considered: the car-body is treated as a rigid body with 6 DOF; each axle is modeled as a rigid body with 2 DOF (yaw and longitudinal translation); and each car wheel has one spin DOF. Similarly, for the trailing unit, the following motions are taken into account: the trailer-body is treated as a rigid body with 6 DOF; the trailer axle is modeled as a rigid body with 2 DOF (yaw and longitudinal translation); and each trailer wheel has one spin DOF. The hitch connection between the car and the trailer results in the CarSim model with 21 DOF. The Magic Formula is used to model the tire/road forces. The geometical parameters and configuration of the car and trailer models are illustrated in [Figure 3](#) and [Figure 4](#).